Fatigue Strength Calculation, General Biaxial State of Stress, Ductile Material

1. Compute each of the following at the worst stressed point (or every candidate worst stressed point if the overall worst stressed point is unclear):

\[
\begin{align*}
\sigma_{\text{bend,mean}} & \quad \sigma_{\text{axial,mean}} & \quad \tau_{\text{torque,mean}} \\
\sigma_{\text{bend,alt}} & \quad \sigma_{\text{axial,alt}} & \quad \tau_{\text{torque,alt}}
\end{align*}
\]

2. Multiply each mean and alternating stress component by its appropriate \(K_f\). If \(\sigma_{\text{axial}}\) stresses exist, further amplify them by \(K_A = 1.4\).

3. Split the state of stress at every potential worst-stressed point into its mean and alternating components:

\[
\begin{align*}
\sigma_{\text{ym}} & \quad (\text{if any}) \\
\sigma_{\text{ya}} & \quad (\text{if any}) \\
\tau_{\text{xyym}} & \quad \tau_{\text{xyya}} \\
\sigma_{\text{xm}} & \quad \sigma_{\text{xa}}
\end{align*}
\]

The following formulas will apply to most cases:

\[
\begin{align*}
\sigma_{\text{xm}} &= K_{f,\text{bend}}\sigma_{\text{bend,mean}} + K_A K_{f,\text{axial}}\sigma_{\text{axial,mean}} \\
\sigma_{\text{xa}} &= K_{f,\text{bend}}\sigma_{\text{bend,alt}} + K_A K_{f,\text{axial}}\sigma_{\text{axial,alt}} \\
\tau_{\text{xyym}} &= K_{f,\text{torque}}\tau_{\text{torque,mean}} \\
\tau_{\text{xyya}} &= K_{f,\text{torque}}\tau_{\text{torque,alt}}
\end{align*}
\]

4. Compute equivalent, or “von Mises”, stresses for the mean and alternating elements using the distortion energy theory:

\[
\begin{align*}
\sigma_{\text{em}} &= \pm \sqrt{\sigma_{\text{ym}}^2 - \sigma_{\text{ym}} \sigma_{\text{ym}} + \sigma_{\text{ym}}^2 + 3\tau_{\text{xyym}}^2} \\
\sigma_{\text{ea}} &= \sqrt{\sigma_{\text{ya}}^2 - \sigma_{\text{ya}} \sigma_{\text{ya}} + \sigma_{\text{ya}}^2 + 3\tau_{\text{xyya}}^2}
\end{align*}
\]

Note: The alternating stress is always taken as a positive number. However, the mean stress may be either positive or negative. The nature of the loads and their relation to the stresses must be examined to determine the correct sign.

5. Compute the endurance limit, \(S_n\), corresponding to loading in pure bending.

6. Apply the fatigue diagram equations using \(\sigma_{\text{em}}\) and \(\sigma_{\text{ea}}\).

7. If \(n < 1\) and finite life is acceptable, determine the fatigue strength, \(S_f\), from the fatigue diagram. Determine the cycles of life using the \(S_f\ versus \ n\ relations.

\[1\text{The alternative “nominal mean stress” method increases only the alternating stresses, which produces a less conservative result.}\]