Modeling of Tiny Hydraulic Cylinders

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ABSTRACT

Objective: To investigate the efficiency of four hydraulic cylinder configurations with cylinder bore size between 1 and 10 mm. The configurations were: (1) no piston seal, no rod seal; (2) no piston seal, rod seal; (3) piston seal, no rod seal; (4) piston seal, rod seal. The influence of operating conditions, geometrical parameters and fluid properties on cylinder force efficiency, volumetric efficiency and overall efficiency were modeled.

Methods: Empirical formulas were used to predict O-ring seal friction and leakage. Analytical solutions were used to predict viscous drag force and leakage of clearance seals. Results: With 10 micron clearance seal, cylinders with configuration (2) have higher overall efficiency than those with configuration (4). The difference increases as bore size decreases, and is significant for bores between 1 and 10 mm. The result reverses with 20 micron clearance seal. The cylinder force efficiency can be greater than one in some cases because of viscous drag forces on the piston.

Discussion: Conventional cylinders have configuration (4) because most fluid power applications are high power with large bore cylinders. Differences between configuration (2) and (4) are small for large bore size. For new fluid power applications such as medical devices, tiny bore size cylinders are needed. Configuration (2) is a useful design option in such applications because it not only saves a piston seal, but also improves cylinder efficiency. Configuration (1) is not feasible for hydraulic systems. However it may be viable for pneumatic systems. Commercial examples of configuration (4) exist.

INTRODUCTION

Hydraulic cylinders are commonly sealed by rubber seals to increase volumetric efficiency and to prevent hydraulic oil leaking into surrounding environment. There is a tradeoff between the cylinder volumetric efficiency and the cylinder force efficiency [1]. The higher the volumetric efficiency, the lower the force efficiency will be. With both cylinder piston and rod sealed by rubber seals, the room for further improving cylinder efficiency is physically limited. Furthermore, rubber seals produce detrimental friction force in tiny devices [7]. Alternatively, clearance seals can be used to replace rubber seals to reduce the sealing friction force [8]. Clearance seals are more favorable in tiny devices since viscous friction force dominates in these seals.

To explore new ways of improving the cylinder efficiency in tiny hydraulic cylinders, four cylinder configurations were conceived and compared side by side. The four cylinder configurations to be modeled are color-coded and labeled as (1) through (4) in Figure 1. The black dots in the figure represent O-ring seals. O-ring seals were chosen as the sealing elements due to its simplicity. Analytical solutions for O-ring seal friction and leakage exist in the literature ([3] – [6]).

![Figure 1: Four cylinder configurations to be modeled](image)

CYLINDER EFFICIENCY MODEL

Since piston seal was the focus of this study, only the outstroke was modelled. Parameters used in the modelling process are summarized in Table 1. The sealed element in the table refers to either piston or rod. The physical meaning of \( \delta \) and \( l \) is illustrated in Figure 2. O-ring squeeze ratio \( \varepsilon' \) is defined in equation (7).
### Table 1: Parameters to be used in the modelling process

<table>
<thead>
<tr>
<th>VAR</th>
<th>PHYSICAL MEANING</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>Operating pressure</td>
<td>MPa</td>
</tr>
<tr>
<td>$D$</td>
<td>Sealed element diameter</td>
<td>mm</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Fluid absolute viscosity</td>
<td>Pa·s</td>
</tr>
<tr>
<td>$U$</td>
<td>Sealed element velocity</td>
<td>mm/s</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Clearance gap size</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>$l$</td>
<td>Sealed element width</td>
<td>mm</td>
</tr>
<tr>
<td>$d$</td>
<td>O-ring cross-section diameter</td>
<td>mm</td>
</tr>
<tr>
<td>$E$</td>
<td>O-ring Young’s modulus</td>
<td>MPa</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>O-ring squeeze ratio</td>
<td>----</td>
</tr>
</tbody>
</table>

**Clearance Seal Model** – The viscous friction force and the leakage flow across a concentric clearance seal can be modelled with the following equations ([1], [2])

$$f_u = \frac{\pi \cdot \delta \cdot P \cdot D}{2} - \frac{\pi \cdot \mu \cdot U \cdot D \cdot l}{\delta} \quad (1)$$

$$q_u = \frac{\pi \cdot P \cdot D \cdot \delta^3}{12 \cdot \mu \cdot l} + \frac{\pi \cdot U \cdot \delta \cdot D}{2} \quad (2)$$

The sub-script $u$ represents the unsealed situation, that is with a clearance seal. The meaning of other parameters is illustrated in Table 1 and Figure 2.

The viscous friction force $f_u$ can be treated as a superimposition of pressure-induced and velocity-induced friction. If sealed element velocity $U$ is zero, then only pressure-induced friction exists, and vice versa. Pressure-induced and velocity-induced gap flow velocity distribution is illustrated in Figure 3. Since the direction of viscous friction force is the same as gap flow velocity gradient [2], the pressure-induced friction is in the same direction as gap flow velocity, and the velocity-induced friction is in the opposite direction as gap flow velocity. Therefore the direction of $f_u$ is determined by the relative magnitude of the operating pressure $P$ and the sealed element velocity $U$. If pressure-induced friction is smaller than velocity-induced friction, then $f_u$ is in the opposite direction as $U$, thus hindering sealed element movement. Conversely, $f_u$ will be in the same direction as $U$, thus helping sealed element movement.

Since both pressure-induced and velocity-induced gap flows are in the same direction as the gap flow velocity, the leakage flow $q_u$ is always in the same direction as $U$.

**Rubber O-ring Seal Model** – Reference [3] gives an analytical solution for rubber O-ring seal friction, as shown in equation (3), where sub-script $s$ represents sealed situation, i.e., with an O-ring seal, $\mu_s$ is the friction coefficient between the O-ring seal and the structural wall, and $r$ is the O-ring cross-sectional radius. Variables $d$, $d_1$, and $d_2$ are defined in Figure 4.

$$f_s = 2 \cdot \pi \cdot \mu_s \cdot d_1 \cdot r \cdot E \cdot \left(1 - \frac{d_1 - d_2}{4 \cdot r}\right) \cdot \sqrt{1 - \left(\frac{d_1 - d_2}{4 \cdot r}\right)^2} \quad (3)$$

To convert this solution to an expression that uses variables defined in Table 1. The following variables are defined.

![Figure 2: Parameters illustration for a clearance seal](image)

![Figure 3: Pressure-induced (1st row) and velocity-induced (2nd row) gap flow velocity distribution](image)

![Figure 4: An O-ring seal before and after installation](image)
\[ d = 2 \cdot r \]  
\[ g = \frac{d_1 - d_2}{2} \]  
\[ D = d_1 \]  
\[ \varepsilon = \frac{d - g}{d} = 1 - \frac{d_1 - d_2}{4 \cdot r} \]  

Substituting equations (4) – (7) into equation (3) following results in

\[ f_s = \pi \cdot \mu_f \cdot D \cdot d \cdot E \cdot \varepsilon \cdot \sqrt{2 \cdot \varepsilon \cdot \varepsilon^2} \]  

In the hydrodynamic lubrication domain, \( \mu_f \) can be expressed as [4]

\[ \mu_f = C \cdot \sqrt{\frac{\mu \cdot U}{P}} \]  

where \( C \) is a constant related to operating conditions.

Moreover, \( \mu_f = 0.3 \sim 0.5 \) for well finished and sufficient lubricated sealed surfaces [3]. If \( \mu_f = 0.4, \mu = 0.1 \text{ Pa} \cdot \text{s}, U = 0.1 \text{ m/s} \) and \( P = 10 \text{ MPa} \) are nominal operating conditions, then

\[ \mu_f = \begin{cases} 
12650 \cdot \sqrt{\mu \cdot U / P} & \text{if } P \neq 0 \\
4 \cdot \sqrt{\mu \cdot U} & \text{if } P = 0 
\end{cases} \]  

Equations (8) and (10) provide a set of equations for O-ring seal friction estimation.

Pressure-energized seals such as O-ring seals are normally designed to operate in a fully lubricated condition [9]. The seal rides on a thin film of lubricant which provides the final sealing barrier, retained in position by the surface tension of the film. Reference [5] gives an experimental formula for O-ring sealing film thickness \( h_c \)

\[ \frac{h_c}{s} = 2.99 \cdot \left( \frac{\mu \cdot U}{\sigma_m \cdot s} \right)^{0.71} \]  

where \( s \) is the O-ring contact width, and \( \sigma_m \) is the maximum O-ring contact pressure. Figure 5 further illustrates the definition of these two variables. The parameters used to achieve this formula spanned a wide range: \( U = 20 \sim 300 \text{ mm/s}, P = 1 \sim 15 \text{ MPa}, \mu = 0.47 \& 0.08 \text{ Pa} \cdot \text{s}, E = 3.9 \sim 20.5 \text{ MPa}, \varepsilon = 0.07 \sim 0.17, \) and \( d = 3 \& 5.5 \text{ mm}. \)

Since the gap flow has a linear velocity distribution at \( h_c \) [3], the average leakage flow velocity equals to half of the sealed element velocity. Therefore the leakage flow across the sealed element is

\[ q_s = \pi \cdot D \cdot h_c \cdot \frac{U}{2} \]  

Substituting equation (11) into equation (12) gives

\[ q_s = 1.495 \cdot \pi \cdot D \cdot \mu_f^{0.71} \cdot U^{1.71} \cdot \sigma_m^{-0.71} \cdot s^{0.29} \]  

In a loaded situation the O-ring contact width \( s \) and the maximum contact pressure \( \sigma_m \) can be expressed as [6]

\[ s = d \cdot (2 \cdot \varepsilon + 0.13) + d \cdot T \]  
\[ \sigma_m = 0.67 \cdot E \cdot (2 \cdot \varepsilon + 0.13) + 3.6 \cdot P / \pi \]  

where

\[ T = \left( \frac{0.39}{1 - \varepsilon} - 0.5 \cdot (2 \cdot \varepsilon + 0.13) \right) \cdot \left[ 1 - e^{-4.6 \cdot P} \right] \]  

Equation (13) – (15) provides a set of equations for O-ring seal leakage estimation.

**Cylinder efficiency model** – Cylinder force efficiency and volumetric efficiency are defined as

\[ \eta_f = \frac{F_{ar}}{F_{ir}} \]  
\[ \eta_v = \frac{Q_{a}}{Q_{a}} \]
where $F_{ar}$ and $Q_{ar}$ are actual rod speed and flow rate into cylinder chamber, and $F_{i}$ and $Q_{i}$ are the ideal rod speed and flow rate into cylinder chamber, defined as

$$F_{i} = P \cdot \frac{\pi \cdot B^2}{4}$$  \hspace{1cm} (18)$$

$$Q_{i} = U_{ar} \cdot \frac{\pi \cdot B^2}{4}$$  \hspace{1cm} (19)$$

where $B$ is cylinder bore size and $U_{ar}$ is actual rod speed. Cylinder overall efficiency is

$$\eta = \eta_f \cdot \eta_v = \frac{F_{ar} \cdot U_{ar}}{P \cdot Q_{ar}}$$  \hspace{1cm} (20)$$

The force efficiency for configuration (1) through (4) can be expressed as

$$\eta_{f1} = \frac{F_{i} + f_{up} + f_{sr}}{F_{i}}$$  \hspace{1cm} (21)$$

$$\eta_{f2} = \frac{F_{i} + f_{up} - f_{sr}}{F_{i}}$$  \hspace{1cm} (22)$$

$$\eta_{f3} = \frac{F_{i} - f_{sp} + f_{sr}}{F_{i}}$$  \hspace{1cm} (23)$$

$$\eta_{f4} = \frac{F_{i} - f_{sp} - f_{sr}}{F_{i}}$$  \hspace{1cm} (24)$$

where

$$f_{up} = \frac{\pi \cdot \delta \cdot P \cdot D_{p} \cdot \mu}{2} - \frac{\pi \cdot \mu \cdot U_{ar} \cdot D_{p} \cdot l}{\delta}$$  \hspace{1cm} (25)$$

$$f_{sr} = - \frac{\pi \cdot \mu \cdot U_{ar} \cdot D_{r} \cdot l}{\delta}$$  \hspace{1cm} (26)$$

$$f_{sp} = 4 \cdot \pi \cdot \sqrt{\mu \cdot U_{ar} \cdot D_{r} \cdot d \cdot E \cdot \varepsilon} \cdot \sqrt{2 \cdot \varepsilon - \varepsilon^2}$$  \hspace{1cm} (27)$$

$$f_{sp} = 12650 \cdot \pi \cdot \sqrt{\frac{\mu \cdot U_{ar}}{P} \cdot B \cdot d \cdot E \cdot \varepsilon \cdot \sqrt{2 \cdot \varepsilon - \varepsilon^2}}$$  \hspace{1cm} (28)$$

where $D_{p}$ and $D_{r}$ represent piston diameter and rod diameter respectively. The plus sign is used for clearance seal friction and the minus sign for O-ring seal friction. The reason is the direction of clearance seal friction depends on the relative magnitude of operating pressure and rod velocity, while the direction of O-ring seal friction is always in the opposite direction of rod speed.

The volumetric efficiency for configuration (1) through configuration (4) can be expressed as

$$\eta_{v1} = \eta_{v2} = \frac{Q_{i}}{Q_{i} + q_{up}}$$  \hspace{1cm} (29)$$

$$\eta_{v3} = \eta_{v4} = \frac{Q_{i}}{Q_{i} + q_{sp}}$$  \hspace{1cm} (30)$$

where

$$q_{up} = \frac{\pi \cdot P \cdot D_{p} \cdot \delta^3}{12 \cdot \mu \cdot l} + \frac{\pi \cdot \mu \cdot U_{ar} \cdot \delta \cdot D_{p}}{2}$$  \hspace{1cm} (31)$$

$$q_{sp} = 1.495 \cdot \pi \cdot B \cdot \mu^{0.71} \cdot U_{ar}^{1.71} \cdot \sigma_{m}^{-0.71} \cdot s^{0.29}$$  \hspace{1cm} (32)$$

Since only outstroke was modelled, leakage across the piston seal determines the volumetric efficiency. Since leakage across both the clearance seal and the O-ring seal is in the same direction as rod speed, the plus sign is used before both leakage terms.

**SIMULATION RESULTS**

Equations (16) – (32) were used to model the efficiency of cylinder configuration (1) through (4). Following nominal values were used in the simulations: $P = 10$ MPa, $\mu = 0.1$ Pa·s, $U_{ar} = 0.1$ m/s, $l = 10$ mm, $d = 1$ mm, $E = 10$ MPa and $\varepsilon = 0.1$. Since configuration (1) and (3) do not have rod seals, they are not feasible for hydraulic cylinders. Following discussions emphasize on configuration (2) and (4).

Figure 6 and figure 7 show the cylinder overall efficiency versus bore size with 20 and 10 micron clearance gap sizes. The results show that the difference between configuration (4) and (2) becomes bigger as bore size decreases, and the difference becomes significant for bore size smaller than 10 mm. With a 20 micron clearance gap size, configuration (4) has higher efficiency than configuration (2), but the situation reverses if 10 micron gap size is used. This means configuration (2) is a better option than configuration (4) if the clearance gap can be made small. The benefits of configuration (2) become more significant in small bore size cylinders.

Figure 8 shows cylinder force efficiency versus bore size. An interesting phenomenon is that the force efficiency of
configuration (1) can be greater than one. This phenomenon does not contradict energy conservation laws because the cylinder overall efficiency is always smaller than one. Clearance seals can generate positive drag force, which assists the piston and rod movement. Because there is a trade-off between force efficiency and volumetric efficiency, a force efficiency being higher than one means that volumetric efficiency is sacrificed, which can be seen in Figure 9.

Figure 6: Cylinder overall efficiency vs. bore size ($\delta = 20\,\mu m$)

Figure 7: Cylinder overall efficiency vs. bore size ($\delta = 10\,\mu m$)

Figure 8: Cylinder force efficiency vs. bore size ($\delta = 10\,\mu m$)

CONCLUSION

Four cylinder configurations were conceived, modeled and analyzed. Empirical formulas were used to model O-ring seals, and analytical solutions were used to model clearance seals. Simulation results showed removing piston seals out can improve hydraulic cylinder overall efficiency if the clearance gap is small. The benefits of removing seals become significant as bore size decreases.

REFERENCES