NUMERICAL ANALYSIS OF HEAT EXCHANGERS USED IN A LIQUID PISTON COMPRESSOR USING A ONE-DIMENSIONAL MODEL WITH AN EMBEDDED TWO-DIMENSIONAL SUBMODEL

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ABSTRACT
The present study presents a one-dimensional liquid-piston compressor model with an embedded two-dimensional submodel. The submodel is for calculating heat conduction across a representative internal plate of a porous heat exchanger matrix within the compression space. The liquid-piston compressor is used for Compressed Air Energy Storage (CAES). Porous-media-type heat exchangers are inserted in the compressor to absorb heat from air as it is compressed. Compression without heat transfer typically results in a temperature rise of a gas and a drop in efficiency, for the elevated temperature leads to wasted thermal energy, due to cooling during subsequent cooling back to ambient temperature. The use of heat exchangers can reduce the air temperature rise during the compression period.

A typical numerical model of a heat exchanger is a one-dimensional simplification of the two-energy-equation porous media model. The present authors proposed a one-dimensional model that incorporates the Volume of Fluid (VOF) method for application to the two-phase flow, liquid piston compressor with exchanger inserts. Important to calculating temperature distributions in both the solid and fluid components of the mixture is heat transfer between the two, which depends on the local temperature values, geometry, and the velocity of fluid through the matrix. In the one-dimensional model, although the axial temperatures vary, the solid is treated as having a uniform temperature distribution across the plate at any axial location. This may be in line with the physics of flow in most heat exchangers, especially when the exchangers are made of metal with high thermal conductivity. However, it must be noted that for application to CAES, the gas temperature in the compression chamber rises rapidly during compression and the core of the solid wall may heat up to a different temperature than that of the surface, depending on the geometry, solid material of the exchanger and fluid flow situation. Therefore, a new, one-dimensional model with embedded two-dimensional submodel is developed to consider two-dimensional heat conduction in a representative solid plate. The VOF concept is used in the model to handle the moving liquid-gas interface (liquid piston).

The model gives accurate solutions of temperature distributions in the liquid piston compression chamber. Six different heat exchangers with different length scales and different materials are simulated and compared.

1. INTRODUCTION
Regenerative heat exchangers are usually made of solid matrices consisting of many flat, internal walls that provide large surface areas for heat transfer. They are often used for the purpose of cooling or thermal energy storage. In the present study, a numerical model is developed for such exchangers being used in a liquid-piston gas compressor. The compressor is for application to Compressed Air Energy Storage (CAES). The
The main advantage of using CAES is to overcome the mismatch between power demand on the electric grid and power generation by intermittent sources, such as wind turbines, by compressing air during low power demand periods and expanding air to output work during high power demand periods [1, 2]. Important to efficient and effective CAES operation is near-isothermal compression, as discussed in [3 – 5], because a rise in the air temperature during compression requires excessive amounts of compression work to be converted into internal energy, which is eventually wasted during the later storage period as the compressed air cools to ambient temperature. Therefore, it is important to reduce the temperature rise during the compression process to reduce the amount of compression work for the same compression pressure ratio. To accomplish this task, heat exchangers, or solid matrices, are inserted into the liquid-piston gas compressor. A schematic of the liquid-piston compressor is shown in Fig. 1. The term liquid piston applies to a rising liquid-gas interface created by pumping liquid into the lower section of the compression chamber. A main advantage of the liquid piston is that it is convenient for insertion of a heat exchanger (solid matrix) into the chamber to cool the air during compression, as the liquid can flow through the gaps of the solid matrix.

![Fig. 1. Schematic of a liquid-piston compressor](image)

Numerical modeling of the heat exchangers (solid matrices) can be categorized into two types of methods, the higher-order CFD simulations and the simplified models. The higher-order CFD approach is a two-dimensional or three-dimensional simulation solving the Volume-Averaged Navier-Stokes Equations [6]. This method assumes that the solid matrix region is a continuum and, thus, the internal walls of the matrix are not being directly modeled, but their equivalent effects are presented by closure terms included into the Navier-Stokes equations. Applications of this method can be found in a number of different studies, such as a liquid rocket propulsion system [7], a solid-piston compressor with porous insert [8], a double-pipe heat exchanger [9], a ceramic matrix used in a solar energy receiver [10], and a liquid piston compressor inserted with metal foam [11]. The second method for simulating this type of heat exchanger is using simplified models, usually one-dimensional. The one-dimensional modeling of a regenerative heat exchanger is well developed for periodic flow conditions that do not involve either compression of a gas or a rising liquid-gas interface. Application of this model can be found in studies such as analysis of metallic regenerative exchangers [12], analysis of entrained fluid in a regenerator [13], and an active magnetic regenerator [14]. In order to simulate the situation where the exchanger matrix is used in a liquid-piston exchanger, the one-dimensional exchanger model has been combined with a VOF (Volume of Fluid) method that handles two-phase flow [15]. This one-dimensional VOF exchanger model has been validated against two-dimensional simulations and has been shown to give quick and accurate solutions of axial distributions of fluid and solid temperatures [15].

One limitation of the aforementioned numerical methods is that the fine details, such as the temperature distribution across a solid exchanger plate, are unknown because both methods are based on a set of volume-averaged equations that treat the matrix region as a continuum. It is, however, important to know the temperature distribution across a plate of the exchanger matrix when it comes to understanding the effects of the exchanger material and shape. Thus, the present study extends the one-dimensional model proposed in [15] to solve a two-dimensional solid conduction equation for a representative exchanger plate. This is coupled with the one-dimensional fluid energy equation. Two types of heat exchangers that consist of flat plate elements will be simulated. One is an interrupted-plate and the other is a honeycomb. The interrupted-plate exchanger features layers of parallel and separated stacked plates so that the plates in two adjacent layers are made perpendicular to one another to interrupt the flow [16], as shown in Fig. 2 (a). The honeycomb matrix features hexagonal cells also stacked in layers to have the interrupting feature, shown in Fig. 2 (b). The shapes of these two exchanger matrices present features of repeating plates. This allows simulating a representative plate using a two-dimensional submodel within the context of the one-dimensional heat exchanger simulation, to understand the axial temperature distribution in the exchanger matrix as well as the heat penetration into the solid plate.

![Fig. 2. Schematics of heat exchanger matrices](image)
2. NUMERICAL MODEL

2.1. Transport Equations

In the proposed numerical model, a one-dimensional fluid domain is coupled with a two-dimensional solid domain through an interfacial heat transfer model between the fluids and the solid wall. For the fluid domain, the one dimension represents axial direction of the compression chamber. For the two-dimensional solid domain, one dimension is along the compressor axis and the other is across the solid plate, perpendicular to the surface. A schematic of the computational domain and discretized cells is shown in Fig. 3. In the actual heat exchanger, the plates are interrupted in a three-dimensional pattern; in the computation domain, only one representative plate with a fixed thickness throughout the entire chamber’s axial direction is modeled. This two-dimensional solid domain is capable of resolving heat penetration into the solid plate interior as well as the temperature distribution in the axial direction. Although it seemingly neglects the interrupting feature of the exchanger, the heat transfer correlations that couple heat transfer between a fluid and the solid surface will incorporate the effect of the interrupting feature of the exchanger plates.

Fig. 3 Schematic of computational domain and grid ( \( \frac{L}{2} \) : half of the plate thickness which includes \( N_y \) nodes; \( L \): chamber length that includes \( N_x \) nodes.)

In the fluid region, a rising liquid-gas interface must be simulated. This is modeled using the VOF approach [17]. A volume fraction variable is defined as a scalar function that gives the fraction of volume occupied by a phase at a location. The sum of volume fractions of both phases (air and water) at any location equals unity.

\[ \alpha_1 + \alpha_2 = 1 \]  

(1)

where subscripts 1 and 2 refer to the air and water phases, respectively. Bulk locations of the water and air phases are presented by the volume fraction variables by solving one continuity equations for each phase. The velocity and temperature fields are shared by all fluid phases.

The continuity equations for the air and water are given by,

\[ \frac{\partial \rho_1 \alpha_1 u}{\partial t} + \frac{\partial \rho_1 \alpha_1 u}{\partial x} = 0 \]  

(2)

\[ \frac{\partial \rho_2 \alpha_2 u}{\partial t} + \frac{\partial \rho_2 \alpha_2 u}{\partial x} = 0 \]  

(3)

The liquid and gas flow through a porous matrix. According to the porous media modeling principles, the energy transport in the fluid must take into account of the porosity and the interfacial heat transfer between the fluid and the solid. Also noted that this is a two-phase flow modeled by the VOF approach, the governing equation for the energy transport in the fluid mixture (both air and water) is given by,

\[ \frac{\partial \rho c_p T}{\partial t} + \frac{\partial \rho c_p u T}{\partial x} = k \frac{\partial^2 T}{\partial x^2} + \alpha_1 \beta_1 T \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \alpha_T h(T_s | y=0 - T) \]  

(4)

where,

\[ \bar{\rho} c_p = \alpha_1 \rho_1 c_{p,1} + \alpha_2 \rho_2 c_2 \]  

(5)

\[ k = \alpha_1 k_1 + \alpha_2 k_2 \]  

(6)

The water is assumed to be incompressible and, thus, the pressure work term in Eq. (4) is effective only in the air phase. The pressure can be related to air temperature and density by the equation of state.

The energy transport in the solid plate is two-dimensional, in both the compressor axial direction (\( x \)) and the direction perpendicular to the solid wall (\( y \)). The governing equation is:

\[ \rho_s c_s \frac{\partial T_s}{\partial t} = k_s \frac{\partial^2 T_s}{\partial x^2} + k_s \frac{\partial^2 T_s}{\partial y^2} \]  

(7)

The energy transport in the solid and fluid are coupled through interfacial heat transfer between the solid surface and the fluid domain,

\[ k_s \frac{\partial T_s}{\partial y} |_{y=0} = h(T_s | y=0 - T) \]  

(8)

Adiabatic boundary condition is applied at the centerline of the solid plate,

\[ k_s \frac{\partial T_s}{\partial y} |_{y= \frac{L}{2}} = 0 \]  

(9)

Other boundary conditions are:

\[ T |_{x=0} = T |_{x=L} = T_s |_{x=0} = T_s |_{x=L} = T_0 \]  

(10)

\[ \alpha_2 |_{x=0} = 1 \]  

(11)

2.2. Velocity Field

The velocity profiles for the water and air are known in this problem. Because water is considered incompressible, the velocity of the water phase is the same as the water inlet velocity, which is a constant in the present study. Previous two-dimensional CFD simulations show that an exchanger matrix when used in a liquid-piston chamber results in a pressure drop along the flow direction, and has a significant effect in smoothing the flow and suppressing the development of
secondary flows [11]. Therefore, it is appropriate to assume that the velocity of air is linearly distributed along the axial direction, matching the water (interface) velocity at the water-air interface, and stagnant at the top cap [16]. In the present VOF modeling approach, the instantaneous location of the water-air interface is calculated by searching for the location that has a volume fraction of 0.5 for either phase. The velocity field of air and water are given by,

\[ u = u(x, t) = \begin{cases} U_0, & \alpha_2 \geq 0.5 \\ U_0 \frac{L-x}{L-x_p}, & \alpha_1 > 0.5 \end{cases} \]  

(12)

where \( x_p \) is the location that corresponds to \( \alpha_2(x_p) = 0.5 \).

### 2.3. Real Gas Modeling

The model will be used for analyzing compression processes from 700kPa to 21MPa. Due to high pressure, a cubic equation of state for real gas based on coefficients recommended in [18] is used to relate air pressure to density and temperature,

\[ p = \frac{\rho RT}{M - \rho b} - \frac{a_p^2}{M^2 + bM\rho} \]  

(13)

where,

\[ b = \frac{0.08664RT}{\rho_c} \]  

(14)

\[ a = \frac{0.427278\gamma^2}{\rho_c} \left(1 + n \left[1 - \left(\frac{T}{T_c}\right)^{0.5}\right]\right)^2 \]  

(15)

\[ n = 0.48 + 1.574\omega - 0.176\omega^2 \]  

(16)

The acentric factor, \( \omega \), is taken to be 0.033. For air, the critical pressure, critical temperature and molecular weight are respectively, 3758MPa, 132.3K and 29g/mol. The universal gas constant is 8.314J/(mol K).

The specific heat for a real gas is modeled using a departure specific heat [18, 19]:

\[ C_{p,1} = C_{p,1,\text{ideal}} - C_{p,1,\text{dep}} \]  

(17)

where the \( C_{p,1,\text{ideal}} \) is for ideal gas. For air, \( C_{p,1,\text{dep}} \) is taken as a function of temperature based on a curve fit of the data from [20]. The departure specific heat is based on the following equation,

\[ C_{p,1,\text{dep}} = -\frac{\rho}{\rho T} + \frac{n}{M} - \frac{\partial T}{M\rho} \ln(1 + \frac{b\rho}{M}) \]  

(18)

### 2.4. Interfacial Heat Transfer Between the Fluids and Solid

When solving the model, an interfacial heat transfer correlation is needed for coupling heat transfer between the solid matrix and the fluids. Two kinds of heat exchanger matrix will be investigated, the interrupted plate and the honeycomb, shown in Fig. 2. For the interrupted-plate exchanger, a correlation based on previous numerous CFD experiments is used [16],

\[ Nu_{Dh} = 8.456 + 0.325Re_{Dh}^{0.625}Pr^{1/3} \]  

(19)

The hydraulic diameter of the interrupted-plate matrix is twice the separation distance of the plates and the Reynolds number is based on the local Darcian velocity. For the honeycomb matrix, since the honeycomb ducts are also laid out in an interrupted fashion, a correlation for heat transfer in an entry region of a hexagonal duct is used [21],

\[ Nu_{i_x} = 2f_r \left( \frac{\Delta Pr}{L_d} \right)^\frac{1}{2} \]  

where the definition and correlation for friction factor are given by,

\[ f_r = \frac{f_w}{2\rho u^2} = \frac{0.564}{1 + (1.664Pr)^{\frac{1}{2}}/2} \]  

(21)

Note that at locations where the volume fraction of either phase is between 0 and 1, the fluid mixture properties, including the density, viscosity, thermal conductivity, and the \( Pr \) number, are volume-fraction-averaged properties, according to the VOF approach.

### 2.5. Numerical Method

The transport equations (Eqs. (2) – (4) and (7)) are solved by finite difference approach. The continuity equations for water and air are discretized using an explicit upwind method,

\[ \frac{a_{2,1}n_{1}^{i+1} - a_{2,1}n_{1}^{i}}{\Delta t} + a_{2,1}n_{1}^{i+1}u_{1}^{i+1} - a_{2,1}n_{1}^{i}u_{1}^{i} n_{1}^{i+1} = 0 \]  

(22)

\[ \frac{a_{1,1}n_{1}^{i+1}p_{1}^{i+1} - a_{1,1}n_{1}^{i}p_{1}^{i}}{\Delta t} + a_{1,1}n_{1}^{i+1}u_{1}^{i+1} - a_{1,1}n_{1}^{i}u_{1}^{i} n_{1}^{i+1} = 0 \]  

(23)

The fluid energy equation is discretized using an explicit scheme, with upwind differencing on the advection term and central differencing on the diffusive term,

\[ \frac{(\rho c_{p,1})_{1}n_{1}^{i+1} - a_{1,1}n_{1}^{i}p_{1}^{i}}{\Delta t} - \frac{(\rho c_{p,1})_{1}n_{1}^{i}p_{1}^{i}}{\Delta t} + \frac{n_{1}^{i+1}u_{1}^{i+1} - n_{1}^{i}u_{1}^{i}}{\Delta x} = a_{1,1}n_{1}^{i+1}u_{1}^{i+1} - a_{1,1}n_{1}^{i}u_{1}^{i} n_{1}^{i+1} \]  

(24)

The solid energy equation is discretized using central differencing in space, and is solved using the Gauss-Seidel method.

\[ \frac{\rho s_{s_i}k_i^{j+1} - T_{s_i}k_i^{j}}{\Delta t} = k_s \left( T_{s_{i-1,j}} k_i^{j+1} - T_{s_{i-1,j}} k_i^{j} + T_{s_{i,j+1}} k_{i,j} - T_{s_{i,j+1}} k_{i+1,j} \right) \]  

(25)
The integer \( k \) represents the Gauss-Seidel iteration step within a time step. The boundary conditions that involve heat flux equations, Eqs. (8) and (9), are discretized with 2nd order differencing.

### 2.6. Grid Independence Verification

Simulations in the present study are done on a liquid-piston compressor for application to a 2nd compression stage of a CAES system. The compression starts with air at 700kPa and 293K. The chamber is initially full of air and has a total length of 0.4826m. Water is pumped into the chamber at a uniform speed, 0.152m/s, to compress the air. The total compression time is 3s. The final air pressure is around 21MPa.

For the grid independence study, a heat exchanger matrix, namely “PL-I-P-7.5_2.75_0.55,” is simulated. The name indicates that it is a plastic (PL) interrupted-plate heat exchanger (I-P) heat exchanger featuring a plate height, plate separation distance, and plate thickness of 7.5mm, 2.75mm, and 0.55mm, respectively. The separation distance is measured between two plate surfaces facing one another. The thermal conductivity, specific heat, and density of the ABS plastic are, respectively, 0.17W/(mK), 1200J/(kgK), and 1060kg/m³. Five grids with progressively refined cell sizes and time step sizes are used for computation. Details of the grids are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1 Computation grids for grid independence study</th>
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<tbody>
<tr>
<td>Grid 1</td>
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<tr>
<td>( N_x )</td>
</tr>
<tr>
<td>( N_y )</td>
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<tr>
<td>Time steps</td>
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The computation results based on different grids, compared in Fig. 4, show that as grid resolution improves, solutions converge to the one based on the finest grid (Grid 5). Grid sets 1 and 2 give less steep air temperature rises next to the interface due to insufficient cell numbers in the x direction. They also calculate larger temperature rises on the solid wall. Grid sets 3, 4, and 5 give very close results. Grid 3 is used in the following.

### 3. CASE STUDIES AND RESULTS

#### 3.1. Heat Exchanger Matrices

Six different heat exchanger matrices are studied. Four are of the interrupted-plate type (Fig. 2 (a)): Metal-I-P-3_2.5_0.4, Metal-I-P-7.5_2.75_0.55, PL-I-P-3_2.5_0.4, and PL-I-P-7.5_2.75_0.55. The naming of the exchangers follows the same rule as introduced in the preceding section; symbols and numbers mean respectively: material, exchanger type (interrupted-plate), plate height, plate separation distance, and plate thickness. Metal, as used in some of the exchangers, refers to steel with a thermal conductivity, specific heat and density of 14W/(mK), 502J/(kgK), and 7954kg/m³, respectively. Two honeycomb matrices with cell sizes of 1/4in and 1/8in (namely Honeycomb1_4 and Honeycomb1_8, respectively) are studied. The cell size is the distance between two parallel walls in a hexagonal duct. Each layer of honeycomb has a length of 7.5mm in the exchanger matrix. The characteristic length, \( l_c \), is the square root of the duct cross-sectional area. The plate thickness values corresponding to the 1/4in and 1/8in honeycomb matrices are, respectively, 0.1427mm and 0.1085mm. The honeycomb matrices are made of aluminum.

Fig. 4. Comparison of results for different grids at 3 seconds with thermal conductivity, specific heat, and density of 205W/(mK), 871J/(kgK), and 2719kg/m³, respectively.

#### 3.2. Temperature Field

The numerical model gives detailed calculations of temperature distributions in a representative solid plate, as well as the axial fluid temperature distribution. Figures 5-10 show that air temperature rises roughly 200K at the end of compression. The temperature rise mostly occurs in the latter period of a compression stroke since, for a constant liquid piston speed, the volume compression ratio becomes larger near the end of compression as the volume of air becomes smaller. In different exchanger matrices, the water temperature is nearly
constant throughout the compression process and immediately above the water-air interface, there is a step rise of air temperature followed by continuously rising temperature that reaches a peak value at a location very close to the top cap of the chamber. The liquid and gas near the interface move together with little relative velocity so it is difficult for convective heat transfer to take place directly from air to water. Though there is a small amount of heat that is conducted from air to water across the interface, given that water has a much larger density and heat capacity. The temperature of air rises by compression. Relative to the air far from the interface, the air temperature near the interface is nearer that of the water temperature because of faster air velocity that results in better heat transfer from air to the exchanger matrix. Near the top cap region, the air temperature reaches a maximum because of the relatively stagnant flow in that region, limiting the amount of heat transfer from air to the solid matrix.

The location of the instantaneous maximum temperature in an exchanger matrix shifts during compression as the water-air interface rises. In general, however, it is located in a region of the matrix that is in contact with air and very near the water-air interface. The reason for this is that once a section of the matrix is immersed in water, it is cooled quickly and the section of the matrix directly above the water-air interface is farther from the cold top cap and it also has been previously heated by air. Thus, this location has the maximum instantaneous temperature throughout the matrix.

The solid temperature distributions are affected by wall thickness and solid material. The two honeycomb matrices have only small temperature rises in the plates (Figs. 5 and 6), mainly because they are made of aluminum with a large thermal conductivity, which conducts the heat quickly to the cooled section at the water side of the interface. Comparison between the interrupted-plate exchangers of the same geometry but different materials (Fig. 7 vs. Fig. 9 or Fig. 8 vs. Fig. 10) shows that the use of ABS plastic, which has a low thermal conductivity, results in a much larger temperature rise on the plate surface than in the core region of the plate, while the stainless steel results in only a slightly larger temperature rise on the surface than in the core region. This is difference between the ABS exchanger and the metal exchanger is caused by a difference in the conductive thermal resistances. Comparison between the interrupted-plate exchangers of the same material but different shapes (Fig. 7 vs. Fig. 8 or Fig. 9 vs. Fig. 10) shows that, when the plate is thicker, the centerline temperature is lower because it takes more time for the heat to penetrate to the core region from the surface. The thermal conductivity also affects the axial temperature distribution in the plate.
The heat penetration depths from the water-air interface location downwards into the plate section that is immersed in water are different for different materials. As shown in Figs. 5-10, the aluminum plate has the longest heat penetration depth downwards from the water-air interface, the stainless steel plate has the medium depth and the ABS plastic has the smallest penetration depth, due to different thermal conductivities.

The bulk air temperature, or the mass-averaged air temperature, at the final compression state, is computed for each simulation case. As shown in Fig. 11, using small exchanger features (small length scales) for the exchanger plate elements is beneficial for improving heat transfer. However, the Honeycomb1_4 matrix is an exception. Its plates are thin, yet the duct size is too large for such thin plates to have a strong heat transfer capability. The Honeycomb1_8, on the other hand, has a smaller duct size and thinner plates and, thus, has very good heat transfer capability. We can also see that, in general, metal (stainless steel) performs better than ABS plastic in the interrupted-plate exchangers, but the effect of the exchanger shape cannot be overlooked, as the plastic-made PL-I-P-3_2.5_0.4 matrix performs almost as well as the stainless-steel-made Metal-I-P-7.5_2.75_0.55 matrix. Finally, we give the porosity and specific surface area values for these matrices to relate to their heat transfer performances. The porosity and specific area values are respectively, 95.7% and 629.9/m for Honeycomb1_4, 93.6% and 1259.8/m for Honeycomb1_8, 86.2% and 768.9/m for Metal/PL-I-P3_2.5_0.4, and 83.3% and 643.1/m for Metal/PL-I-P-7.5_2.75_0.55. One can immediately see that the low temperature rise with the Honeycomb1_8 matrix is also attributable to its very large specific surface area as a result of fine plate features, while the high air temperature rise with the Honeycomb1_4 matrix is attributable to its high porosity and long fluid-to-solid transfer length.
3.3. Thermodynamic Compression Efficiency

The thermodynamics efficiency will be analyzed for the simulated cases. The goal of compression is to store energy in the form of compressed gas for later work extraction. The storage energy is defined as the amount of work output as the compressed air undergoes an isothermal expansion process. Thus, it is given by

$$E_s = P_0 V_0 \left[ \ln(\zeta) - 1 + \frac{1}{\zeta} \right]$$  \hspace{1cm} (26)

The cost has two parts, the compression work and the cooling work. The compression work during compression from \( t = 0 \) to \( t = t_f \) is given by

$$W_{\text{comp}} = \int_0^{t_f} (P + P_f) U_0 A dt$$  \hspace{1cm} (27)

The instantaneous thermodynamic pressure of the air in the chamber is given by,

$$P = \int \frac{a_1 p dx}{\int \frac{a_1 d x}}$$  \hspace{1cm} (29)

Another compression work term, associated with the term, \( P_r \), is the work done to overcome the resistance of the exchanger matrix. For the interrupted-plate exchanger, the following formulation is used,

$$P_r = \int_0^{t_f} \frac{Q}{K} \epsilon u + \frac{F}{\sqrt{R}} (\epsilon u)^2 \right] dx$$  \hspace{1cm} (30)

where \( K \) and \( F \) are respectively \( 2.764 \times 10^{-7} m^2 \) and \( 0.02210 \) for Metal/PL-I-P-3_2.5_0.4, and \( 3.316 \times 10^{-7} m^2 \) and \( 0.09955 \) for Metal/PL-I-P-7.5_2.75_0.55, according to [16]. For the honeycomb matrices, the following formulation is used,

$$P_r = \int_0^{t_f} 1/2 \rho u^2 f_r dx$$  \hspace{1cm} (31)

where \( f_r \) is given by Eq. (21). Note that Eqns.(30) and (31) are integrated along the entire fluid domain and the fluid properties are mixture properties in line with the VOF approach. After compression, the compressed air cools to the initial temperature; “cooling work” is done to decrease its volume while maintaining its pressure. This cooling work has been addressed in [5 and 15]. It is given by:

$$W_{\text{cool}} = (P_f - P_0) (V_f - V_0) \frac{P_0}{P_f}$$ \hspace{1cm} (32)

Therefore, the efficiency of a compression process is,

$$\eta = \frac{E_s}{W_{\text{cool}} + W_{\text{comp}}}$$  \hspace{1cm} (33)

The compression efficiencies with different exchanger matrices are computed and shown in Fig. 12. Exchangers with lower air temperature rises generally have higher efficiency, because lower temperature rise requires less compression work and has less thermal energy loss during the storage period. The overall difference in efficiency among the simulated exchangers is less than 4%.

![Fig. 12. Comparison of compression efficiencies among different exchanger matrices](image)

4. CONCLUSIONS

A One-D heat exchanger model with a submodel that calculates two-dimensional heat conduction in a representative exchanger plate is developed. The VOF method is used for modeling of the two-phase flow in the liquid piston chamber. The model allows one to quickly and accurately solve for transient temperature distributions in the fluid and solid as well as understanding heat penetration across the plate.

The model is applied to simulate six different exchanger matrices. It is found that the air temperature rises along the chamber’s axial direction from the water-air interface towards the top cap, and the peak air temperature is located very close to the top cap. The temperature distribution in a representative plate element of the exchanger is affected by the plate material and thickness. The instantaneous maximum solid temperature is usually at a location directly above the water-air interface. The temperature distribution across the plate is influenced by the material. With high-thermally-conductive material, the temperature variation across the plate is small; with low-thermally-conductive material, this variation is large, resulting in a significant temperature difference between the plate surface and the core of the plate.

As shown by the simulations, it is often not one single
parameter that determines the heat transfer performance of an exchanger but a group of parameters, including the material, length scales of the plate elements, specific surface area, and porosity. The present study has developed a valuable numerical tool to obtain quick and accurate solutions for heat exchanger matrices while taking all these parameters into account. It is useful for heat exchanger design and analysis.

NOMENCLATURE

\( c_p \) Constant-pressure specific heat
\( D_h \) Hydraulic diameter
\( E_s \) Storage energy
\( F \) Forchheimer coefficient
\( f_r \) Friction coefficient
\( g \) Gravitational acceleration
\( h \) Surface heat transfer coefficient
\( K \) Permeability
\( \kappa \) Thermal conductivity
\( \kappa \) Gauss-Seidel iteration step
\( L \) Chamber length
\( L_d \) Duct length
\( l_c \) Square root of the cross-sectional area of a duct
\( N \) Number of optimization nodes
\( N_{x} \) Number of computation nodes in the x-direction
\( N_{y} \) Number of computation nodes in the y-direction
\( N_{t} \) Nusselt number
\( P \) Averaged thermodynamic pressure of air
\( P_r \) Pressure resistance of the porous inserts
\( p \) Local pressure
\( Pr \) Prandtl number
\( \Phi \) Universal gas constant
\( T \) Local fluid temperature
\( T_0 \) Initial temperature; wall temperature
\( T_s \) Local solid temperature
\( t \) Time
\( t \) Plate thickness
\( U_0 \) Liquid piston velocity
\( u \) Local velocity of fluid mixture
\( W_0 \) Compression work
\( x_p \) Instantaneous location of liquid piston surface

Greek Symbols
\( \alpha \) Volume fraction
\( \varepsilon \) Porosity
\( \zeta \) Air pressure ratio
\( \mu \) Dynamic viscosity
\( \rho \) Density
\( \tau_w \) Wall shear stress

Subscripts
1 Air phase
2 Water phase
c Critical state
f Values at the end of compression
i Node index in the x-direction
j Node index in the y-direction
s Solid

ACKNOWLEDGEMENTS

This work is supported by the National Science Foundation under grant NSF-EFRI #1038294, and University of Minnesota, Institute for Renewable Energy and Environment (IREE) under grant: RS-0027-11. The authors would like to thank also the Minnesota SuperComputing Institute for the computational resources used in this work.

REFERENCE