Innovation feedback

The innovation is the output prediction error:

\[ \nu := y - C\hat{x} = -Ce \]

Therefore,

\[ \nu(s) = Y(s) - C\hat{X}(s) \]
\[ = Y(s) - CT_1(s)U(s) - CT_2(s)Y(s) \]
\[ = (1 - CT_2(s))Y(s) - CT_1(s)U(s) \]

where

\[ T_1(s) = (sI - A + LC)^{-1}B \]
\[ T_2(s) = (sI - A + LC)^{-1}L \]

In transfer function form:

\[ \frac{L(s)}{E(s)}U(s) = -\frac{P(s)}{E(s)}Y(s) \]
\[ G(s) = \frac{B_o(s)}{A_o(s)} = \frac{C \text{Adj}(sI - A)B}{\text{det}(sI - A)} \]

\[ E(s) = \text{det}(sI - A + LC) \]
\[ F(s) = \text{det}(sI - A + BK) \]
\[ L(s) = \text{det}(sI - A + LC + BK) \]
\[ P(s) = K \text{Adj}(sI - A)L \]
\[ \frac{P(s)}{L(s)} = K [sI - A + LC + BK]^{-1} L \]

Then, it can be shown (see Goodwin P545) that the innovation

\[ \nu(s) = \frac{A_o(s)}{E(s)} Y(s) - \frac{B_o(s)}{E(s)} U(s) \]

Consider now that observer state feedback augmented with innovation feedback,

\[ u = \nu - K \hat{x} - Q_u(s) \nu \]

where \( Q_u(s) \nu \) is \( \nu \) filtered by the stable filter \( Q_u(s) \).
(to be designed). Then,

\[ \frac{L(s)}{E(s)} U(s) = -\frac{P(s)}{E(s)} Y(s) - Q_u(s) \left[ \frac{A_o(s)}{E(s)} Y(s) - \frac{B_o(s)}{E(s)} U(s) \right] \]

The nominal sensitivity functions, which define the robustness and performance criteria, are modified affinely by \( Q_u(s) \):

\[ S_o(s) = \frac{A_o(s)L(s)}{E(s)F(s)} - Q_u(s) \frac{B_o(s)A_o(s)}{E(s)F(s)} \quad (14) \]
\[ T_o(s) = \frac{B_o(s)P(s)}{E(s)F(s)} + Q_u(s) \frac{B_o(s)A_o(s)}{E(s)F(s)} \quad (15) \]

For plants that are open-loop stable with tolerable pole locations, we can set \( K = 0 \) so that

\[ F(s) = A_o(s) \]
\[ L(s) = E(s) \]
\[ P(s) = 0 \]
so that

\[ S_o(s) = 1 - Q_u(s) \frac{B_o(s)}{E(s)} \]

\[ T_o(s) = Q_u(s) \frac{B_o(s)}{E(s)} \]

In this case, it is common to use \( Q(s) := Q_u(s) \frac{A_o(s)}{E(s)} \) to get the formulae:

\[ S'_o(s) = 1 - Q(s)G_o(s) \quad (16) \]

\[ T'_o(s) = Q(s)G_o(s) \quad (17) \]

Thus the design of \( Q_u(s) \) (or \( Q(s) \)) can be used to directly influence the sensitivity functions.
For instance, using Eqs.(28)-(29):

**Minimize nominal sensitivity** $S(s)$:

$$Q_u(s) = \frac{L(s)}{B_o(s)} F_1(s)$$

**Minimize complementary sensitivity** $T(s)$:

$$Q_u(s) = -\frac{P(s)}{A_o(s)} F_2(s)$$

where $F_1(s)$, $F_2(s)$ are close to 1 at frequencies where $\|S(s)\|$ and $\|T(s)\|$ need to be decreased.