MEASUREMENT OF DYNAMIC PROPERTIES OF MATERIALS

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ABSTRACT

It is often necessary to measure the dynamic properties of materials such as the loss factor and the modulus of elasticity in order to provide valid data for numerical analysis of structures. This is especially the case when structures with damping treatments are to be optimized. Viscoelastic materials are widely used for damping treatment purposes in order to reduce resonant vibrations in many applications. Therefore, various experimental techniques have been developed in the past for the estimation of these properties. Among these techniques, ‘Oberst Beam Method’ is one of the standard test methods for measuring dynamic properties of materials (ASTM E756 – 93).

This paper presents a brief description of a methodology based on the Oberst Beam Method and demonstrates its application for the identification of the dynamic properties of ‘self-supporting’ and ‘non-self-supporting’ materials. A unique feature of this work is that the so-called Line-Fit Method – commonly used in modal analysis of Frequency Response Functions – is employed during the process of identification of material properties. Also, sample results are presented in this paper to demonstrate the accuracy of the identification procedure using a homogeneous steel (Oberst) beam and a composite beam comprising a steel beam coated with bitumen.

KEYWORDS: Oberst Beam Method, Line-Fit Algorithm, Loss Factor, Young’s Modulus
INTRODUCTION

Damping treatment is a standard practice in many industries for controlling excessive noise and vibration levels. The level of noise reduction due to damping treatment depends on the structure itself, the detailed nature of the excitation sources, properties of damping material as well as the type and location(s) of the damping treatment. Therefore, all these factors are to be considered while modeling and/or optimizing a system.

There are various damping mechanisms that can be utilized in damping treatment applications. One of these mechanisms, based on the distortion of rubber-like materials, is analyzed thoroughly in the past [1]. The rubber-like materials, often called “viscoelastic” materials, dissipate energy when subjected to alternating stresses. Their elastic and damping properties are slowly varying functions of both the frequency of the alternating stresses and the temperature of the material. Methods are now available for calculating the vibration-damping properties of the rubber-like materials [1] [2].

This paper presents a brief description of an experimental technique based on Oberst Beam Method (ASTM E756 – 93) and demonstrates its application for the identification of the vibration-damping properties of ‘self supporting’ and ‘non-self supporting’ materials [2]. The so-called Line-Fit Method is employed for the estimation of natural frequencies and damping levels during the process of identification of material properties [3] [4]. Bitumen is selected, mainly due to widespread applications of this material in domestic appliances for damping treatment, to demonstrate the procedure for non – self supporting damping materials. This includes the comparison of the two test results involving a metal bar and the composite bar. Sample results included in this paper highlight the overall accuracy that can be achieved in practice.

THEORY

Modulus of Elasticity :

Modulus of elasticity is a well-known property that defines the relationship between stress and strain in the linear region of the Hook’s diagram. The measurement of the modulus of elasticity using the Oberst Beam Method is based on analytical solution of the bending vibrations of beams with the clamped–free boundary conditions. This can be summarised in a few steps as follows. First, according to the Bernoulli-Euler beam theory [6], natural frequencies of an homogenous beam in bending vibrations are given by;

\[ \omega_n = \left( \beta_n L \right)^2 \frac{EI}{mL^4} \]  

(1)
where $L$ is the free length of the beam, $E$ is the elasticity modulus of the material, $I$ is the mass moment of inertia of the beam, $m$ is the mass per unit length and $\beta_n L$ are the constant values which are given for the first five bending modes in Table 1. By substituting the appropriate expressions for $\omega_n$, $m$ and $I$ into Eq (1), it is possible to obtain:

$$E = \frac{12 \rho L^4 f_n^2}{H^2 C_n^2}$$

(2)

where $\rho$ is the density of the material, $H$ is the thickness of the beam, $f_n$ is the natural frequency [Hz] and $C_n$ are the constants that can be computed from:

$$C_n = \frac{(\beta_n L)^2}{2\pi}$$

(3)

the numeric values of which are listed in Table 1 for the first five modes.

<table>
<thead>
<tr>
<th>Mode No (n)</th>
<th>$\beta_n L$</th>
<th>$C_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.875</td>
<td>0.560</td>
</tr>
<tr>
<td>2</td>
<td>4.694</td>
<td>3.507</td>
</tr>
<tr>
<td>3</td>
<td>7.855</td>
<td>9.820</td>
</tr>
<tr>
<td>4</td>
<td>10.996</td>
<td>19.242</td>
</tr>
<tr>
<td>5</td>
<td>14.137</td>
<td>31.809</td>
</tr>
</tbody>
</table>

**Loss Factor:**

Damping is generally characterized by the amount of energy dissipated under steady harmonic motion. The most common measure of this dissipation is the loss factor $\eta$ which can be defined as the ratio of the average energy dissipated per radian to the peak potential energy during a cycle.

$$\eta = \frac{W}{2\pi U}$$

(4)

where $W$ is the energy dissipated per cycle and $U$ is the peak potential energy.

The equation of motion for the forced vibration of a SDOF system with structural damping is given by

$$m\ddot{x} + (k + id)x = f(t)$$

(5)

Assuming harmonic motion leads to the expression for receptance as

$$\frac{X}{F} = \alpha(\omega) = \frac{1/k}{1 - (\omega/\omega_n)^2 + i\eta}$$

(6)

where $X$ and $F$ are complex amplitudes of the response and the force respectively. The same
procedure can easily be extended for MDOF systems as
\[
[M][\ddot{x}] + ([K] + i[D])\dot{x} = \{f\}e^{i\omega t}
\]  
(7)
leading to the definition of the receptance matrix for harmonic motion :
\[
[a(\omega)] = ([K] + i[D] - \omega^2[M])^{-1}
\]  
(8)
or in a more suitable form for the Line-Fit algorithm, the individual elements of the
receptance matrix is
\[
\frac{X_j}{F_k} = \alpha_{jk}(\omega) = \sum_{r=1}^{N} \frac{(a + ib)_{jk}}{\omega_r^2 - \omega^2 + i\omega^2\eta_r^2}
\]  
(9)
where the index r indicates mode number. Eq. (9) is the receptance function related to co-
ordinates j and k of a MDOF system with structural damping. In this paper, Eq.(9) is used in
the implementation of Line-Fit [4] algorithm in order to estimate the natural frequencies and
loss factors using the measured data. It should be noted that ASTM E756 – 93 standard
suggests using half-power-bandwidth method for this task. However, it is known that the
"Line-Fit" procedure has better performance than half-power-bandwidth method in estimating
the modal properties, especially for lightly damped systems.

IMPLEMENTATION

As stated, "Line-Fit" procedure is utilised in this paper for the estimation of natural
frequencies and loss factors using measured data. It is therefore appropriate here to give a
brief summary of this procedure. Further details can be found in reference [4].

The Line-Fit procedure is based on SDOF assumption and relies on the use of dynamic
stiffness data (1/ receptance). Starting with the receptance equation in the vicinity of a natural
frequency, i.e.,
\[
\alpha(\omega)_{\omega = \omega_r} = \frac{A_r}{\omega_r^2 - \omega^2 + i\omega\eta_r} + R
\]  
(10)
a new form of receptance term is defined so as to cancel out the residual effects R as
\[
\alpha'(\omega) = \alpha(\omega) - \alpha(\Omega)
\]  
(11)
where \(\Omega\) is a fixed frequency selected around a natural frequency. This receptance term is
inverted and a new function called \(\Delta(\omega)\) is defined;
\[
\Delta(\omega) = \frac{\omega^2 - \Omega^2}{\alpha'(\omega)}
\]  
(12)
The function consists of real and imaginary components;

$$\Delta(\omega) = RE(\Delta) + i IM(\Delta)$$  \hspace{1cm} (13)

which are linear functions of $\omega^2$ due to the Eq. (12). Following a set of equations below

$$RE(\Delta) = m_R \omega^2 + c_R \quad IM(\Delta) = m_i \omega^2 + c_i$$

$$m_R = n_R \Omega^2 + d_R \quad \quad m_i = n_i \Omega^2 + d_i$$

$$n_R = a_r \quad \quad \quad n_i = -b_i$$

$$d_R = -a_r (\omega_r^2) - b_i (\omega_r^2 \eta_r) \quad \quad d_i = b_i (\omega_r^2) - a_r (\omega_r^2 \eta_r)$$  \hspace{1cm} (14)

the Line-Fit algorithm establishes the natural frequencies and loss factors as;

$$\omega_r^2 = \frac{d_R}{(p \eta_r - 1) n_R}, \quad \eta_r = \frac{(q - p)}{(1 + pq)}$$ \hspace{1cm} (15)

where,

$$p = \frac{n_i}{n_R}, \quad q = \frac{d_i}{d_R}$$ \hspace{1cm} (16)

and the modal constant, $A_r$ is given by

$$A_r = a_r + i b_r$$ \hspace{1cm} (17)

where,

$$a_r = \frac{\omega_r^2 \cdot (p \eta_r - 1)}{(1 + p^2) d_R}, \quad b_r = -p a_r$$ \hspace{1cm} (18)

Once the dynamic properties of the Oberst beam (i.e. self-supporting material) as well as the natural frequencies and loss factors for the composite beam are determined, the dynamic properties of the damping material are extracted using Eqs. (19) and (20).

$$E_{dm} = \frac{(\alpha - \beta) + \sqrt{(\alpha - \beta)^2 - 4T^2(1-\alpha)}}{2T^3} E_{obserst}$$ \hspace{1cm} (19)

$$\eta_{dm} = \frac{(1 + 4MT)(1 + 4MT + 6MT^2 + 4MT^3 + M^2T^4)\eta_{composite}}{MT(3 + 6T + 4T^2 2MT^3 + M^2T^4)} \hspace{1cm} (20)$$

$$\alpha = \left( f_c / f_n \right)^2 (1 + DT), \quad \beta = 4 + 6T + 4T^2$$ \hspace{1cm} (21)

where $E_{dm}$, $\eta_{dm}$ are elasticity modulus and loss factor for the damping material $f_c$ is the natural frequency of the composite beam, $f_n$ is the natural frequency of the oberst beam, $M$ is the elasticity modulus ratio, $D$ is the density ratio and $H$ is the thickness ratio of the composite beam to the oberst beam.
NUMERICAL SIMULATION

The procedure outlined above for the determination of the dynamic properties of ‘self supporting’ and ‘non-self supporting’ materials is implemented. Then, the implementation was verified by means of numerical simulations before it was put in practical use. This was achieved by creating individual Finite Element (FE) models for both Oberst and composite beams as illustrated in Fig. 1. In both models, two-dimensional 2nd order continuous elements with plane-stress condition were used so as to simulate the theory that Oberst Beam Method is based on. An important feature of the FE code [5] used for the analysis is its’ ability to perform complex eigen solution in order to determine the modal properties of assemblies with non-proportional damping distribution. FE code required the geometry, material properties and boundary conditions.

The main steps involved during this verification process were (i) specification of material properties for the FE models, (ii) generation of data (FRF) that would otherwise come from measurements and (iii) comparison of whether the known answers with those obtained using the procedure developed for the determination of material properties. Results corresponding to Oberst and the composite beams are summarized in Tables 2 and 3, respectively. It is seen that the results for the Oberst beam (Table 2) are excellent. However, results in Table 2 indicate that the error is increasing with frequency, reaching to a level of about 4% of 5th mode. A possible explanation for this discrepancy is that in contrast to the FE model the Oberst beam method ignores shear deformation and rotary inertia effects.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Specified Oberst Beam Properties (FE Model)</th>
<th>Estimated Oberst Beam Properties (Line-Fit Algorithm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>126.53</td>
<td>195.00</td>
</tr>
<tr>
<td>3</td>
<td>354.24</td>
<td>195.00</td>
</tr>
<tr>
<td>4</td>
<td>694.10</td>
<td>195.00</td>
</tr>
<tr>
<td>5</td>
<td>1147.00</td>
<td>195.00</td>
</tr>
</tbody>
</table>

Figure 1. FE models for a) Oberst Beam b) Composite beam (top 4 layers of elements represent damping material.)
Table 3. Comparison of the specified and estimated properties of damping material.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Specified Damping Material Properties ( FE Model )</th>
<th>Estimated Damping Material Properties ( Line-Fit Algorithm )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency [Hz]</td>
<td>Elasticity [GPa]</td>
</tr>
<tr>
<td>2</td>
<td>131.03</td>
<td>2.00</td>
</tr>
<tr>
<td>3</td>
<td>366.74</td>
<td>2.00</td>
</tr>
<tr>
<td>4</td>
<td>716.50</td>
<td>2.00</td>
</tr>
<tr>
<td>5</td>
<td>1181.38</td>
<td>2.00</td>
</tr>
</tbody>
</table>

EXPERIMENTAL RESULTS

The measurement setup utilizing non-contact excitation and measurement transducer are schematically shown in Fig. 2a. Also shown are some sample measurements obtained using a composite beam (a steel beam coated with bitumen), Fig. 2b. It must be stated that the sample results presented here correspond to a fairly thick, coating applied to bitumen applied to the Oberst beam, causing large levels of damping in the composite beam. The reason for this was to simulate the upper bounds for experimental errors.

![Experimental Setup](image)

Figure 2. a) Experimental Setup, b) Typical measurements for composite beam.

The repeatability and the reproducibility of the measurements are assessed and the results are summarized in Tables 4 and 5. It is seen that, for this particular example, variability of the properties of damping material can be about ±5% for the loss factor and over ±10% for the modulus of elasticity. Therefore, some kind of averaging is recommended to reduce the level of uncertainty in estimated parameters.

CONCLUDING REMARKS

The Oberst Beam Method suggested in the standard ASTM E756 – 93 for the identification of the dynamic properties of ‘self supporting’ and ‘non-self supporting’ materials is implemented. The so-called Line-Fit Method – often used in modal analysis of Frequency
Response Functions – is employed during the process of identification of material properties so as to achieve higher accuracy in estimating the modal properties of the system under test. The procedure is validated via numerical simulations. Also, sample test results are presented in this paper to demonstrate the accuracy of the identification procedure using a homogeneous steel (Oberst) beam and a composite beam comprising a steel beam coated with bitumen. It is concluded that although there could be some degree of uncertainty in identified material properties, higher level of accuracy can be achieved if repetitive measurements are made and mean values are estimated.

Table 4. Repeatability of the experimental results.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Oberst Beam</th>
<th>Composite Beam</th>
<th>Damping Material</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Elasticity E_{Oberst}</td>
<td>Frequency</td>
</tr>
<tr>
<td></td>
<td>[Hz]</td>
<td>[GPa]</td>
<td>[Hz]</td>
</tr>
<tr>
<td>2</td>
<td>129.6 ± 0.1%</td>
<td>204.6 ± 0.1%</td>
<td>115.9 ± 0.7%</td>
</tr>
<tr>
<td>3</td>
<td>361.9 ± 0.1%</td>
<td>203.4 ± 0.2%</td>
<td>322.6 ± 0.5%</td>
</tr>
<tr>
<td>4</td>
<td>708.7 ± 0.1%</td>
<td>203.1 ± 0.2%</td>
<td>626.6 ± 0.4%</td>
</tr>
</tbody>
</table>

Table 5. Reproducibility of the experimental results.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Oberst Beam</th>
<th>Composite Beam</th>
<th>Damping Material</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Elasticity E_{Oberst}</td>
<td>Frequency</td>
</tr>
<tr>
<td></td>
<td>[Hz]</td>
<td>[GPa]</td>
<td>[Hz]</td>
</tr>
<tr>
<td>2</td>
<td>129.4 ± 0.5%</td>
<td>203.9 ± 1.1%</td>
<td>116.4 ± 1.2%</td>
</tr>
<tr>
<td>3</td>
<td>361.6 ± 0.5%</td>
<td>203.1 ± 0.9%</td>
<td>323.5 ± 1.0%</td>
</tr>
<tr>
<td>4</td>
<td>708.0 ± 0.5%</td>
<td>202.8 ± 1.0%</td>
<td>629.5 ± 1.1%</td>
</tr>
</tbody>
</table>

REFERENCES


