Step response and impulse response

Step response:

a). Initial conditions are all zero.

b). The input is a unit step.

Example: Constant force acting on a spring-mass-damper system.

Input:

Output:

or for over-damped systems:
**Impulse response:**

a). Initial conditions are all zero.

b). The input is a unit step impulse.

In the transfer function domain:

\[ Y(s) = G(s)U(s) \]

\( Y(s) \): Output \quad U(s) \): Input

Impulse input: \( U(s) = 1 \)

Hence: \( Y(s) = G(s) \)

\[ y(t) = \mathcal{L}^{-1}\{ G(s) \} = g(t) \]

Why do we need to find/study impulse response?

1) When \( U(s)=1, u(t) \) can be considered to be a random input signal containing all possible frequency components. (Can be better understood after we finish learning frequency response)

2) Convolution Theorem:

\[ y(t) = \int_{0}^{t} g(t - \tau)u(\tau)d\tau \]

Note: \( Y(s) = G(s)U(s) \not\Rightarrow y(t) = g(t)u(t) \)

**Example:** Find the unit impulse response for a second-order system described by the equation:

\[ \ddot{y} + 2\zeta \omega_n \dot{y} + \omega_n^2 y = \omega_n^2 u(t) \quad \text{.......... ①} \]

Take the Laplace transform of ①, I.C.s are zero:

\[ s^2 Y(s) + 2\zeta \omega_n sY(s) + \omega_n^2 Y(s) = \omega_n^2 U(s) \]
\[ Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} U(s) \]  \[ \cdots \cdots \cdots (2) \]

Impulse input: \[ U(s) = 1 \]

\[ Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2 (1 - \zeta^2)} \]

\[ = \frac{\omega_n}{\sqrt{(1 - \zeta^2)}} \cdot \frac{\omega_n \sqrt{(1 - \zeta^2)}}{(s + \zeta\omega_n)^2 + \omega_n^2 (1 - \zeta^2)} \]

\[ = \frac{\omega_n}{\sqrt{(1 - \zeta^2)}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{(1 - \zeta^2)} t \]

The impulse response given in the book on page 270 is incorrect. The impulse response of this system decays to zero and looks like this:

Impulse response of a stable system with \( n > m \) (order of denominator > order of numerator) is bounded and \( \to 0 \) as \( t \to \infty \)