1. Consider the circuit shown below with input voltage, \( v(t) \), and output voltage, \( v_o \).

![Circuit Diagram]

A. (2 pts.) Find the state-variable equations for the system. Use the energy variables, \( e_c \) and \( i_L \), as states. Include an equation for the output, \( v_o \).

B. (1 pts.) Put the state-variable equations in matrix form:

\[
\dot{x} = Fx + Gu \\
y = Hx + Ju
\]

where \( x = \begin{bmatrix} e_c \\ i_L \end{bmatrix} \) is the state vector, \( u = \begin{bmatrix} v_i(t) \end{bmatrix} \) is the input, and \( y = \begin{bmatrix} v_o \end{bmatrix} \) is the output. \( F, G, H, \) and \( J \) are matrices.

For problems 2 and 3, the unity-gain feedback, proportional closed-loop control is considered. The configuration is shown below.

![Feedback Control Diagram]

\( G(s) \) is the plant, \( r(t) \) is the reference and \( d(t) \) is the disturbance. \( y \) is the output, \( e \) is the error, and \( u \) is the control input. \( K \) is the proportional gain.
2. \[ G(s) = \frac{Y(s)}{U(s)} = \frac{1}{(s+1)^2(s+2)} \]

A. (1 pt.) Find the differential equation relating \( u \) and \( y \).

B. (3 pts.) Find the range of \( K \) (both positive and negative) for which the system is stable.

C. (2 pts.) We wish to use the Ziegler-Nichols ultimate sensitivity method to find parameters for a P.I.D. controller. Find the ultimate gain, \( K_u \), and the ultimate period, \( P_u \), for this method. Restrict your consideration to positive gains.

3. \[ G(s) = \frac{Y(s)}{U(s)} = \frac{1-s}{s(1+s)} \]

A. (2 pts.) Find the closed-loop transfer function, \( G_{cl}(s) = \frac{Y(s)}{R(s)} \), for proportional control. Find the characteristic equation and identify the undamped natural frequency, \( \omega_n \), and the damping ratio, \( \zeta \), as a function of \( K \).

B. (3 pts.) Find the open-loop step response, that is, find \( y(t) \) when \( u(t) = 1(t) \). The system is initially at rest and \( d(t) = 0 \). Sketch the step response.

C. (1 pts.) We wish to use the Ziegler-Nichols process reaction curve method to find parameters for a P.I.D. controller. Find the required parameters, \( R \) and \( L \), from the step response.

D. (1 pts.) Find the recommended value of \( K \) for proportional control using the Ziegler-Nichols process reaction curve parameters. What are the resulting values of \( \omega_n \) and \( \zeta \)?

E. (2 pts.) What is the system type with regard to input tracking? What would the steady-state error be for \( r(t) \) being a unit step, a unit ramp, or a unit parabola?

F. (2 pts.) What is the system type with regard to disturbance rejection? What would the steady-state error be for \( d(t) \) being a unit step, a unit ramp, or a unit parabola?