Color Reproduction Stabilization with Time-Sequential Sampling

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Abstract

Color reproduction stabilization is a control process to ensure that the printed output matches the desired color tones. In this paper we consider the stabilization of single color tone separation. The mapping of input-tone to output-color is characterized by the color tone reproduction curve(TRC). By compensating for any variations of the Xerographic printing process, the TRC stabilization controller aims to achieve identity TRC maps. This problem is critical to ensure high fidelity color printing. To make TRC stabilization viable in an actual Xerographic printing process, there are two major issues to address; 1) there exist only small number of actuators to stabilize a potentially infinite dimensional TRC; 2) only a small number of color sensor patches can be printed and measured at a time. We will look into TRC stabilization controller based on optimal controller. To address the second problem, we proposed the use of time-sequential sampling to fulfill the sparse spatio-temporal sampling requirement of sensing the TRC. We also proposed using Kalman filtering approach to reconstruct the time-sequential sampled signal. Two types of time-sequential sampling strategies - lexicographical and bit reversed strategies are analyzed and their suitability for TRC stabilization discussed. The time-sequential sampling and Kalman filter forms a linear time invariant observer for the TRC stabilization controller. This gives a straightforward implementation of the TRC stabilization control with time-sequential sampling. Experimental and simulation results verified the adequacy of the approach taken in realizing a practical color reproduction stabilization control system.

Introduction

Xerographic color printing is a process in which a digital color image is reproduced on paper. High quality color printout is not only judge by exact spatial reproduction, but also in maintaining high fidelity and consistency in the color reproduction. In this paper we forgo the spatial reproduction and concentrate on achieving consistency in the color reproduction. In this respect, a Xerographic color printer can be represented by the color reproduction function; CRC : $\mathcal{C} \rightarrow \mathcal{C}$, desired color $\mapsto$ output-color, where $\mathcal{C}$ is a 3-dimensional color space. An ideal printer is one that gives identity mapping between desired and output-color. A typical CMYK Xerographic color printer reproduce a particular color by overlaying different tones of CMYK (Cyan, Magenta, Yellow and blacK) colorants. The different CMYK tones subtract unwanted spectral components from "white" light while transmitting other required wavelengths to produce the desired color. The printing of each of the color seperation is characterized by the tone reproduction curve; TRC : $[0, 1] \rightarrow \mathcal{C}$, desired tone $\mapsto$ output-color, where $tone = 1$ gives the solid colorant.

The Xerographic printing process is subjected to both controllable and uncontrollable processes. The uncontrollable processes such as humidity, temperature, material age etc. give rise to inconsistency in the color reproduction. To maintain a constant and stable process i.e. to ensure that CRC is close to identity at all times, the controllable processes are regulated. Available controllable actuators in the Xerographic process includes laser power, corotron voltage and development/bias voltages. In this paper consideration will be made for a simplified problem. Instead of looking at stabilizing the CRC, we will be looking at stabilizing individual color separation TRCs. Achieving consistency of the TRC for individual color separation is synonymous to achieving consistency in the overall color reproduction. Both the TRC and CRC stabilization control pose significant problems for sensing and control. This is due largely to limited sensing capabilities and the potentially infinite dimensional mapping of TRC and CRC. A modest discretization ,$p = 21$ of the TRC would gives $21^4 = 194K$ desired color to be kept track of for the CRC control problem. Even by considering a single color separation, we are face with controlling a p-dimensional TRC with limited actuators ($\ll p$). A solution to this control problem was previously addressed in [1] using a curve-fitting technique. In this paper we will consider another solution to this problem based on optimal controller.

To address the limited sensing problem, time-sequential sampling is used to increase the utility of available feedback information. Time-sequential sampling was investigated in the 1980's and 1990's for video and time varying imaging applications [2][3][4]. For time varying images, time-sequential sampling refers to sampling the images at different spatial locations at different sampling instances. The benefit of using time-sequential sampling is the ability to reduce the sampling rate beyond the Nyquist rate while retaining the same information content. This is accomplished by trading off temporal bandwidth with spatial bandwidth. The method maximizes the available information from the small number of sampling patches (3 to 5 sampling patches every few photoreceptor belt cycles) and allows for the time-varying CRC or TRC to be captured(and subsequently reconstructed faithfully).

This paper gives the implementation of the TRC stabilization control with time-sequential sampling. It is the basis of realizing a realistic approach in stabilizing the xerographic color printing process.

Problem Formulation

The time-varying $TRC(k) \in \{\mathcal{F} : [0,1] \rightarrow \mathcal{C}\}$ gives the input tone to output color characterization of xerographic printing process. The mapping $\mathcal{F}$ is potentially infinite dimensional and is nonlinear. However we can approximate the TRC by a finite number of p-tones (p large). We can also assume that the mapping $\mathcal{F}$ can be adequately described by a linear plant with uncertainty. This gives:

$$\begin{aligned} y(u(k), d(k)) &= \hat{\phi}(I + \Delta(k)W_u)\bar{u}(k) + y(u_o, d(k)) \\ TRC(k) &= C \cdot y(u(k), d(k)) \end{aligned} \tag{1}$$

where $TRC(u(k), d(k))$ is the tonal-temporal signal with compact spectral support subjected to changes in the xerographic actuators, $u(k) \in \Re^m$ and disturbances, $d(k) \in \Re^p$. Also, $\bar{u}(k) := u(k) - u_o$, where u_o is the nominal control input. $\hat{\phi}$ is the nominal sensitivity function, $\Delta(k)$ is the multiplicative uncertainty, W_u is the matrix of given uncertainty weights. $C \in \Re^{M \times p}$ is the indicator matrix which gives the M-tone measurements of the discretized TRC. For the time sequential sampling, assume that at each sampling instant k, n-tones given by $\alpha(k) = [\alpha_1(k), \alpha_2(k), ..., \alpha_n(k)]$ are printed and measured. $\alpha(k)$ gives the time-sequential sampling pattern. The time sequentially sampled TRC is defined by:

$$TRC_s(k) = TRC(k)\,[\alpha(k)] \tag{2}$$

We take α to be M-periodic i.e., $\alpha(k + M) = \alpha(k)$ for all $k \in \mathcal{Z}$ and $n = 1$. For lexicographic sampling $\alpha(k) = \text{mod}(k, M)$ and for bit-reversed sampling $\alpha(k)$ is given by reversing the order of the significants bits for the binary representation of the index-k. The idea of the bit-reversed order is that it is roughly an even sampling of the time-tone space. The lexicographical and bit-reversed sampling pattern with $M = 21$ in the time-tone space is given in Figure 1.

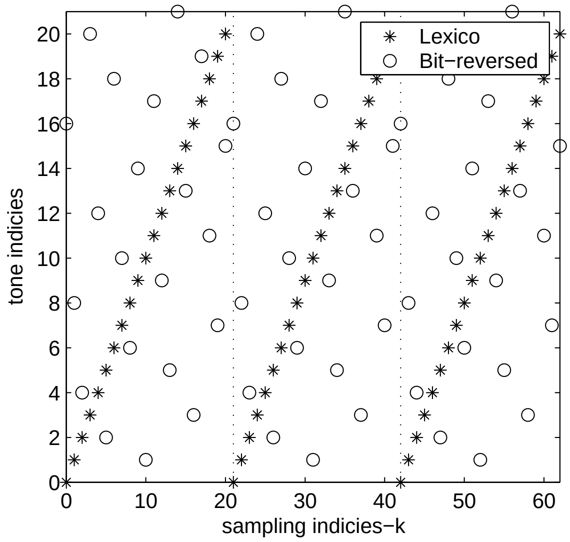

Figure 1: Lexicographical and bit-reversed time sequential sampling sequence with $M = 21$ points

To realize a practical implementation of the TRC stabilization control the following issues are addressed:

1. Since only limited sensing capabilities are available, the TRC is time-sequentially sampled to maximize the available information. To reconstruct the time-sequentially sampled tonal-temporal signal, Kalman filter is used in place of a low-pass filter. Kalman filter is used because it allows causal low-pass implementation and provide a method to incorporate information on changes in the control inputs in the low-pass reconstruction.

2. The control objective is to achieve the nominal TRC where there is no disturbance i.e. at each $tone_i$, $i = 1, 2, ..., p$, as $k \rightarrow \infty$:

$$\begin{aligned} &y(u(k), d(k))[tone_i] \rightarrow \\ &\qquad y^*(u = u_o, d = 0)[tone_i] \end{aligned} \tag{3}$$

where u_o gives the nominal control. With the number of actuators, m being much less that the number of p-tones to be control, it is not possible to make (3) hold. To make sure that the p-tones TRC is well behaved, curve-fitting is employed. In this paper we will consider a linear-quadratic controller with integrator dynamics.

Time-Sequential Sampling and Reconstruction by Kalman Filter

Let the TRC measurements error, $\Delta TRC(k) := TRC(k) - TRC^*$, where $TRC^* := C \cdot y(u_o, 0)$ Consider the linear uncertain xerographic plant given in eq.(1), we arrive at:

$$\Delta TRC(k) = C\hat{\phi}(I + \Delta(k)W_u)\bar{u}(k) + \bar{d}(k) \quad (4)$$

where $\bar{d}(k) := C \cdot y(u_o, d(k)) - TRC^*$. Assume that the TRC is well represented by its values at $p = M$-tones. To exhibit its tonal spectral contents, the TRC measurement errors is modelled by its DFT so that:

$$\Delta TRC(k) = G \cdot x(k) \quad (5)$$

where $k \in \mathcal{Z}^+$ is the index, $G \in \Re^{M\times M}$ is a matrix of Fourier basis function and $x(k) \in \Re^M$ is the vector of Fourier coefficients. Substituting (5) into (4), we have:

$$G \cdot x(k)(k) = \hat{\phi}(I + \Delta(k)W_u)\bar{u}(k) + G \cdot x_d(k)$$

where $x_d(k) := G^{-1}\bar{d}(k)$. The disturbance dynamics, $x_d(k)$ is modelled as having a compact spectral support and the time-sequentially sampled signal is subject to measurement noise $n(k)$:

$$\begin{aligned} x_w(k+1) =& A_w x_w(k) + B_w w(k) \\ \Delta TRC_s(k) =& C_\alpha(k) G C_w x_w(k) + C_\alpha(k)\hat{\phi}(I + \\ & \Delta(k)W_u)\bar{u}(k) + C_\alpha G D_w w(k) + n(k) \end{aligned} \quad (6)$$

where $w(k)$ and $n(k)$ is zero-mean white noise sequence with covariance R_{ww} and R_{nn} respectively. $C_\alpha(k) \in \Re^{1\times M}$ is a M-periodic sampling matrix for the time-sequential sampling pattern $\alpha(k)$. The system given by A_w, B_w, C_w, D_w gives the desired compact spectral support for the disturbance signal.

Because span$\{C_\alpha(k), k = 0, 1, ..., M-1\} = \Re^M$, (6) is a M-periodic observable linear system and admits a M-periodic Kalman filter in steady state:

$$\begin{aligned} \hat{x}_w(k+1) =& A_c(k)\hat{x}_w(k) + B_c(k)\Delta TRC(k) \\ \Delta\widehat{TRC}(k) =& \tilde{G} \cdot \hat{x}_w(k+1) + \hat{\phi}(I + \Delta(k)W_u)\bar{u}(k) \end{aligned} \quad (7)$$

where

$$\begin{aligned} A_c(k) &= A_w(I - L(k)C_\alpha(k)\tilde{G}) \\ B_c(k) &= A_w L(k) C_\alpha(k) \\ \tilde{G} &= G C_w \end{aligned}$$

and $L(k)$ is the periodic Kalman filter gain obtained by solving the periodic Riccati equation:

$$\begin{aligned} \bar{P}(k+1) =& A_w[\bar{P}(k) - L(k)C_\alpha(k)\tilde{G}\bar{P}(k)]A_w^T + \\ & B_w R_{ww} B_w^T \\ L(k) =& \bar{P}(k)\tilde{G}^T C_\alpha^T(k)\left[R_{nn} + C_\alpha(k)\tilde{G}\bar{P}(k)\tilde{G}^T C_\alpha^T(k)\right]^{-1} \\ \bar{P}(k) =& \bar{P}(k+M) \end{aligned} \quad (8)$$

For further analysis of time-sequential sampling with reconstruction using Kalman filter, refer to [5].

TRC Stabilization Controller

The high dimensionality of the TRC coupled with limited actuation does not permit us to keep track of all the color tones. Moreover, we have to consider uniformly sampling of only a finite M-tones. We consider an optimal control approach to ensure each of these M-tones achieve the nominal TRC in a least-squared sense. We also impose an integrator dynamics on the optimal control formulation to ensure certain q-tones ($q \le m$) achieve (3). Hence the optimal control problem is to find the control $u(k)$ based on the measured TRC, $TRC(k)$, such that the following quadratic performance index(QPI), J is minimized:

$$\begin{aligned} J =& \frac{1}{2}\sum_{k\in\mathcal{Z}} \Delta TRC_i^T(k) Q_i \Delta TRC_i(k) + \frac{1}{2}\sum_{k\in\mathcal{Z}} u^T(k) R u(k) \\ & + \frac{1}{2}\sum_{k\in\mathcal{Z}} \Delta TRC^T(k) Q \Delta TRC(k) \end{aligned}$$

where the integrator dynamics is given by $\Delta TRC_i(k+1) = \Delta TRC_i(k) + C_i \Delta TRC(k)$ with $C_i \in \Re^{q\times p}$ is the indicator matrix for the selected q-tones to fulfill (3). Q_i, R and Q is the weighting matrices. Taking $\Delta(k) = 0$, the linear-quadratic state feedback follower-controller for the given QPI for system (4) can be solve by using the backward-sweep solution [6]. The optimal control from the solution gives a feed-forward control sequence, $K_2\bar{d}(k)$ and a feedback sequence, $K_1 \Delta TRC_i(k)$ as follows:

$$\bar{u}(k) = -K_1 \Delta TRC_i(k) + K_2 \bar{d}(k) \quad (9)$$

where

$$\begin{aligned} K_1 =& Z_{ww}^{-1} Z_{xw}^T \\ K_2 =& -Z_{ww}^{-1}(\hat{\phi}^T C^T C_i^T S_B C_i + \hat{\phi}^T C^T Q - \hat{\phi}^T C^T C_i)^T F_B \end{aligned}$$

with $Z_{ww} := R_u + \hat{\phi}^T C^T C_i^T S_B C_i C\hat{\phi}$; $Z_{xw} := S_B C_i C\hat{\phi}$; $R_u = R + \hat{\phi}^T C^T Q C\hat{\phi}$. S_B is obtained from the solution of the discrete algebraic Riccatti equation.

$$\begin{aligned} & I^T S_B I - S_B - S_B C_i C\hat{\phi}\times \\ & (R_u + \hat{\phi}^T C^T C_i^T S_B C_i C\phi)^{-1}(S_B C_i C\hat{\phi})^T + Q_i = 0 \end{aligned}$$

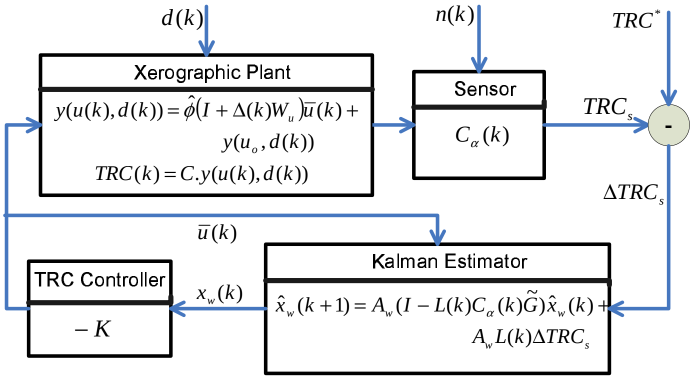

Figure 2: *Kalman estimator of time-sequentially sampled TRC and TRC stabilization controller mechanization*

and,

$$F_B = (Z_{xw} Z_{ww}^{-1} \hat{\phi}^T C^T C_i^T)^{-1} \times$$
$$(Z_{xw} Z_{ww}^{-1} (\hat{\phi}^T C^T C_i^T S_B C_i + \hat{\phi}^T C^T Q) - S_B C_i)$$

Figure 2 shows the schematic the TRC stabilization with time sequential sampling. The well known separation principle ensure that the estimator dynamics and controller dynamics have no effect on each other. This means that the controller and estimator can be designed separately yet used together. Hence, the controller as stated in (9) can be obtained from the estimated states, $\hat{x}_w(k+1)$ of the Kalman estimator. From (7), we have:

$$\Delta TRC_i(k) = C_i \cdot (\tilde{G} \cdot \hat{x}_w(k+1) + \hat{\phi}\bar{u}(k))$$
$$\bar{d}(k) = \tilde{G} \cdot \hat{x}_w(k+1)$$

Another interesting approach to TRC stabilization was previously given in [1] by using a robust static controller. The linear plant with uncertainty as given in (4) is assumed to be essentially a static system. This is a reasonable assumption because the disturbances of the xerographic printing process is slow-varying. Based on this assumption, a causal robust static controller that takes into account unknown disturbances and uncertainty is implemented.

Simulation and Experiments

Both simulation and experimental results are presented on the proposed TRC stabilization controller with time sequential sampling. The performance of using time sequential sampling is compared to that of full sampling. We also discuss the implication of changing the time-sequential sampling pattern. Experimental results shows the successful implementation of the proposed approach subjected to actual disturbance to the xerographic printer.

Simulation

The behavior of the xerographic plant is simulated by taking the model of the form of (1). The TRC is discretized uniformly into $p = 21$ points and the measurements points is also taken as $M = 21$ points. The nominal sensitivity matrix $\hat{\phi}$ is obtained experimentally by performing least-square fitting of the data into the linear model. The behavior of the system without plant perturbation is considered i.e. $\delta = 0$. The disturbance $\bar{d}(k)$ dynamics is taken to have a compact spectra support $\{(u, f)|(u/U)^2 + (f/W)^2 \leq 1\}$ where U and W are the highest tonal and temporal frequencies in $\bar{d}(k)$. In our study, we used a sampling interval of $T = 0.4s$ and the tonal range is $tone_i \in$ [0,1]. With $M = 21$, this gives a tonal temporal Nyquist frequencies of (u_N, f_N) = (10.5cycles/tone, 0.06Hz).

We consider two cases of sampling: full sampling where all M-measurement points are used at each sampling instant, k and time sequential sampling where only one tone is sampled at each sampling instant according to a prescribed sampling pattern i.e. lexicographical or bit-reversed sampling sequences. The TRC measurements are corrupted by noise with dynamics of $n(k+1) = 0.3n(k) + 0.07 \cdot rand(\eta), \eta = M$ for full sampling and $\eta = 1$ for time sequential sampling. The root mean square of the TRC errors, $(\sum_k \|\Delta TRC(k)\|_2^2)^{1/2}$ for all M-points is take as the measure of performance of the TRC stabilization controller.

Simulations was carried out at different tonal-temporal support frequencies (U, W) within the range of $\{(U, W)|1 \leq U \leq u_N, 0.01 \leq W \leq 6f_N\}$ using the optimal TRC stabilization controller with two integral fix points at $tone_1$ and $tone_{18}$ of the $M = 21$ points TRC. The comparable RMS of time-sequential sampling (lexicographical sampling sequence) and full sampling as shown in Figure 3 and 4 means that we are able to achieve good TRC stabilization by just sampling one TRC tone at each time step. Both Figure 3 and 4 also shows the expected response in increasing the tonal-temporal support frequencies of the disturbance signal – the higher the tonal-temporal support frequencies, the higher the RMS of TRC error. The higher tonal-temporal disturbance frequencies makes it hard for the TRC controller to curve-fit the measured TRC to the desired TRC.

Figure 5 compare the difference in RMS value between lexicographical and bit-reversed sampling sequences. Although the bit-reversed sampling sequence yield better signal reconstruction as reported in [5] it does not give overall good performance in achieving smaller TRC error. Essentially the bit-reversed sequence gives better performance for high tonal-temporal disturbance frequencies. Hence the bit-reversed sampling sequence is preferred for our application where sensing is expected to be highly sparse.

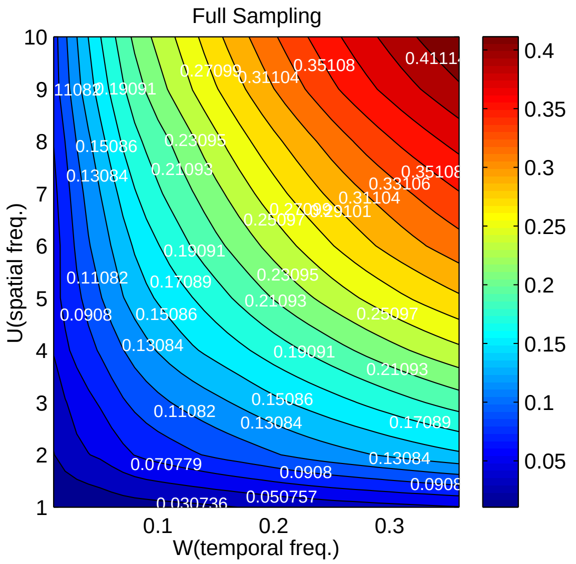

Figure 3: *RMS of TRC error using full sampling for different tonal-temporal frequencies support* (U, W)

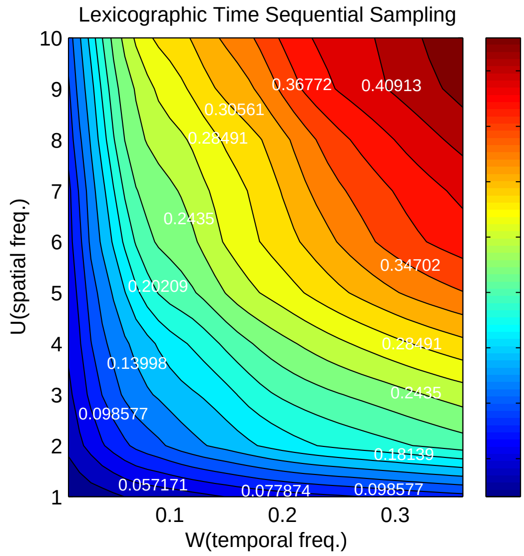

Figure 4: *RMS of TRC error using lexicographical time sequential sampling for different tonal-temporal frequencies support* (U, W)

Experiment

The proposed TRC stabilization system was also experimentally tested on a Xerox Phaser 7700 xerographic printer. Currently we do not have direct access to the xerographic actuators. To evaluate the TRC stabilization controller with full and time sequential sampling, a virtual printer model is used to generate the response (color image) due to changes in the actuator inputs. The output response is then printed using the physical printer. By calibrating the printer such that it is an identity map at nominal condition, we can capture the effect of the actual disturbances on the performance of the TRC stabilization system. The output response is in the form of a single colorant wedge of 21 different tones i.e. $M = 21$. The disturbances was artificially induced by introducing interference in the path of the laser. Sensing of the color wedge is performed using a scanner that has been calibrated using a spectrophotometer. The optimal controller with two integral fix points at $tone_2$ and $tone_{18}$ was used.

Figure 6 and 7 show the effectiveness of the proposed TRC stabilization system using both full and time sequential sampling. The TRC stabilization using both sampling approach results in the convergence of the TRC to the nominal TRC with each time step. Considering that only one tone is sampled at each time step, the time sequential sampling perform relatively well compare to that achieved using full sampling.

Conclusion

This paper addressed two main problems in realizing a practical TRC stabilization controller in maintaining consistency in color reproduction. The first problem of under actuation is resolved using a curve-fitting optimal control approach. Similar approach to this problem has been previously reported in [1] using static robust controller. The second problem relating to limited sensing capability is resolved using time sequential sampling. Both simulation and experimental results shows the effectiveness of the proposed approach. In particular we demonstrated the comparable stabilization property in using time sequential sampling in place of full sampling. Time sequential sampling substantially lower the TRC sensing requirements and this is important in actual implementation where available print area should be devoted to customer images and not in printing sensor patches.

The next step would be to expand the idea to cover stabilization of not only one single color separation as addressed here, but to all different color combinations. The same basic idea as proposed here should applied.

Acknowledgement

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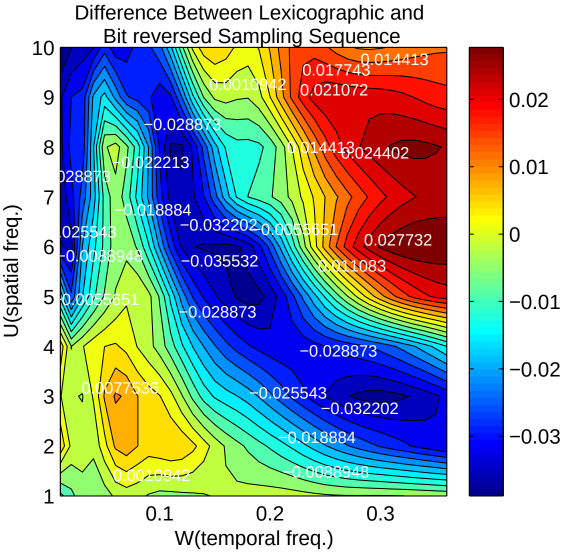

Figure 5: *Difference in RMS of TRC error of lexicographical and bit-reversed time sequential sampling for different tonal-temporal frequencies support* (U, W)

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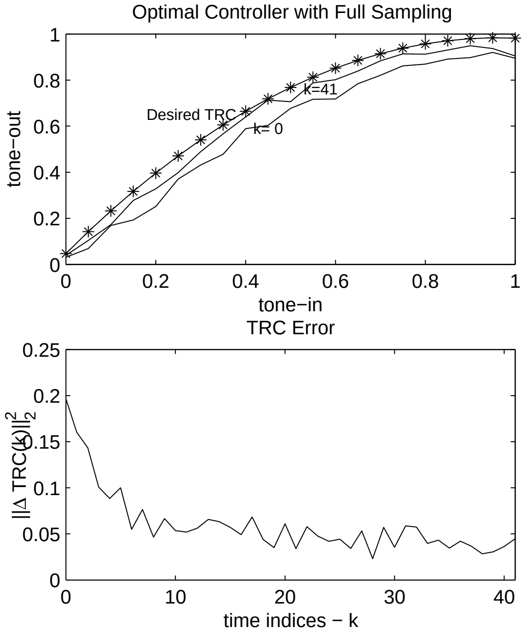

Figure 6: *Response of TRC stabilization control subjected to induced disturbance with full sampling. The curve marked with * is the desired TRC*

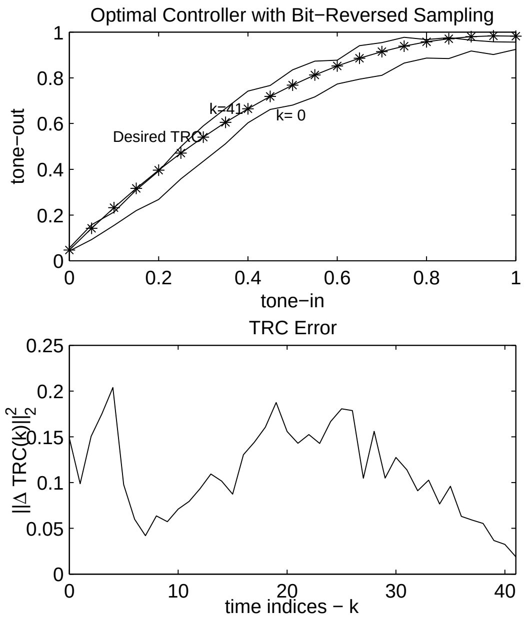

Figure 7: *Response of TRC stabilization control subjected to induced disturbance with time sequential sampling (Lexicographical sequence). The curve marked with * is the desired TRC*