Write a function to perform a golden section optimization in a specified search direction in a multi-dimensional search space. As in Part 1, the golden section optimization function is kept “generic” by isolating the actual merit function in a separate “C”, “C++” or FORTRAN function. A suggested input/output chart for the multi-dimensional golden search function is provided below\(^1\):

- initial upper limit on the search vector length multiplier \((\lambda U_{\text{init}})\)
- final lower limit on the search vector length multiplier \((\lambda L_{\text{final}})\)
- desired fractional reduction in the interval of uncertainty \((f)\)
- name of the function containing the merit function \((\text{MERIT})\)
- dimension of search space \((n_{\text{dim}})\)
- reference point (vector) \((x_{\text{ref}})\)
- search direction (vector) \((s)\)
- final upper limit on the search vector length multiplier \((\lambda U_{\text{final}})\)
- successful completion flag \((\text{multgold} \text{ in } \text{C or C++}; \text{IOK in FORTRAN})\)

Test the operation of your one-dimensional Golden Section search function by writing a simple mainline and test merit function to find the minimum of function:

\[
M = x_1^4 x_2^2 + 9 x_1^4 - 6 x_2 (x_1^4 + 1) + x_1^2 + x_2^2 - 2 x_1 + 11
\]

starting from reference point:

\[
\begin{align*}
  x_1 &= 5.0 \\
  x_2 &= 5.0
\end{align*}
\]

and using the steepest descent search direction:

\[
\hat{S} = -502 \hat{e}_1 - 626 \hat{e}_2
\]

While the test case has two dimensions, ensure that your “multgold” function will work for any number of dimensions with no modifications (other than specifying a different “n_{\text{dim}}”).

The multi-dimensional golden section function should be fully documented but the test mainline and merit function need not be. Please turn in a listing of your test program and a record of a successful run of your test program\(^2\). (15 pts)

\(^1\) If you wish, you may also include an input argument that enables the user to print intermediate results from “multgold” or to prevent any printing from “multgold”. This might be useful to troubleshoot the optimization of challenging merit functions. If you code your “multgold” in FORTRAN, you will need to add at least one output argument to enable variable dimensioning of the vector where the design variables are set to evaluate the merit function.

\(^2\) The print can be created using the “script” command as described in Section 6 of the “An Introduction to SGI UNIX Computing” tutorial.