In this second half of the course, you will be designing, implementing and testing control systems on an electro hydraulic actuator setup. Systems dynamics and control concepts will be used routinely. These exercises aim to refresh your memory about these. Be sure to review your ME 3281 notes and textbooks, especially if you do not feel comfortable with any of these exercises.

Due Date: - October 24 or during your scheduled lab period.

1. Sketch the following functions and find their Laplace transforms.

   a) Step function : \( x(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \)

   [The step function is denoted by \( 1(t) \) or sometimes by \( U(t) \)]

   b) Ramp : \( x(t) = t \cdot 1(t) \)

   c) Quadratic : \( x(t) = t^2 \cdot 1(t) \)

   d) Impulse : \( x(t) = 1(t) \)

   e) Sinusoid : \( x(t) = [20 \sin(2\omega_1 t) + 5 \cos(\omega_1 t)] \cdot 1(t) \)

   f) Exponential : \( x(t) = 10e^{3t} \cdot 1(t) \)

In all of the cases shown above, multiplication by \( 1(t) \) means that the signal is zero for \( t<0 \). Sometimes we will be sloppy and not write it precisely (as long as we are clear that the value of the signal is 0 for \( t<0 \)). Thus, from now on, we will follow the convention i.e. \( x(t) = 4e^{3t} \) to denote the signal in (f).

2. Find the inverse Laplace transform for:

   (a) \( X(s) = \frac{2}{s+4} \)

   (b) \( X(s) = \frac{2}{(s+3)(s+1)} \)

   (c) \( X(s) = \frac{s+1}{s^2+9} \)
3. Solution of differential equations:

Take the laplace transform for the following differential equation:

\[ \dot{x} + 3x = 2 \cdot 1(t) \]

Where 1(t) is the step function and \( x(t=0)=1 \).

4. Calculation of the response of a system using inverse lapace transforms:

Recall that if the transfer function of a system is \( G(s) \), then assuming zero initial conditions, the laplace transform of the output of the system is \( Y(s) \) in response to the input \( R(s) \) is

\[ Y(s) = G(s)R(s) \]

Compute the response of the system with \( G(s) = \frac{s+1}{3s+2} \) and zero initial conditions, when input is the step function, i.e. \( r(t)=1(t) \).

5. Effect of poles on the response of the system.

Below are plots of the free responses (i.e. response due to initial conditions) of five systems. The systems are all second order, but they have different poles \( p_1 \) and \( p_2 \). Pair up the various responses due to initial conditions below (\( \alpha, \beta, \gamma, \delta, \mu \)) with the combinations of pole pairs (A, B, C, D, E). Be sure to include a brief comment explaining why you think so for each case.

- A: \(-0.1 \pm 15i \)
- B: \(-4, -5 \)
- C: \(-3 \pm 5i \)
- D: \(-3, 0.5 \)
- E: \(-0.4 \pm 3i \)
6. Concept of frequency responses:

\[ G(s) = \frac{6}{s + \sqrt{3}} \]

Compare the magnitudes and phases of \( G(s=j\omega) \) for \( \omega = 0.8 \) and for \( \omega = 0.3 \). Given a sinusoidal input:

\[ u(t) = 8 \sin(2t) \]

What is the output \( y(t) \) when sufficient time has passed? Do the same thing for \( u(t) = \cos(0.4 \ t) \).
7. Block diagram reduction:

Derive the close loop transfer function, i.e. \( \frac{Y(s)}{R(s)} \) for the following system:

\[
\begin{align*}
\text{r}(t) & \quad \frac{5s}{s + 2} \quad + \quad u(t) \quad \frac{s + 4}{2s + 5} \quad \text{y}(t) \\
\end{align*}
\]

8. Bode Plot:

Draw the magnitude and phase plot for the following transfer function by hand (show detailed steps). Determine the values of the asymptotes in limit and comment on corner frequencies. After completing the hand sketches, verify your result using Matlab. Turn in your answer together with Matlab code and plots.

\[
H(s) = \frac{s + 5}{s + 9}
\]