NUMERICAL MODELING OF THREE DIMENSIONAL HEAT TRANSFER AND FLUID FLOW THROUGH INTERRUPTED PLATES USING UNIT CELL SCALE

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ABSTRACT

Interrupted-plate heat exchangers are used as regenerators for absorbing and releasing thermal energy such as in a Compressed Air Energy Storage (CAES) system in which the exchanger absorbs energy to cool the air being compressed. The exchanger features layers of thin plates in stacked arrays. In a given layer, the plates are parallel to one another and parallel to the exchanger axis. Each successive layer is rotated to have its plates perpendicular to the layer below but still parallel to the exchanger axis. As flow passes from one layer to the next, new thermal boundary layers develop, beneficial to effective heat transfer. The interrupted-plate heat exchanger, also seen as a porous medium, demonstrates strong anisotropic behavior when flow approaches the plates other than axially. Pressure drops and heat transfer coefficients are dependent upon the attack angle. Mathematical models for anisotropic pressure drop and heat transfer are proposed based on numerical experiments on a Representative Elementary Volume (REV), which represents a unit cell of the interrupted-plate medium. The anisotropic pressure drop is modeled by the traditionally used Darcy and inertial terms, with the addition of another term representing mixing effects. Heat transfer between fluid and plates is formulated in terms of Nusselt number vs. Reynolds number and mean flow angle. These models can be used when solving the volume-averaged Navier-Stokes equations for global-scale flow through the interrupted-plate arrays, by assuming that the porous medium region is a continuum. The global-scale analysis is used for the design and optimization of the medium.

KEY WORDS: Porous media, Numerical simulation, Heat exchanger, Convection, Interrupted plate

1. INTRODUCTION

The present study presents modeling of three-dimensional anisotropic heat transfer and flow resistance properties of an interrupted-plate medium that is made for the purpose of heat absorption. The design idea of the present interrupted-plate heat exchanger (heat-absorbing porous medium) originated from a microfabricated segmented-involute-foil regenerator used for a Stirling engine [1, 2]. The segmented-involute-foil structure features layers of thin foils that are stacked perpendicularly to each other. Therefore, as flow passes through the foil layers, new thermal boundary layers are created, which is beneficial to heat transfer. The same idea of generating new thermal boundary layers is applied to the present design of the interrupted-plate medium. A schematic of the interrupted plate medium is shown in Fig. 1. One application of such medium is Compressed Air Energy Storage where it is used in a liquid-piston chamber to absorb heat from air being compressed [3]. Due to its shape, the interrupted plate medium represents a three-dimensional, highly anisotropic porous medium.

Numerical simulations of fluid flow and heat transfer through porous media are usually done by solving volume-averaged transport equations. The volume-averaging technique was introduced to average the Navier-Stokes equations in [4], and was extended to the energy transport equation in [5]. Volume-averaging is performed on a representative elementary volume (REV) of the porous medium and, therefore, the pore-scale activities need not be solved directly by the transport equations; their effects are represented by

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source terms resulting from volume-averaging in the transport equations. The volume-averaging operation is defined by,

\[
\chi^{f} = \frac{1}{V_{REV,f}} \int_{V_{REV,f}} \chi \, dV
\]  

(1)

or,

\[
\chi^{s} = \frac{1}{V_{REV}} \int_{V_{REV}} \chi \, dV
\]  

(2)

where \( \chi \) represents pore-scale velocity, temperature, or pressure. The volume-averaged transport equations are given by,

\[
\rho \nabla \cdot \vec{u}^{f} = 0
\]  

(3)

\[
\rho \frac{\partial \vec{u}^{f}}{\partial t} + \rho \nabla \cdot (\vec{u}^{f} \vec{u}^{f}) = -\nabla p^{f} + \nabla \cdot \vec{e}^{f} + \rho \vec{g} + \vec{S}_{m}
\]  

(4)

\[
\frac{\rho \varepsilon (c_{p} T^{s})}{\partial \theta} + \rho \nabla \cdot (\vec{u}^{f} c_{p} (T^{f} + T^{s} - T^{f})) = \varepsilon \nabla \cdot (k_{f} \nabla T^{f} - \rho \nabla \cdot c_{p} \vec{u}^{f} T^{f} + h_{v} (T^{s} - T^{f}) + \rho \varepsilon \frac{\partial \theta}{\partial t})
\]  

(5)

\[
\rho_{s} (1 - \varepsilon) \frac{\partial}{\partial \theta} (c_{s} T^{s} - \rho \nabla \cdot \vec{u}^{s} T^{s} - h_{v} (T^{s} - T^{f})
\]  

(6)

These governing equations have been used in solving flow and heat transfer problems for applications such as metal foam filled pipes [6], packed beds [7], a double-pipe heat exchanger [8], a ceramic structure in a solar receiver [9], and a metal-foam-filled liquid piston compressor [10]. Volume averaging the transport equations results in the source terms, \( \vec{S}_{m} \), \( h_{v} (T^{s} - T^{f}) \) and \( -\rho \nabla \cdot c_{p} \vec{u}^{f} T^{f} \), which represent respectively: flow resistance, interfacial heat transfer between the fluid and solid, and thermal dispersion. The present study investigates closure models for the momentum source term \( \vec{S}_{m} \), and the interfacial heat transfer term \( h_{v} (T^{s} - T^{f}) \), which are dominant terms in the momentum and energy transport equations.

(a) Global view                          (b) Unit cell

Fig. 1 Interrupted Plate Medium

The momentum source term represents resistance of the porous medium to the flow. It has been traditionally modeled using a viscous and an inertial term [11]. Another term proportional to \( -(\rho \mu |\vec{u}|) \vec{u} \) was added to capture transitional effects based on experiments on steady and oscillating flows through wire screens [12]. The complete model, for an isotropic porous medium, is given by,

\[
\vec{S}_{m} = \frac{\mu \vec{a}_{i}}{K_{f}} - \rho b |\vec{u}| \vec{a}_{i} - \frac{H}{K_{3/4}} (\rho \mu |\vec{u}|)^{1/3} \vec{a}_{i}
\]  

(7)

The interfacial heat transfer term is modeled using dimensionless correlations based on Nusselt, Reynolds, and Prandtl numbers. Such correlations for isotropic open cell porous media and packed beds have been developed in [13-15].
For anisotropic porous media, the viscous effect is modeled by a permeability tensor, which was shown to be a diagonal matrix [16]. The inertial effect is modeled by an inertial tensor, which has non-zero off-diagonal terms that capture effects of velocity components in perpendicular directions on the pressure drop. For an orthotropic medium consisting of repeating square rods, an inertial tensor was modeled in the following way [17].

\[
\bar{b} = \begin{bmatrix}
    b_{f_1} & b_{f_1} \cos \alpha \cos \beta & b_{f_3} \cos \alpha \cos \gamma \\
    b_{f_1} \cos \alpha \cos \beta & b_{f_2} & b_{f_2} \cos \beta \cos \gamma \\
    b_{f_3} \cos \alpha \cos \gamma & b_{f_2} \cos \beta \cos \gamma & b_{f_3}
\end{bmatrix}
\]  

(8)

Numerical simulations on the REV models of anisotropic porous media have been done to determine correlations for the direction-dependent pressure drop and heat transfer [18-19]. Yet, the porous media investigated in these references are two-dimensional and quasi-three-dimensional media, and have distinctively different shapes from the present interrupted-plate medium. The present study will develop closure models for the permeability and inertial tensors, and the interfacial heat transfer correlation for the three-dimensional, anisotropic interrupted-plate medium. The essentials of this model can be applied on other geometries of the general form of Fig. 1.

2. UNIT CELL MODEL

The anisotropic pressure drop and heat transfer of the interrupted-plate medium are determined by using the microscopic flow details obtained from simulations on a unit cell, also called an REV, of the medium. The Reynolds number and flow directions are varied in these simulations. A total of 42 simulation runs have been calculated. In this section, the procedure of carrying out these simulations is discussed. In the following sections, these simulation results are used to characterize the interrupted-plate medium in terms of the direction-dependent hydro-thermal properties. These characterizations are presented by closure models for \( \delta_m \) and \( h_T(\epsilon T_s^s - \partial \psi^f) \) in the momentum and energy transport equations.

2.1 Periodic Flow and Governing Equations  Discussions will be given to the governing equations and boundary conditions for calculating a periodic flow field in an REV of a porous medium. The reason for setting a periodic flow calculation in the REV is that the macroscopic flow in the porous medium is a periodic, fully developed flow after a short entrance region. The flow region, as shown by the transparent region of the REV in Fig. 1. (b), is the computational domain. Three Reynolds numbers are studied in the simulations runs, 1, 181 and 8309. The total mass flow rate through an REV is prescribed according to the Reynolds number, which is based on a characteristic pore size,

\[
Re_L = \frac{\rho |\bar{u}_L| L}{\mu}
\]  

(9)

Laminar flow transport equations are solved when Reynolds number is 1 or 181; the time-averaged transport equations are solved when Reynolds number is 8309. The time-averaged continuity, momentum, and energy equations are given as follows.

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  

(10)

\[
\rho \frac{\partial (u_i u_j)}{\partial x_j} = \frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right] + \frac{\partial}{\partial x_j} \left( -\rho \bar{u}_i \bar{u}_j \right)
\]  

(11)

\[
\rho \frac{\partial}{\partial x_j} \left[ u_j \left( c_p T + \frac{u_i u_i}{2} \right) \right] = \frac{\partial}{\partial x_j} \left[ \kappa \frac{\partial T}{\partial x_j} + u_i \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right] + c_p \mu \frac{\partial \bar{T}}{\partial x_j}
\]  

(12)

The Boussinesq hypothesis is used to relate the Reynolds stress to the mean velocity gradient,

\[
\frac{\bar{u}_i \bar{u}_j}{3} = \mu e \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij}
\]  

(13)
The Transition SST model is used for solving the turbulent viscosity and turbulent kinetic energy. The model couples the SST $k - \omega$ model with two other transport equations, one for the intermittency, and one for the momentum thickness Reynolds number, which is used as a transition onset criterion. Governing equations of the model are developed in [20]. Discussions on the Transition SST modeling equations will be omitted in the present paper.

Simulation runs with different flow angles are set by prescribing different directions of pressure gradient. To discuss three-dimensional vectors, we define some angles. For a mean velocity vector $\vec{u} = (u_x, u_y, u_z)$, its direction can be characterized using either the set $(\alpha, \beta, \gamma)$ or the set $(\phi, \theta)$, which are defined as:

$$
cos \alpha = \frac{u_x}{|\vec{u}|}, \quad cos \beta = \frac{u_y}{|\vec{u}|}, \quad cos \gamma = \frac{u_z}{|\vec{u}|}, \quad tan \phi = \frac{cos \beta}{cos \alpha}, \quad sin \theta = cos \gamma
$$

Schematics of these angles are shown in Fig. 2. Using the same definition, angles (with subscript $p$) can be defined to describe the direction of a pressure gradient vector. In the REV simulations, the pressure gradient is decoupled into a macroscopic (global-scale) pressure gradient, caused by the repeating interrupant of the porous medium to the flow, and a microscopic (pore-scale) pressure gradient, caused by the flow field variation inside an REV. If the macroscopic pressure gradient in the porous medium has a direction defined by the unit vector $(cos \alpha_p, cos \beta_p, cos \gamma_p)$, and the microscopic pressure field is represented by $\mathcal{p}$, then the total pressure gradient is:

$$
\frac{\partial \mathcal{p}}{\partial x_i} = \left[ \frac{\partial \mathcal{p}_m}{\partial x_i} \right] (cos \alpha_p, cos \beta_p, cos \gamma_p) + \frac{\partial \mathcal{p}}{\partial x_i}
$$

In the computation, the pressure gradient term in Eq. (11) is substituted by Eq. (14). Angles $\alpha_p$, $\beta_p$ and $\gamma_p$ are varied in different simulation runs. The pore-scale pressure gradient, $\frac{\partial \mathcal{p}}{\partial x_i}$, and the magnitude of the global-scale pressure drop $\left| \frac{\partial \mathcal{p}_m}{\partial x_i} \right|$ are calculated by solving the continuity and momentum equations iteratively.

![Fig. 2](image1.png)

**Fig. 2** Schematic of two sets of angles used to characterize the direction of a vector

Periodic boundary conditions are applied on the inlet and outlet boundaries of the REV domain,

$$
\Gamma|_{x=-\ell} = \Gamma|_{x=\ell}, \quad \Gamma|_{y=-\ell} = \Gamma|_{y=\ell}, \quad \Gamma|_{z=-\ell} = \Gamma|_{z=\ell}, \quad \Gamma = u_i, \mathcal{p}
$$

On the interfacial boundary between the fluid and solid, no-slip velocity and uniform-wall-heat-flux boundary conditions are applied. The imposed uniform-wall-heat-flux $q''$ causes a macroscopic temperature drop $\frac{\partial \mathcal{q}_m}{\partial x_i}$, along the macroscopic flow direction. The macroscopic flow direction can be calculated from the continuity and momentum equations. Let this direction be defined by unit vector, $(cos \alpha, cos \beta, cos \gamma)$, then the periodic thermal boundary condition dictates,
\[ \tau_{i=1} - \tau_{i=2} = \frac{q' r a y}{\rho c_l |\sigma|^2} \cos \alpha \]  

\[ \tau_{y=\phi+\frac{\pi}{2}} - \tau_{y=-\phi+\frac{\pi}{2}} = \frac{q' r a y}{\rho c_l |\sigma|^2} \cos \beta \]  

\[ \tau_{x=\phi+\frac{\pi}{2}} - \tau_{x=-\phi+\frac{\pi}{2}} = \frac{q' r a y}{\rho c_l |\sigma|^2} \cos \gamma \]  

In the calculations, the uniform heat flux has a negative value, which physically means that the fluid is heating the plate. The bulk temperatures at all inlet boundaries are defined as constant. Geometric features of the interrupted plate medium are:

\[ \ell = 7.5 \text{mm}, \; 2\delta = 2.75 \text{mm}, \; t = 0.55 \text{mm}, \; \epsilon = 0.8333, \; a_v = 643.1/m, \; L = 4.0833 \text{mm} \]

2.2 Numerical Procedure The transport equations are solved by the finite volume method using the commercial CFD software ANSYS Fluent. The first-order implicit formulation is used for transient discretization; the second-order upwind scheme is used for discretizing advective terms; central differencing is used for discretizing diffusive terms. The SIMPLE algorithm [21] is used for pressure-velocity coupling. The convergence criteria for residuals of all equations is set to \(10^{-9}\). The computational domain is discretized into rectangular hexahedral cells. The cell size gradually decreases as the walls are approached. The total numbers of grid cells, respectively for 181 and 8309 Reynolds numbers are: 707,840 and 1,469,139. Twenty different flow directions are computed for each of these two Reynolds numbers. The maximum value of the dimensionless wall distance \((y^+)\) of the first cell adjacent to the wall is between 2 and 6 for simulations with 8309 Reynolds number. In addition, two flow directions, one along the \(x\) axis and one along the \(y\) axis, are computed for a Reynolds number of 1, using 707,840 grid cells.

For grid-independence verification, the CFD run with an 8309 Reynolds number and with mean flow in the \(x\)-axis direction is studied after refining the mesh to 2,584,833 cells. The calculated Nusselt number and pressure drop (normalized on two times the mean flow kinetic energy) for 1,469,139 and 2,584,833 grid cell simulations are, respectively: 180.6 and 182.4, -0.0956 and -0.0969.

3. FLOW RESISTANCE

In this section, an anisotropic pressure drop model is developed for the interrupted plate, based on data from CFD runs of different flow approaching angles and Reynolds numbers. First, a theoretical background on the modeling will be given. Then, detailed discussions will be given on resolving the coefficients in the modeling equation. Reference [17] proposed a symmetric inertial tensor (Eq. (8)) for the orthotropic porous medium investigated. A more generalized form of the inertial tensor is,

\[ \bar{b} = \begin{bmatrix} b_{11} & b_{12} \cos \alpha \cos \beta & b_{13} \cos \alpha \cos \gamma \\ b_{21} \cos \alpha \cos \beta & b_{22} & b_{23} \cos \beta \cos \gamma \\ b_{31} \cos \alpha \cos \gamma & b_{32} \cos \beta \cos \gamma & b_{33} \end{bmatrix} \]  

The physical reason for this form of tensor is explained in the following. To facilitate discussion, let subscripts \(i\) and \(j\) stand for different directions along Cartesian axes. The diagonal element captures the relation between the inertia and pressure drop of the same \(i\)th or \(j\)th directions, and are therefore independent of flow angle. An off-diagonal element, \(b_{ij} \) (\(i \neq j\)) of this tensor represents the effect of the \(j\)th directional inertia on the pressure drop in the \(i\)th direction, and, therefore, the element, \(b_{ij} \) (\(i \neq j\)), depends on velocity components in the \(i\)th and \(j\)th direction. For example, when \(i = 1\) and \(j = 2\), the element \(b_{12}\) is represented by a coefficient \(b_{12}\) multiplied by dimensionless velocity components in the \(x\) and \(y\) directions, given respectively by \(\cos \alpha\) and \(\cos \beta\). After multiplying the inertia tensor with velocity vector, the element \(b_{12}\) results for application in a pressure drop term in the \(x\) direction. This term is caused by the inertia of the \(y\)-velocity component \((u_2)^2\):
\[ S_{m,\text{int},12} = -\rho b_{12} \cos \alpha \cos \beta |\vec{u}| u_2 = -\rho b_{12} \frac{a_i}{|\vec{u}|} u_2^2 \quad (19) \]

It is expected also that as the \( x \) directional velocity component, \( \frac{a_i}{|\vec{u}|} \), increases, the more effectively \( u_2^2 \) results in a pressure drop component in the \( x \) direction. In a generalized form, the pressure drop in the \( i \)th direction that is caused by the inertial effect in the \( j \)th direction (\( i \neq j \)) should be given by:

\[ S_{m,\text{int},ij} = -\rho b_{ij} \frac{a_i}{|\vec{u}|} u_j^2 \quad (20) \]

This justifies the expression of off-diagonal elements in the tensor \( \bar{b} \) as given by Eq. (18). Furthermore, due to the special geometric feature of the interrupted plates, identical periodicities are along the \( Oy \) and \( Oz \) directions (see Fig. 2), which dictates that:

\[ b_{13} = b_{12} \quad b_{31} = b_{21} \quad b_{23} = b_{22} \quad b_{33} = b_{22} \]

Therefore, the inertial tensor has the form:

\[ \bar{b} = \begin{bmatrix} b_1 & b_{12} \cos \alpha \cos \beta & b_{12} \cos \alpha \cos \gamma \\ b_{21} \cos \alpha \cos \beta & b_2 & b_{23} \cos \beta \cos \gamma \\ b_{21} \cos \alpha \cos \gamma & b_{23} \cos \beta \cos \gamma & b_2 \end{bmatrix} \]

In the study of [12], the term \( -\frac{H}{K^{3/4}} (\rho \mu |\vec{u}|) \hat{z} |\vec{u}| \) (see Eq. (7)) is added in the expression to capture the transitional and mixing effects of flow through wire screen. This term can be rewritten as,

\[ -\frac{H}{K^3 \beta} (\rho \mu |\vec{u}|) \hat{z} |\vec{u}| = -\rho \frac{H^{3/2}}{K^{3/2}} Re^{-0.5} |\vec{u}| |\vec{u}| \quad (21) \]

To model anisotropic behavior, a term of similar meaning, but in a tensor form, must be developed. The interrupted plate medium has long, flat plates, the effects of which on the flow are substantially different from those of thin wires. When the mean flow is not parallel to the plates, there could be strong mixing effects, much more significant than when the mean flow is parallel to one of the plates. Therefore, for the interrupted-plate medium, this tensor that captures mixing effects is proposed to have zero diagonal components and non-zero off-diagonal components. Without loss of generality, we use \( H \) to represent the term, \( -\frac{H^{3/2}}{K^{3/2}} \). The pressure drop in the \( i \)th direction that is caused by mixing of the \( j \)th and \( i \)th velocity components is proposed to have the following general form:

\[ S_{m,\text{mix},ij} = -\rho H_{ij} \frac{a_i}{|\vec{u}|} u_j \quad (22) \]

In addition, using the same reasoning of identical periodicities along the \( Oy \) and \( Oz \) directions, the pressure drop term caused by anisotropic mixing effects can be modeled as:

\[ \tilde{S}_{m,\text{mix}} = -\rho \begin{bmatrix} 0 & H_{12} R_{L}^{m12} \cos \alpha & H_{12} R_{L}^{m12} \cos \alpha \\ H_{21} R_{L}^{m21} \cos \beta & 0 & H_{23} R_{L}^{m22} \cos \beta \\ H_{21} R_{L}^{m21} \cos \beta & H_{23} R_{L}^{m22} \cos \beta & 0 \end{bmatrix} |\vec{u}| |\vec{u}| \quad (23) \]

The tensor, which multiplies the velocity vector on the RHS of Eq. (23), will be referred to as a mixing-effect tensor. After non-dimensionalizing pressure, velocity, and coordinates using respectively, \( \rho |\vec{u}|^2 \), \( |\vec{u}| \), and \( L \), the pressure drop due to resistance of the interrupted-plate medium is
\[ \nabla^* p^* = \frac{L}{\rho |\mathbf{u}|^2 \nabla p} \]

\[
\begin{bmatrix}
\frac{L^2}{K_1} & 0 & 0 \\
0 & \frac{L^2}{K_2} & 0 \\
0 & 0 & \frac{L^2}{K_2}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_1^* \\
\mathbf{u}_2^* \\
\mathbf{u}_3^*
\end{bmatrix}
- \begin{bmatrix}
Lb_1 \\
Lb_2 \\
Lb_3
\end{bmatrix}
\begin{bmatrix}
\mathbf{L}_{12} \cos \alpha \cos \beta \\
\mathbf{L}_{12} \cos \beta \cos \gamma \\
\mathbf{L}_{23} \cos \alpha \cos \beta \\
\mathbf{L}_{23} \cos \beta \cos \gamma \\
\mathbf{L}_{33} \\
\mathbf{L}_{33}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_1^* \\
\mathbf{u}_2^* \\
\mathbf{u}_3^*
\end{bmatrix}
- \frac{1}{Re} \begin{bmatrix}
H_{12} Re_{L}^{m_{12}} \cos \alpha \\
H_{12} Re_{L}^{m_{12}} \cos \alpha \\
H_{23} Re_{L}^{m_{23}} \cos \beta \\
H_{23} Re_{L}^{m_{23}} \cos \beta \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_1^* \\
\mathbf{u}_2^* \\
\mathbf{u}_3^*
\end{bmatrix}
\]

(24)

3.1 Flow along the Ox and Oy Axes  The Ox and Oy coordinates (see Fig. 2) are aligned with the plates. When the mean flow, \( \langle \mathbf{u} \rangle \), is along either of these directions, the pressure gradient points to the same direction as the mean flow direction. Thus, studying flows in these two directions can resolve the diagonal components of the tensors representing viscous and inertial effects.

Three CFD runs are computed for each of the Ox and Oy directions. The Reynolds numbers are 1, 181, and 8309. The velocity vectors on the boundary surfaces of the REV domain and the velocity streamlines are shown in Figs. 3 and 4 respectively for the mean flow going in the Ox and Oy directions. Both figures show flow fields with Reynolds number equaling 8309. 因为板的厚度相对较小，当平均流沿 Ox 方向流动时，大部分局部流体运动也在同方向；只有很小的区域在前缘和后缘附近的滞留区才会被看到。当平均流沿 Oy 方向流动时，板面在大部分 REV 区域受到垂直影响，导致强烈的滞留区域，但对不接触板面的地方影响不大。

(a) Velocity vectors on REV boundary surfaces  
(b) Streamlines and wall temperature 

Fig. 3 Velocity and wall temperature distribution \((Re_L = 8309, \theta = 0, \varphi = 0)\)

In both of these cases, the pressure drop is caused only by the diagonal terms in the resistance tensors representing viscous and inertial effects. The pressure drop expressions are:

\[
\left( \frac{\partial p^*}{\partial x} \right)_{Ox} = -\frac{1}{Re L K_1} Lb_1
\]

(25)

\[
\left( \frac{\partial p^*}{\partial y} \right)_{Oy} = -\frac{1}{Re L K_2} Lb_2
\]

(26)
Computed pressure drop data from a total of six CFD runs with mean flows along the $Ox$ and $Oy$ directions are used to find the permeability and inertial coefficients in Eqns. (25) and (26). By applying least square fit, the results are:

\[ K_1 = 4.79 \times 10^{-7} \text{m}^2, \quad K_2 = 2.59 \times 10^{-7} \text{m}^2, \quad b_1 = 24.69/\text{m}, \quad b_2 = 23.85/\text{m} \]

![Velocity vectors on REV boundary surfaces](a) (b) Streamlines and wall temperature

**Fig. 4** Velocity and wall temperature distribution ($Re_L = 8309, \theta = 0, \phi = 90^\circ$)

### 3.2 Flow Parallel to the yOz Plane

The effect of the $y$-velocity component on the pressure drop in the $z$ direction is revealed by studying situations in which the mean flow is approaching at an angle between the $Oy$ and $Oz$ axes while being parallel to the $yOz$ plane. Note that the effect of the $y$-velocity component on the pressure drop in the $z$ direction is the same as the effect of the $z$-velocity component on the pressure drop in the $y$ direction, due to identical geometrical periodicity in the $Oy$ and $Oz$ directions.

Four CFD runs are computed with different mean flow angles for each of the two Reynolds numbers, 181 and 8309. The velocity vectors on the boundary surfaces of the REV domain and the velocity streamlines for the mean flow going in the direction of $\theta = 64^\circ, \phi = 90^\circ$ with 8309 Reynolds number are shown in Fig. 5. Even though the mean flow is at an angle between the $Oy$ and $Oz$ directions, most of the local fluid in the REV tends to flow parallel to the plates, instead of following the mean flow direction.

![Velocity vectors on REV boundary surfaces](a) (b) Streamlines and wall temperature

**Fig. 5** Velocity and wall temperature distribution ($Re_L = 8309, \theta = 64^\circ, \phi = 90^\circ$)

Studying flows at different approaching angles, which are all parallel to the $yOz$ plane, allows one to resolve the lower right, 2 by 2 sub-matrices in Eq.(24). The pressure drop equation for these flow situations
is characterized by:

\[
\begin{bmatrix}
\frac{\partial p^*}{\partial y^*} \\
\frac{\partial p^*}{\partial z^*}
\end{bmatrix}
_{yoz} = -\frac{1}{Re_L} 
\begin{bmatrix}
\frac{L^2}{K_2} & 0 \\
0 & \frac{L^2}{K_2}
\end{bmatrix}
\begin{bmatrix}
u_{2y} \\
\nu_{3z}
\end{bmatrix} - \begin{bmatrix}
Lb_2 & Lb_{23}\cos\beta\cos\gamma \\
Lb_{23}\cos\beta\cos\gamma & Lb_2
\end{bmatrix}
\begin{bmatrix}
u_{1x} \\
\nu_{3z}
\end{bmatrix}
- \frac{0}{H_{23}Re_L^{m_{23}}\cos\beta} \begin{bmatrix}
u_{2y} \\
\nu_{3z}
\end{bmatrix}
\]

(27)

Computed pressure drop values from the eight CFD runs with mean flows all parallel to \( yOz \) plane follow Eq. (27). Using a least square fit, the coefficients can be calculated:

\[
b_{23} = -16.16/m, \quad H_{23} = 4.846\times10^{-2}Re_L^{0.08914}
\]

3.3 Flow Parallel to the \( xOy \) Plane

The effects of the \( y \)-velocity component on the pressure drop in the \( x \) direction, as well as the \( x \)-velocity component on the pressure drop in the \( y \) direction are revealed by studying flow fields where the mean flow direction are characterized by different \( \phi \) angles and are all parallel to the \( xOy \) plane. Six CFD runs with different mean flow directions are computed under these situations for each of the two Reynolds numbers, 181 and 8309.

The velocity vectors on the boundary surfaces of the REV domain and the velocity streamlines of the case with the mean flow going in the direction of \( \theta = 0 \) and \( \phi = 18^\circ \) with 8309 Reynolds number are shown in Figs. 6. Next, compare Fig. 6. (b) and Fig. 5. (b). In both cases, the mean flows are directed to approach at an angle towards the plates. The difference is: in Fig. 5. (b), the fluid is allowed to go in both \( Oy \) and \( Oz \) directions and, therefore, most of the fluid in the REV is able to escape from direct impinging onto the plate to going parallel along the plates in \( Oy \) and \( Oz \) directions; in Fig. 6. (b), dictated by the mean flow direction, the fluid in the REV cannot go along the \( Oz \) direction, therefore, part of the fluid has to impinge on the vertical plates as shown in Fig. 6. (b), creating a strong separation zone behind these plates, which leads to large pressure drop in the \( Ox \) direction. As a result of this difference, the magnitudes of the coefficients \( b_{12} \) and \( b_{21} \) are larger than the magnitude of \( b_{23} \). The same comparison can be made for \( H_{12} \), \( H_{21} \) and \( H_{23} \).

![Velocity vectors on REV boundary surfaces](image1)

![Streamlines and wall temperature](image2)

**Fig. 6** Velocity and wall temperature distribution \((Re_L = 8309, \theta = 0, \phi = 18^\circ)\)

Studying flows with different approach angles that are all parallel to the \( xOy \) plane allows resolving the upper left 2 by 2 sub-matrices in Eq.(24). The pressure drop equation for these flow situations reduces to:
\[
\begin{bmatrix}
\frac{\partial p^*}{\partial x^*} \\
\frac{\partial p^*}{\partial y^*}
\end{bmatrix}_{x_0 y} = - \frac{1}{Re_L} \begin{bmatrix}
\frac{L^2}{K_1} & 0 \\
0 & \frac{L^2}{K_2}
\end{bmatrix} \begin{bmatrix}
\langle u_1^* \rangle \\
\langle u_2^* \rangle
\end{bmatrix} - \begin{bmatrix}
Lb_1 & Lb_1 \cos \beta \cos \gamma \\
Lb_2 \cos \beta \cos \gamma & Lb_2
\end{bmatrix} \begin{bmatrix}
\langle u_1^* \rangle \\
\langle u_2^* \rangle
\end{bmatrix}
\]

\[
- \begin{bmatrix}
H_{21} R_{m2}^L \cos \beta \\
0
\end{bmatrix} \begin{bmatrix}
\langle u_1^* \rangle \\
\langle u_2^* \rangle
\end{bmatrix}
\]

(28)

Results from these twelve CFD runs are used to calculate the coefficients by least square fitting. They are:

\[
b_{12} = -63.12/m, \quad b_{21} = 221.61/m, \quad H_{12} = 0.6252 R_{21}^{-0.04198}, \quad H_{21} = 2.271 R_{21}^{0.02699}
\]

3.4 Flow with Mixed Angles In the previous discussions, CFD runs on the REV have been made to investigate different situations when the mean flow is parallel to one of the plates. Additional CFD runs are made for cases in which the mean flow in the REV is with a mixed angle, which means that neither \( \phi \) nor \( \theta \) is 0 or 90°. One of such cases is shown in Fig. 7. Even though the mean flow is pointing at a specific direction, most of the local fluid inside an REV may not follow this direction, and may go in very different directions to avoid as much direct impingement on the wall as possible. This feature is beneficial for heat transfer, as the amount of mixing that leads to heat transfer is determined by the complex pore-scale fluid activities.

Twenty cases on the REV with different mean flow directions have been computed for Reynolds numbers of 181 and 8309. The viscous, inertial, and mixing coefficients have been obtained using data from REV simulations. The analytical model for anisotropic pressure drop, Eq. (24), has been closed. The predicted solution calculated from the model and the solutions from all REV simulations are plotted in Fig. 8 for comparison. Overall, the model captures anisotropic pressure drop in the interrupted-plate medium.

From these simulation results, the following features of the anisotropic pressure drop of the interrupted-plate medium can be observed. For a very small \( \theta \), as \( \phi \) increases from 0 to 90°, (1) the \( x \)-direction pressure drop increases slightly when \( \phi \) reaches around 30°, due to an increase in viscous effect in the \( x \)-direction, and then decreases to 0 as \( \phi \) increases to 90°, due to decrease in the \( x \)-direction inertial effect, and (2) the \( y \)-direction pressure drop increases as \( \phi \) increases from 0 to around 45°, due to increasing of flow separation caused by flow impinging on the plate, and then decreases to smaller values, due to an increasing size of the flow stagnation region. For a large \( \theta \), the effects of changing \( \phi \) on the pressure drop values in the \( Ox \) and \( Oy \) directions are small because the local fluid in the REV is allowed to go in the \( Oy \) direction to avoid direct impingement onto the plate, which would otherwise lead to flow separation or stagnation. For a very small values of \( \phi \), as \( \theta \) increases from 0 to 90°, the \( z \)-direction pressure drop first...
increases and then decreases, due to impingement of flow onto the plates first causes separation and then stagnation. For a large value of $\phi$, the effect of $\theta$ on the $z$-direction pressure drop is small because local fluid in the REV can avoid directly impinging upon the plates while maintaining the mean flow direction.

Fig. 8 Comparison of anisotropic pressure drop between model (curved surfaces) and CFD solutions (points)

4. INTERFACIAL HEAT TRANSFER

Results of the REV calculations are analyzed for interfacial heat transfer between the fluid and the wall. The wall temperature distributions for different mean flow directions are shown in Figs.3 – 7 (b). Because the wall has a uniform negative heat flux, the fluid is heating the wall and, thus, locations on the wall with high temperature values indicate high local heat transfer coefficient. Observing these figures, one sees that wall heat transfer effect is strong next to the regions where the flow is active. The weakest local heat transfer effect on the wall is found next to the stagnant fluid regions in Fig. 4 (b). For particular mean flow directions, local fluid inside an REV at certain location can go in a variety of directions leading to complex mixing effects that enhance heat transfer, such as the case shown by Fig. 7 (b).

In this section, we develop a correlation for anisotropic heat transfer. The Nusselt number is a function of Reynolds number, as well as angles that characterize the mean flow direction; physically, these angles relate
the pore-scale mixing effect to the macroscopic heat transfer. We first define a volumetric heat transfer coefficient,

\[
h_v = \frac{q^{\text{avg}}}{\int_A \frac{h_{\text{wall}} T dA}{T_f - T_L}}
\]  

(29)

Then, the Nusselt number is defined as,

\[
Nu = \frac{h_v L^2}{k}
\]  

(30)

Studies in [17, 18] used the following expression to model anisotropic heat transfer.

\[
Nu = \left( c_1 n^2 \cos^2 \alpha + c_2 n^2 \cos^2 \beta + c_3 n^2 \cos^2 \gamma \right) \frac{1}{Pr} + \left( d_1 n d \sin^2 \gamma + d_2 n d \cos^2 \gamma \right) \frac{1}{Pr^{0.6}} \text{Re}^{0.6} \text{Pr}^{0.6}
\]  

(31)

The term \( \left( c_1 n^2 \cos^2 \alpha + c_2 n^2 \cos^2 \beta + c_3 n^2 \cos^2 \gamma \right) \frac{1}{Pr} \) represents conduction with very small Reynolds number. In the present study, focus is given to relatively high Reynolds numbers; thus, the conduction term is modeled as independent of the mean flow direction. Some of the porous media heat transfer correlations found in [14] have neglected the conduction effect for convenience of normalizing the Nusselt number on Prandtl number. In the present study, we approximately model the conduction term to make it proportional to \( Pr^{0.6} \) and independent of mean flow direction. The second term on the RHS of Eq. (31), which depends on Reynolds number and mean flow angles, represents heat transfer due to fluid inertial and mixing effects. For the two-dimensional and quasi-three-dimensional porous media studied in [16-17], angle \( \gamma \) dominates. In the present study, a more generalized form for this term is adopted by including all independent angles using trigonometric functions in a quadratic form. Also, considering the fact that angles \( \varphi \) and \( \theta \) have similar effects on volumetric heat transfer, the proposed heat transfer model for the interrupted plate medium is,

\[
\frac{Nu}{Pr^{0.6}} = c + d_1 (cos^2 \varphi + cos^2 \theta) \text{Re}^{n_{d_1}} + f_1 (sin^2 \varphi + sin^2 \theta) \text{Re}^{n_{f_1}} + d_2 (cos \varphi + cos \theta) \text{Re}^{n_{d_2}} + f_2 (sin \varphi + sin \theta) \text{Re}^{n_{f_2}}
\]  

(32)

Using results from forty REV simulations for two Reynolds numbers, 181 and 8309, we calculate the coefficients in Eq. (32) by a least-square fit. They are:

\[
c = 7.012, \quad d_1 = -2.766, \quad f_1 = -0.530, \quad d_2 = 2.685, \quad f_2 = 0.444
\]

\[
n_{d_1} = 0.387, \quad n_{f_1} = 0.637, \quad n_{d_2} = 0.476, \quad n_{f_2} = 0.682
\]

Results calculated from the model are compared with CFD solutions in Fig. 9.
The directional heat transfer of the interrupted plate medium has the following characteristics. (1) For a given $\theta$, (especially for one between 0 and 65°), heat transfer increases as $\varphi$ increases from 0 to around 40°, and then decreases as $\varphi$ continues to reach 90°. The reason for the increase in heat transfer when $\varphi$ increases from 0 to around 40° is the increasing effect of flow impinging on the plates leading to increasingly complex pore-scale fluid movement in different directions. One such representative case is shown by Fig. 6 (b). The reason for the decrease in heat transfer when $\varphi$ increases from 40° to around 90° is that impingement of flow at a more and more directed angle onto the plate leads to more and more fluid stagnating. One such representative case is shown by Fig. 4 (b), where the flow is impinging on the plates at a perpendicular angle causing large stagnation zones, thus less heat transfer. (2) Due to identical geometric periodicity in the $Oy$ and $Oz$ directions, for a given $\varphi$, (especially between 0 and 65°), heat transfer also increases as $\theta$ increases from 0 to around 40° and then decreases as $\theta$ continues to reach 90°. (3) Because of the aforementioned aspects, the highest heat transfer effect occurs when the mean flow approaches in a direction with both $\varphi$ and $\theta$ being around 40°. The reason for this is that at this direction, the pore-scale fluid is most agitated, forming complex fluid movement in many different directions inside the REV, even though the mean flow is pointing in one specific direction. This situation is similar to the flow field illustrated in Fig. 7 (b). The heat transfer is enhanced by the agitated pore-scale fluid.

5. CONCLUSIONS

The present study investigates anisotropic pressure drop and heat transfer for an interrupted plate medium. It is done with numerical simulations. The results could be used in other heat exchangers generally of the geometry of Fig. 1. Flows through the REV of the medium with different mean flow angles are simulated. Because the medium consists of long, flat plates, impingement of flows at different angles onto the plates, which lead to substantially different pore-scale fluid activity are responsible for the features observed in the directional effects on pressure drop and heat transfer. The mean flow angle is characterized by $\theta$, the angle between the mean flow and its projection on the $xOy$ plane, and $\varphi$, the angle between the projected vector on the $xOy$ plane and $Ox$ axis. The directional pressure drop is largest when either $\theta$ or $\varphi$ is around 45° while the other is 0. The reason for this is that the largest flow separation effect is realized at this mean flow direction. If either $\theta$ or $\varphi$ is smaller than 45° while the other is 0, the flow separation effect is smaller; if either $\theta$ or $\varphi$ is greater than 45° while the other is 0, stagnation zones in the REV play important roles in suppressing fluid movement and, thus, reducing pressure drop; if either $\theta$ or $\varphi$ is around 45° while the other is greater than 0, the pore-scale fluid attempts to run parallel to the plate to avoid direct impingement on the plate, leading reduced flow separation effects. Heat transfer is greatest when both $\theta$ and $\varphi$ are around 40°. The reason for this is, again, due to the interaction between fluid and plates, the pore-scale fluid is agitated the most at this mean flow direction; the fluid inside an REV runs in many different possible directions, resulting in enhanced heat transfer. A particular point worth mentioning is that for a given Reynolds number, the largest heat transfer and the largest pressure drop do not occur for the same mean flow direction. This means that in heat exchange applications, heat transfer can be maximized by carefully positioning the interrupted plate medium at a particular angle such that the mean flow has approach angles, $\theta$ and $\varphi$, of around 40°, with reduced cost in maximum pressure resistance.

NOMENCLATURE

- $a_v$: surface area per volume (m$^{-1}$)
- $b$: inertial coefficient (m$^{-1}$)
- $b_f$: inertial coefficient (m$^{-1}$)
- $b_{f1}$: anisotropic inertial coefficient for x direction (m$^{-1}$)
- $b_{f2}$: anisotropic inertial coefficient for y direction (m$^{-1}$)
- $b_{f3}$: anisotropic inertial coefficient for z direction (m$^{-1}$)
- $c$: conduction effect (-)
- $d_1$, $d_2$: coefficients for mixing on heat transfer (-)
- $f_1$, $f_2$: heat transfer
- $\bar{b}$: anisotropic inertial matrix (m$^{-1}$)
- $\delta$: half distance between plates (m)
- $H$: pressure resistance term due to mixing effects (-)
- $h_v$: volumetric heat transfer coefficient W/(m$^3$K)
- $K$: permeability (m$^2$)
- $L$: representative pores size (m)
- $\ell$: plate length (m)
- $m$: Reynolds number exponent (-)
characterizing anisotropic mixing on pressure drop

Reynolds number exponent

characterizing anisotropic mixing on heat transfer

periodic pore-scale pressure (Pa)

momentum source term (Pa/m)

thickness (m)

angle between mean velocity vector and x axis (-)

angle between mean velocity vector and y axis (-)

angle between mean velocity vector and z axis (-)

porosity (-)

angle between mean velocity vector and xOy plane

thermal conductivity (W/(mK))

angle between the projection of mean velocity vector on the xOy plane and x axis

a flow variable (velocity, temperature or pressure) or Pa


t

\(\alpha\)

\(\beta\)

\(\gamma\)

\(\epsilon\)

\(\theta\)

\(\kappa\)

\(\varphi\)

\(\chi\)

\(\text{REV}\)

\(\text{REV}, f\)

\(s\)

\(*\)

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references


