APPENDIX B: ERROR ANALYSIS

One method of error analysis that can be used for engineering applications is the root-sum-square (RSS) method. This should be familiar to you from the junior labs. This section briefly outlines the method and its consequences for the Thermal Environmental Engineering laboratory report data analysis.

Definitions:

\( \Delta = \) uncertainty in precision, usually reported as a relative or percentage error, or as an absolute error
\( x, y, z = \) any measured quantities such as pressure, temperature, flow rate, etc.
\( w = \) the resultant value from the measured quantities

If:

\[ w = f(x, y, z) \]

According to the RSS method, the error in \( w \) is defined by:

\[
(\Delta w)^2 = (\Delta x \frac{\partial w}{\partial x})^2 + (\Delta y \frac{\partial w}{\partial y})^2 + (\Delta z \frac{\partial w}{\partial x})^2
\]

The partial derivatives depend on what the function, \( f \), is. The uncertainties, \( \Delta \), can be estimated by ± \( \frac{1}{2} \) the smallest division of the measuring instrument. If enough measurements are taken, the standard deviation of those measurements can be used as the uncertainty.

Example:

\[ w = \frac{x}{\sqrt{y}} = xy^{-1/2} \]

\[ \frac{\partial w}{\partial x} = y^{-1/2} \quad \text{and} \quad \frac{\partial w}{\partial y} = -\frac{1}{2}xy^{-3/2} \]

therefore:

\[
(\Delta w)^2 = (\Delta xy^{-1/2})^2 + (\Delta y\left(-\frac{1}{2}\right)xy^{-3/2})^2
\]
Dividing both sides by $w^2 (=xy^{-1/2})$ greatly simplifies this and also defines the 'easy way' to do the root-sum-square error analysis if $w$ results from a PRODUCT of measured quantities:

$$
\left( \frac{\Delta w}{w} \right)^2 = \left( \frac{\Delta x}{x} \right)^2 \left( \frac{y^{-1/2}}{x} \right)^2 + \left( \frac{\Delta y}{y} \right)^2 \left( \frac{-\frac{1}{2} x y^{-3/2}}{x y^{-1/2}} \right)^2 = \left( \frac{\Delta x}{x} \right)^2 + \left( \frac{1}{2} \frac{\Delta y}{y} \right)^2
$$

More generally, if:

$$w = x^a y^b z^{-c}$$

then:

$$
\left( \frac{\Delta w}{w} \right)^2 = \left( a \frac{\Delta x}{x} \right)^2 + \left( b \frac{\Delta y}{y} \right)^2 + \left( \frac{c \Delta z}{-z} \right)^2
$$

Comments:

1) The error in any quantity resulting from a PRODUCT of measurements can be expressed easily in relative, or %, form.
2) Unfortunately, if the function defining $w$ contains any sums, canceling of terms cannot be done and Equation 1 must be used.
3) Partial derivatives do not need to be computed in this case.
4) The sign of the exponent doesn't matter, since it gets squared.

Back to the original example:

Suppose that:

$$
x = 400 \pm 2
$$
$$
y = 30 \pm 1
$$

then:

$$
\left( \frac{\Delta w}{w} \right) = \sqrt{\left( \frac{2}{400} \right)^2 + \left( \frac{1}{2} \cdot \frac{1}{30} \right)^2} = \sqrt{0.005^2 + 0.012^2} = 0.013 = 1.3\%
$$

It is easy to see which measurement contributes the most error. In this case it is the measurement of $y$. The error can be expressed as a relative (%) error or as an absolute error:

$$w = 73.0 \pm 1.3\% = 73.0 \pm 0.9$$
INDEPENDENT SOURCES OF ERROR

Example: measuring the temperature of flowing water

Independent sources of error may include:

1) measurement by a thermocouple
2) actual variations in the water temperature

therefore, the total error in the water temperature is:

\[
(\Delta T)^2 = (\Delta T_{\text{thermocouple}})^2 + (\Delta T_{\text{actual}})^2
\]

USING ERROR ANALYSIS IN DESIGNING TESTS

Error analysis also is very useful in designing experiments or tests. Using the same example, suppose you want the percentage error in \( w \) to be less than 3%.

Then:

\[
3\% = 0.03 \geq \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{1}{2} \frac{\Delta y}{y}\right)^2}
\]

\((\Delta x/x)\) and \((\Delta y/y)\) are the quantities that can be controlled when designing the test.

The choice of equipment determines what \( \Delta x \) and/or \( \Delta y \) are, and (sometimes) the choice of the duration of the test determines what \( x \) and/or \( y \) are. For instance, if \( x \) represents time, using electronic timing makes \( \Delta x \) very small and the duration of the test, \( x \), may be short. On the other hand, if a stopwatch is used, \( \Delta x \) is larger. Therefore the duration of the test must be increased to make \((\Delta x/x)\) sufficiently small.