Energetically Passive Multi Degree-of-Freedom Hydraulic Human Power Amplifier with Assistive Dynamics

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Abstract—This paper develops a control approach for a multi-degree of freedom (DoF) hydraulic human power amplifier (HHPA) which is a tool with which a human interacts physically. The control objective is to use the hydraulic actuators to amplify the human force while ensuring that the closed loop system is energetically passive with respect to a scaled human and environment input power, so as to enhance safety and coupling stability. A multi-DoF virtual velocity coordination approach is used to recast the force control problem as one of coordination between the velocities of a fictitious inertia and of the HHPA. To aid the human to perform specific tasks, additional passive assistance dynamics are incorporated to include a passive velocity field controller (PVFC) for path guidance, and an artificial potential field for obstacle avoidance. Control of the hydraulic actuators is defined by a passivity based controller which considers the natural energy storage of the hydraulic actuators to fully account for the nonlinear pressure dynamics. The controllers specify the required flows which are then satisfied by either a servo-valve or an energy efficient hydraulic transformer. Experimental results demonstrate good control performance and effective task assistance.

Index Terms—Human power augmentation, exoskeleton, energetic passivity, passive decomposition, hydraulic transformer, natural energy storage, passive velocity field control, obstacle avoidance.

I. INTRODUCTION

THE goal of the human power amplifier is to enable a human operator to physically interact with the machine as if it is an extension of his/her body while amplifying the applied human effort. The control objective is similar to that of a wearable exoskeleton except that the operator uses the machine as a tool that he/she can hold onto or let go of whenever he/she desires. In both cases, because of the physical connection to the machine, direct haptic and motion feedback are provided to the operator for intuitive operation while reducing the physical efforts required.

Since the HHPA interacts physically with both the human operator and its physical environment, coupling stability and safety of both human and physical environments are paramount. One approach to improving safety is to impose on the control so that the machine behaves like an energetically passive device (with an appropriate power scaling) to its human and the physical environments (formal definition will be provided in section II-C). An energetically passive system is one that only stores and dissipates (scaled) energy but does not generate energy of its own. Since most physical environments and humans can be considered energetically passive systems [1], the physical interactions with these environments are guaranteed to be stable by the passivity theorem which states that the coupling between a passive and a strictly passive system is necessarily stable [2], [3]. Even with active environments, the coupled system can only become unstable if the environments generate an infinite amount of energy.

To be powerful, hydraulic actuation is preferred to electric actuation because hydraulic actuators are typically an order of magnitude more force/torque dense and power dense than electric actuators. In this paper, we develop the control for a two degree-of-freedom hydraulic human power amplifier (HHPA) setup in Fig. 1. An issue with hydraulic actuation, however, compared to electric motors, is that its force/torque are not directly controlled but are the result of pressure dynamics which are in turn controlled by the flows into the actuators.

The contributions of this papers are:

1) development of a virtual coordination approach for nonlinear multi-DoF HHPA that fully accounts for the nonlinear nature of the compressible hydraulic actuation that ensure energetic passivity;
2) incorporation of passive dynamics for task guidance and obstacle avoidance;
3) experimental validation of the control implemented using an energy efficient hydraulic transformer instead of a high bandwidth but inefficient servo-valve.

In the virtual velocity coordination approach first proposed in [4], a hydraulic actuator is modeled as a combination of an ideal velocity source and a nonlinear spring. The latter captures the compressibility effects of the fluid medium. Instead of controlling the actuator to track the desired force directly, which was shown to lack robustness [5], the controller coordinates the velocities of the system and of a fictitious virtual mass whose dynamics are influenced by the hydraulic actuator and...
the human force. The control law for achieving coordination is accomplished via a passive decomposition [6]–[8] into a shape system and a locked system. This approach is more robust as the controller structure itself enforces the passivity property. This approach was also applied to a hydraulically actuated patient transfer device in [9].

In the present paper, the control of HHPA is extended beyond our early work in [4] in several ways. Firstly, in [4], the control was developed for each individual degree-of-freedom assumed to be decoupled from each other. Here, the fully coupled dynamics of a multi-DoF HPA are considered. Secondly, we develop human-machine shared control strategies by rendering useful passive dynamics to assist the human to execute specific tasks more easily. In particular, guidance for moving in preferable directions, such as to follow a contour, is achieved by incorporating the Passive velocity field controller (PVFC) [10]; and obstacle avoidance is achieved by incorporating potential fields [11] to prohibit the machine from entering prohibited zones. These assistive dynamics are imposed while ensuring that the HHPA remains energetically passive. Thirdly, in [4], the hydraulic actuator is modeled as a combination of an ideal kinematic actuator and an empirically defined “nonlinear spring” that represents the fluid compressibility; and a linear affine parameterization based control is used. In the present paper, the spring model is replaced by a physical model of the fluid compressible energy which properly accounts for the volume variation in the actuator as well as the pressure dependent bulk modulus (if so desired) [12]. The use of this model results in a more robust, nonlinear passivity based shaped system control. In addition, a modification of the virtual coordination control is proposed in this paper to robustly enforce energetic passivity. Finally, the control approach is demonstrated experimentally with an energy efficient but lower bandwidth hydraulic transformer [13], [14] rather than with a servo-valve as in our previous work [4].

Of the exoskeletons in the open academic literature (see [15], [16] for reviews), the Berkeley Lower Extremity Exoskeleton (BLEEX) [17] is one of the most prominent. BLEEX is hydraulically actuated and targets mainly able bodied persons and uses only motion sensors on the exoskeleton to impart a positive feedback. This approach is also adopted by several other research groups [18]–[20]. However, motion sensing alone cannot distinguish between human applied force and external environment forces that are not explicitly modeled, including uncertain gravitational loads. Effects due to these external load would be amplified unintentionally by the control law. Consequently in [17], [18], the amounts of weight bearing assistance to be provided are pre-determined (i.e. not estimated from sensing) and treated differently from scalings of inertial effects [21]. Similarly, in [20], the joint torque estimator in the fictitious gain controller requires knowledge of the gravitational potential energy function. In our hydraulic HPA, an intermediate approach is taken in that the human applied force is measured. Since the point of interaction is well defined (at the handle), the human intent, distinct from the applied environment forces, can be determined accurately. Inertia effects and gravitational loads and other environment forces can then be compensated in a uniform manner by “simply” scaling the applied human force to achieve similar effects as the fictitious gain in [20], or the admittance/energy shaping in [19], [21].

The close attention paid to the hydraulic actuation in this paper is in contrast to most academic exoskeletons that use electric motor for actuation. Those that are hydraulic actuated (e.g. [17], [22]) are controlled by throttling valves and do not address the critical issue of pressure dynamics that ultimately produce the actuation force. An exception are [9] which uses an electric motor driven pump/motor, and a recent paper [23] on a 1-DoF hydraulic exoskeleton in which the valve controlled hydrualics are treated in details in a cascade motion control structure with direct force tracking.

Preliminary results in this paper appeared in [24] and in [25], [26] where single DoF passivity based force amplification control and multiple DoF assistive dynamics rendering were first presented respectively. The present paper presents the multi-DoF, passivity based, HHPA control with assistive dynamics in a comprehensive and complete manner. In particular, this paper includes detailed proofs and analysis, thorough discussion of energetic passivity, and more extensive experimental validations. The result for robustly enforcing energetic passivity is completely new.

The rest of paper is organized as follow. System models and control objectives are stated in section II. The reformulation of the force control problem into a coordination control problem is presented in section III. Shape system coordination control and locked system guidance are presented in sections IV and V respectively. Closed loop energetic passivity is discussed in section VI. Experimental results are given in section VII. Section VIII contains concluding remarks.

II. SYSTEM DESCRIPTION AND CONTROL OBJECTIVES

The HHPA being considered (Figs. 1-2) has two degrees of freedom (DoF): reach motion - translation of a beam (A) relative to another beam (B), pitch motion - rotation of the beams about a pivot. The generalized coordinates are \( q = [\theta_p, x_p]^T \) where \( \theta_p \) describes the angular position of the pitch movement...
and $x_p$ describes the linear position of the reach movement. The pitch (angular) motion is actuated by a linear hydraulic actuator whereas the reach (linear) motion is actuated by a hydraulic motor via a pulley-and-belt mechanism. The linear forces applied by the hydraulic actuator and by the pulley-and-belt are measured by force sensors. The human operates the HPA via a handle instrumented with a two degree-of-freedom force sensor.

### A. Mechanical system

The dynamics of the HPA are given by:

$$M_p(q)\ddot{q} + C_p(q, \dot{q}) = F_{human} + F_{env} + F_a$$  \hspace{1cm} (1)

where $M_p(q) ∈ \mathbb{R}^{2×2}$ is the symmetric and positive definite inertia matrix, $C_p(q, \dot{q}) ∈ \mathbb{R}^{2×2}$ is the Coriolis matrix such that $M_p(q) - 2C_p(q, \dot{q})$ is skew-symmetric; $F_{human}$ is the generalized force/torque; $F_{env}$ is the force exerted by the environment, including gravitational force; $F_a$ is the generalized actuator force/torque given by:

$$F_a = \begin{bmatrix} T_T \\ F_x \end{bmatrix} = \begin{bmatrix} J_A(\theta_p)F_\theta \\ \frac{1}{r_m}T_x \end{bmatrix}$$  \hspace{1cm} (2)

where $T_T$ and $F_x$ are the torque and force applied to the pitch and reach directions. They are related to the pitch hydraulic cylinder force $F_\theta$ and the reach hydraulic motor torque $T_x$ by the Jacobian $J_A(\theta_p)$ given by:

$$J_A(\theta_p) = \frac{h \cdot d \sin(\theta_p)}{\sqrt{h^2 + d^2 - 2h \cdot d \cos(\theta_p)}}$$

and the pulley radius $r_m$. The generalized pitch and reach motions are related to the linear velocity of the hydraulic cylinder and the angular velocity of the hydraulic motor by:

$$\dot{x}_p = J_A(\theta_p)\dot{\theta}_p; \quad r_m\dot{\theta}_x = \dot{x}_p \hspace{1cm} (3)$$

Pitch and reach motions ($\theta_p$ and $x_p$) are monitored via encoders; and their velocities are obtained via direct differentiation (i.e. $s/(s/\lambda + 1)$ where $\lambda$ is large). The actuator forces $F_\theta$, $F_x ∈ \mathbb{R}$ and the human force $F_{human} ∈ \mathbb{R}^2$ are measured via force sensors. The environment force $F_{env} ∈ \mathbb{R}^2$, however, is not measured.

### B. Hydraulic system

The reach and pitch motions are actuated respectively by a hydraulic motor and a hydraulic cylinder. Although one is controlled by a servo-valve and the other by an energy efficient hydraulic transformer [14], for the purpose of this paper, we can consider the flows into the motor and the cylinder as the control inputs which are faithfully provided by the servo-valve and transformer. For details of control using hydraulic transformers, see [13], [27].

1) Reach actuation: The reach axis hydraulic motor torque is:

$$F_x = \frac{1}{r_m}T_x = (P_{m1} - P_{m2}) \frac{D_m}{2\pi r_m}$$  \hspace{1cm} (4)

where $D_m$ is the fixed displacement of the motor, $P_x$ is the pressure differential across the two motor ports, and $Q_x$ is the hydraulic motor flow. The dynamics of the pressure difference are:

$$\dot{P}_x = \frac{\beta}{V_m} \left( Q_x - \frac{D_m \dot{x}_p}{2\pi r_m} \right)$$  \hspace{1cm} (5)

where $V_m$ is the effective fluid volume of the motor.

2) Pitch actuation: The pitch hydraulic torque $T_\theta$ is given by (Fig. 3):

$$T_\theta = J(\theta_p)F_\theta = J(\theta_p)(A_1P_\theta - A_2P_T)$$  \hspace{1cm} (6)

where $A_1$ and $A_2$ are the cap side and piston side areas, $P_\theta$ and $P_T$ are the supply and tank pressures on the cap and rod sides of the actuator. The pressure dynamics are given by:

$$\dot{P}_\theta = \frac{\beta}{V_1(x_\theta)} \left( Q_\theta - A_1J_A(\theta_p)\dot{\theta}_p \right)$$  \hspace{1cm} (7)

where $Q_\theta$ is the pitch axis control input which is the flow into the cap side chamber,

$$V_1(x_\theta) = V_{10} + A_1x_\theta$$  \hspace{1cm} (8)

is the fluid volume in the cap-side chamber and the hose, which is dependent on the linear displacement of the cylinder $x_\theta$, and $\beta$ is the fluid bulk modulus.

\[1\] $V_{m1}^{-1} := V_{m1}^{-1} + V_{m2}^{-1}$ where $V_{m1,2}$ are the hose volumes of either side of the motor.
After coordination:

\[ \begin{align*}
F_d & = M \ddot{q} + F_{\text{env}} \\
F_{\text{human}} & = (\rho + 1)F_{\text{human}}
\end{align*} \]

\[ M \phi(q) \]

\[ F_{\text{guide}} \]

\[ \begin{align*}
F_{\text{guide}} & \rightarrow M \phi(q) \\
M \phi(q) & \rightarrow F_{\text{env}} \\
(\rho + 1)F_{\text{human}} & \rightarrow M \phi(q)
\end{align*} \]

Fig. 4. Hydraulic human power amplifier coupled with a virtual inertia via the fluid spring. They become a common mechanical tool after coordination.

C. Control Objectives

For the HHPA plant dynamics given by (1)-(7) with control inputs being the reach- and pitch-axis flows \( Q = [Q_x, Q_y]^T \), the goals are to enable the generalized actuator force \( F_a \) in (2) to:

1) exert \( \rho > 0 \) times the applied human force \( F_{\text{human}} \) so that the human would feel that he/she is interacting with an inertia and an environment force that are attenuated by \( (\rho + 1) \) times;

2) apply assistive dynamics to provide task specific guidance, such as to follow a preferred contour and/or to avoid obstacles.

To enhance safety, the HHPA is to behave, in its interactions with the human and physical environments, like an energetically passive system with a power scaling of \( \rho + 1 > 1 \); i.e. there exists \( c^2 > 0 \), s.t. for all human force \( F_{\text{human}}(\cdot) \) and environment force \( F_{\text{env}}(\cdot) \), and for all time \( t > 0 \),

\[ \int_0^t (\rho + 1)q^T F_{\text{human}} + q^T F_{\text{env}} d\tau \geq -c^2 \]

(9)

Here, the supply rate consists of \( (\rho + 1)q^T F_{\text{human}} \) and \( q^T F_{\text{env}} \) which are the scaled power exerted by the human and the physical environment. (9) expresses that \( c^2 \) is the maximum net scaled energy that the human and physical environment can extract from the HHPA.

These objectives will be satisfied, in comparison with (1), if \( F_a = F_d \) given by:

\[ F_d = \rho F_{\text{human}} + F_{\text{guide}} \]

(10)

and \( F_{\text{guide}} \) which installs the guidance dynamics is passive:

\[ \int_0^t q^T F_{\text{guide}} \, d\tau \leq -c_1^2 \]

The challenges are that \( F_a \) is only indirectly controlled by the pressure dynamics.

III. Virtual Coordination Control Approach

Although the control goal is for the actuator force \( F_a \) to track the desired force \( F_d \) in (10), as shown in [5], a feedback controller that adds on the force error directly leads to a positive velocity feedback that does not generally observe the energy passivity requirement and can suffer from robustness issues in the presence of uncertainty, slow sampling or feedback noise. Instead, in this paper, the virtual coordination approach in [4], which converts the problem into one of coordinating the velocities of two coupled mechanical systems is advocated. Besides avoiding the positive velocity feedback, this approach can also be interpreted physically as an interconnection of passive components. The controller can exploit the intrinsically passive structure to make the system more robust and safer to operate.

The compressible fluid in the hydraulic cylinder and motor actuators can be interpreted as 2-port springs, with a mechanical port (connected to the inertia \( M_p \) of the HHPA) and a fluid port such that the two ends of the fluid spring can be compressed by the motion of the cylinder piston (or motor shaft), and by the injection of fluid. By controlling the fluid flow, the fluid port is made to be connected, via a kinematic transformation, to a small virtual inertia \( M_v \in \mathbb{R}^{2 \times 2} \) which is in turn acted on by the desired force \( F_d \) as given by (10). Fig. 4 illustrates this physical interconnection. In this way, the dynamics of a virtual inertia \( M_v \) (implemented as part of the controller) and the flow into the hydraulic actuators are:

\[ M_v \dot{q}_v = F_d - F_a + w \]

(11)

\[ Q = [Q_x, Q_y] = \begin{bmatrix} A_1J_A(\theta_p) & 0 \\ 0 & \frac{B_{\text{man}}}{2\pi r_m} \end{bmatrix} \dot{q}_v + \dot{Q} \]

(12)

\[ \dot{J}(q) \]

where \( q_v = [\theta_v, x_v]^T \) is the generalized coordinate for the virtual inertia, \( F_a \) is the generalized actuator force, \( Q \) are the flows to the actuators, and \( w \) and \( \dot{Q} \) are the additional control to achieve coordination. Note that (11) and (12) without \( w \) or \( \dot{Q} \) implement the dynamics of the virtual inertia coupled to the fluid spring.

If the virtual inertia \( M_v \) and the actual inertia \( M_p(q) \) are perfectly coordinated such that \( \dot{q}_v(t) \equiv \dot{q}(t) \) (i.e. they become a single rigid inertia), then comparing (1) and (11), and with \( w \) defined such that \( w \rightarrow 0 \) when coordinated, the resulting dynamics become:

\[ M_L(q) \ddot{q} + C_p(q, \dot{q}) \ddot{q} = (\rho + 1)F_{\text{human}} + F_{\text{env}} + F_{\text{guide}} \]

(13)

where \( M_L(q) = M_v + M_p(q) \) is the apparent inertia. Hence the required human force amplification is fulfilled and after coordination \( \dot{q}_v(t) \equiv \dot{q}(t) \), the closed loop system is energetically passive as defined in (9). The remaining control tasks are therefore to

1) define the additional hydraulic flow \( \dot{Q} \) to ensure that this coordination indeed occurs,

2) define \( F_{\text{guide}} \) to provide task guidance; and

3) ensure that the passivity property (9) is satisfied even during the transient.

In section III-A, the HHPA system coupled with the virtual inertia is decomposed into two passive subsystems: a shape system and a locked system to facilitate, respectively, the design of the energetically passive coordination and guidance controls. Specifically, coordinating the virtual and actual inertias is equivalent to regulating the shape system coordinates.
at 0, and the guidance control can be designed for to achieve a desired behavior for the locked system. One way of considering an energetically passive coordination control is to mimic an infinitely stiff spring between the virtual and actual inertias. These two aspects are described in sections IV and V. The remaining task of ensuring the passivity property (9) is discussed in section VI.

A. Passive decomposition into locked and shape systems

The coupled system of (1) and (11) is given by

\[
M_p(q)\ddot{q} + C_p(q, \dot{q})\dot{q} = F_{human} + F_{env} + F_a
\]

\[M_v\dot{\theta}_v = F_d - F_a + w
\]

where the generalized coordinates for the physical system are \(q = [\theta_p, x_p]^T\) and for the virtual system are \(q_v = [\theta_v, x_v]^T\).

Since we are interested in the coordination between \(\dot{q}\) and \(\dot{\theta}_v\), i.e., \(V_E := \dot{q} - \dot{\theta}_v \rightarrow 0\) and in ensuring that the desired dynamics in (13) are not disturbed by the coordination control, the passive decomposition transformation [6]–[8] is used:

\[
\begin{bmatrix}
V_L \\
V_E
\end{bmatrix} = S(q)
\begin{bmatrix}
I - \phi(q) & \phi(q) \\
-I & I
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{\theta}_v
\end{bmatrix}
\]

(15)

where \(\phi(q) = [M_p(q) + M_v]^{-1} M_v\) and \(M_L(q) = M_p(q) + M_v\) is the so-called locked system inertia. Since \(V_E\) corresponds to the relative speed, it is referred to as the shape system velocity, and as \(V_L = \dot{q} = \dot{\theta}_v\) when \(V_E = 0\), \(V_L\) is referred to as the locked system velocity. It is in fact the velocity of the center of mass of the combined virtual and actual systems. The system dynamics in the shape and locked system coordinates are:

\[
M_L(q)\ddot{V}_L + C_L(q, \dot{q})\dot{V}_L + C_{LE}(q, \dot{q})V_L = F_d + F_{env} + F_{human} + w
\]

\[M_E(q)\ddot{V}_E + C_E(q, \dot{q})\dot{V}_E + C_{EL}(q, \dot{q})V_E = F_a + \phi(F_{env}) + \phi(F_{human}) - (I - \phi)(F_d + w)
\]

(16)

where \(F_a\) is the generalized hydraulic actuator force in (2) that is controlled indirectly by the flow input \(Q\) via pressure dynamics (5) and (7). The transformed locked and shape systems’ inertias are (see [6] for a similar derivation):

\[
M_L(q) = M_p(q) + M_v
\]

\[M_E(q) = (I - \phi(q))^T M_v (I - \phi(q))
\]

\[C_L(q, \dot{q}) = S^{-T} M_p(q) d/dt (S^{-1}) + S^{-T} \begin{bmatrix} C(q, \dot{q}) & 0 \\ 0 & 0 \end{bmatrix} S^{-1}
\]

\[C_{EL}(q, \dot{q}) = S^{-T} M_v d/dt (S^{-1}) + S^{-T} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} S^{-1}
\]

(19)

Note that the locked and shape systems are coupled through the \(C_{LE}(q, \dot{q})V_E\) and \(C_{EL}(q, \dot{q})V_L\) terms. Defining the decoupling control as:

\[
w = C_{LE}(q, \dot{q})
\]

\[F_a = F_{a1} + C_{EL}(q, \dot{q})V_L
\]

and the decoupled system dynamics become:

\[
M_L(q)\ddot{V}_L + C_L(q, \dot{q})V_L = F_d + F_{env} + F_{human}
\]

\[M_E(q)\ddot{V}_E + C_E(q, \dot{q})V_E = F_{a1} + F_{E1} + F_{E2}
\]

(20)

where \(F_{E1}\) and \(F_{E2}\) are defined in (17). Note that decoupling in (21) is necessary only for multi-DoF nonlinear mechanical systems.

The passive decomposition transformation \(S(q)\) in (15) is defined with the property that:

\[
\frac{1}{2} V_L^T M_L(q)V_L + \frac{1}{2} V_E^T M_E(q)V_E = \frac{1}{2} \dot{q}^T M_p(q)\dot{q} + \frac{1}{2} \dot{q}_v^T M_v \dot{\theta}_v
\]

(24)

i.e. the kinetic energy of the system is preserved if \(M_L(q)\) and \(M_E(q)\) are considered the inertias of the locked and shape systems. This gives rise to the result that (22) and (23) can be considered individual passive mechanical systems as shown in the Proposition 1 below (See Appendix A for proof):

**Proposition 1** The inertia matrices \(M_L(q)\) and \(M_E(q)\) in (18)-(19) are positive definite, and the matrices,

\[
\dot{M}_L(q) - 2C_L(q, \dot{q}); \dot{M}_E(q) - 2C_E(q, \dot{q})\]

(25)

are skew-symmetric. Hence, the systems (22) and (23) satisfy the passivity properties: \(3c_L, c_E \) s.t. \(\forall t \geq 0\) and all inputs,

\[\int_0^t \dot{V}_L^T(\tau)(F_d(\tau) + F_{env}(\tau) + F_{human}(\tau)) \cdot d\tau \geq -c_L^2\]

\[\int_0^t \dot{V}_E^T(\tau)(F_{a1}(\tau) + F_{E1}(\tau) + F_{E2}(\tau)) \cdot d\tau \geq -c_E^2\]

Furthermore, no mechanical power is required for the decoupling control (21), i.e.

\[0 = V_L^T C_{LE}(q, \dot{q})V_E + V_E^T C_{EL}(q, \dot{q})V_L\]

The significance of this result is that if the shape and the locked systems remain passive after applying their respective controls, the HHPA will also be energetically passive.

IV. SHAPE SYSTEM CONTROL - COORDINATION

In this section, we define a controller that regulates \(V_E \rightarrow 0\) which in turn coordinates the virtual and actual inertias. The control law estimates and compensates for the unknown environment force and provides some pressure damping. What makes this problem challenging is the presence of the nonlinear fluid spring. Our approach is to utilize the natural energy storage of the fluid spring [12] in the Lyapunov function.

Combining Eqs.(4),(5),(6),(7),(21),(23), the shape system dynamics are:

\[
\dot{M}_E(q)\ddot{V}_E + C_E(q, \dot{q})V_E = F_{a1} + F_{E1} + F_{E2}
\]

\[F_a = F_{a1} + C_{EL}(q, \dot{q})V_L = J(q)P
\]

\[\dot{P} = B_v(q) (Q - J^T(q)\dot{q})
\]

(27)

(28)

where \(F_{E2}\) contains measurable or known terms and \(F_{E1}\) is potentially unknown as defined in (17) and

\[
B_v(q) := \begin{bmatrix}
\beta / V_l(x_0) & 0 \\
0 & \beta / V_m
\end{bmatrix}, \quad J(q) := \begin{bmatrix}
J(\theta_p)A_1 & 0 \\
0 & \frac{D_{m}}{2\pi r_m}
\end{bmatrix}
\]
A. Flow control input $Q$

The shape system flow input will be given by:

$$ Q = \bar{J}(q)\dot{q}_v + B_v^{-1}(q)\dot{p}_d - \Lambda_p \dot{\tilde{p}} $$  

where $\dot{\tilde{p}} = P - \bar{p}_d$ is the pressure error, and the desired pressure and actuator force are:

$$ P_d = \bar{J}^{-1}(q)F_{a,d} + \left[ PT_A2/A_1 0 \right]^T $$

$$ F_{a,d} = C_{EL}(\dot{q},q)V_L - \Delta V_E - \hat{F}_{E1} - F_{E2} $$

$\hat{F}_{E1}$ is an estimate of $F_{E1}$. $\Lambda_p = \text{diag}(\lambda_{p,\theta}, \lambda_{p,z})$, and $\Lambda$ are positive definite gain matrices.

The estimate for the external force, $\hat{F}_{E1}$, is obtained from the adaptation algorithm,

$$ \hat{F}_{E1} = \sigma V_E + \hat{F}_{E1} $$

where $\hat{F}_{E1}$ is the best estimate of the derivative of $F_{E1}$ and $\sigma > 0$.

Notice that the flow input (29) is consistent with the interaction with the virtual mass in (12) with $\dot{Q} = B_v^{-1}(q)\dot{p}_d - \Lambda_p \dot{\tilde{p}}$.

B. Derivation and analysis

To derive the control law (29)-(31), notice that the desired pressure in Eq. (30) will generate the following shape velocity dynamics:

$$ M_E(q)\ddot{V}_E + C_E(q,\dot{q})V_E = -\Delta V_E - \hat{F}_{E1}(t) + \hat{F}_{a} $$

where $\hat{F}_{a} = F_a - F_{a,d}$ is the error in delivering the desired force in (31). Consider a Lyapunov function consisting of the shape system kinetic energy and $F_{E1}$ estimation error energy:

$$ W_1 = \frac{1}{2}V_E^T M_E V_E + \frac{1}{\sigma} \hat{F}_{E1}^T \hat{F}_{E1} $$

Its time derivative is given by:

$$ \dot{W}_1 = -V_E^T \Delta V_E - V_E^T \hat{F}_{E1}(t) + \frac{\hat{F}_{E1}^T}{\sigma} (\dot{\hat{F}}_{E1} - \hat{F}_{E1}) + V_E^T \hat{F}_{a} $$

(34)

The proposed adaptation algorithm in Eq. (32) gives rise to:

$$ \dot{W}_1 = -V_E^T \Delta V_E + V_E^T \hat{F}_{a} $$

(36)

If $\hat{F}_{E1}$ is not available, an extra term $-V_E^T \hat{F}_{E1} \hat{F}_{a}$ will be present. However, if $\hat{F}_{E1}(t)$ is slowly varying such that $\hat{F}_{E1}/\sigma$ is small, this term can be ignored.

The force error could be written in terms of pressure error using $F_a = \bar{J}(q)\bar{P}$. To account for the pressure error, the Lyapunov function is augmented to become:

$$ W_2 = W_1 + V_1(x_\theta)\dot{W}_V(\tilde{P}_0) + \frac{V_m}{2\beta} \tilde{P}_z^2 $$

(37)

where $\dot{W}_V(\tilde{P})$ is the volumetric pressure error energy density associated with compressing the fluid from pressure $p_d$ to $p_d + \tilde{p}$ as defined in [12]. For constant bulk modulus $\beta$, it is independent of $p_d$ and is given by:

$$ \dot{W}_V(\tilde{P}) := \beta \left[ \dot{\bar{p}} - \left( \frac{1}{\beta} + \frac{\dot{\tilde{p}}}{\beta} \right) \right]. $$

(38)

The pressure error energy terms for the pitch and reach axes take different forms because the fluid volume for the rotary (reach) actuator does not change with the rotary motion whereas the fluid volume for the cylinder actuator (pitch) does change with cylinder movement according to (28). However, from Taylor expansion of (38), they are similar in that:

$$ V_1(x_\theta)\dot{W}_V(\tilde{P}_0) \approx V_1(x_\theta) \frac{\dot{P}_d^2}{2\beta} + H.O.T. $$

**Proposition 2** The pitch and reach pressure error energy functions satisfy:

$$ \frac{d}{dt} \left[ V_1(x_\theta)\dot{W}_V(\tilde{P}_0) + \frac{V_m}{2\beta} \tilde{P}_z^2 \right] $$

$$ = \dot{\bar{P}} \left[ \left( 1 + \frac{W_\theta}{\tilde{P}_0} \right)^T Q - \dot{J}^T(q)\dot{q} - \frac{V_1(x_\theta)}{B(\tilde{P}_0)} \frac{V_m}{\beta} \frac{\dot{P}_d}{\tilde{P}_0} \right] $$

(39)

where

$$ \frac{1}{B(\tilde{P}_0)} = \frac{\dot{P}_d/\beta - 1}{\tilde{P}_0} \approx \frac{1}{\beta} + \frac{\tilde{P}_d}{2\beta} + H.O.T. $$

(40)

and $(1 + \dot{W}_V(\tilde{P}_0)/\tilde{P}_0) > 0$.

**Proof:** By direct computation and using (28), (8) and (3), we have

$$ \frac{d}{dt} \left[ V_1(x_\theta)\dot{W}_V \right] = \left[ \dot{\bar{P}} + \dot{W}_V \right] Q_\theta - \dot{P}_A \dot{x}_\theta - \tilde{P} \frac{V_1(x_\theta)}{B(\tilde{P}_0)} \frac{\dot{P}_d}{\tilde{P}_0} $$

$$ \frac{d}{dt} \left[ \frac{V_m}{2\beta} \tilde{P}_z^2 \right] = \tilde{P}_x \left( \frac{Q_x - \tilde{P}_d}{\beta} \frac{D_m}{2\pi v_m} \dot{x}_p \right) $$

The desired relation (39) is a re-organization of these terms.

**Remark 1** Proposition 2 shows that the hydraulic cylinder and motor system is a passive 3-port system with a hydraulic port, a mechanical port, and a port relating to desired pressure. This means that the second and third terms of (37) indeed represent the energy stored in the fluid springs. Interestingly, the hydraulic port supply rate for the hydraulic cylinder is $Q_0 \bar{P}_0 (1 + \dot{W}_V/\bar{P})$ instead of the more conventional $Q_0 \bar{P}_0$ for the hydraulic motor. The difference is due to the fluid volume being constant in the hydraulic motor but varying in the hydraulic cylinder. For details, please see [12].

We are now ready to define the flow input $Q$. From (39), if we approximate

$$ 1 + \dot{W}_V/\bar{P}_0 \approx 1; \quad B(\bar{P}_0) \approx \beta $$

the approximation errors are of the order $\bar{P}_0$, the flow input:

$$ Q_1 = J^T(q)\dot{q}_v + B_v(q)\dot{p}_d $$
would generate a term $-\hat{P}^T J^T(q) V_E = -V_E^T \hat{F}_a$ to cancel out the force error term in (36). Let

$$Q = Q_1 - \Lambda_p \hat{P}$$

where the second term with $\Lambda_p = \text{diag}(\lambda_{p,\theta}, \lambda_{p,x})$ being positive definite will be used to compensate for the approximation error. Then,

$$\dot{\hat{W}}_2 = -V_E^T \Lambda V_E + \hat{F}_a^T \left[1 + \frac{W_2}{\rho \hat{P}} \ 0 \ 1\right] \hat{Q}_1$$

$$+ \frac{1}{\beta} \left[1 - \frac{1}{B(P_0)}\right] V_1(x_0) \hat{P}_d + \tilde{W}_V(\hat{P}_0) Q_{1,\theta}$$

Note that the last two terms are quadratic in $\hat{P}_0$. Thus, with $\lambda_{p,\theta}$ (possibly time-varying) in $\Lambda_p$ sufficiently large, we have for some $\lambda_{p,1} > 0$,

$$\dot{\hat{W}}_2 \leq -V_E^T \Lambda V_E - \lambda_{p,1} \| \hat{P} \|^2.$$  

Applying Barbalat’s lemma, this analysis shows that $V_E \to 0$ and $\hat{P} \to 0$.

**Theorem 1** With the input flow control law and the environment force estimator (29)-(32), assuming that $\dot{F}_{E1}$ is well estimated, the velocity coordination error $V_E = \dot{q} - \dot{q}_v \to 0$ and the pressure error $\hat{P} = P - P_{a,d} \to 0$ asymptotically. Furthermore, $V_L \to \dot{q}$, i.e. the locked system velocity converges to velocity of the HPA.

**Remark 2** 1) With the natural energies of the hydraulic actuators in the Lyapunov function (37), the shape system control results in a natural passive interconnection between the mechanical system and the hydraulic system. Experimental study in [12] reveals that this method requires fewer parameter tuning (than conventional arbitrarily defined quadratic terms in $\hat{P}$) and has better performance especially in the presence of measurement noise.

2) The natural energy function used in (37) assumes that the bulk modulus $\beta$ is a constant. The general form for pressure dependent bulk modulus [12], can also be used.

3) Although the control is written in terms of pressure feedback, it can be implemented using the actuator force sensing (as is the case in our experiments).

V. LOCKED SYSTEM CONTROL - GUIDANCE $F_{guide}$

The decomposed locked system represents the dynamics of the system when the virtual and actual parameters are coordinated. Hence it can be used for the design of guidance dynamics. With $F_d = \rho F_{human} + F_{guide}$, the locked system dynamics are given by (22):

$$M_L(q) \ddot{q} + C_L(q, \dot{q}) = F_{guide} + (\rho + 1) F_{human} + F_{env}$$

(41)

$$F_{guide} = F_{PVFC} + F_{OA}$$

Here, two types of guidance are considered for the design of $F_{guide}$: 1) with $F_{PVFC}$, the HPA is guided towards a preferred direction of motion encoded with a velocity field; 2) with $F_{OA}$, the HHPA is prevented from entering prohibited regions. The former will be provided using a passive velocity field controller (PVFC) [6], [10] and the latter will be provided via an artificial potential field. In both cases, the guidance dynamics will have intrinsic passivity properties to enhance system safety.

A. Passive velocity field control (PVFC)

A general approach to task guidance is to encode the task as a desired velocity field $q \rightarrow V(q)$ which specifies a desired velocity at each configuration. Instead of using time to specify the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position. For HHPA applications, there are likely preferred path of motion for the desired motion, task encoding as velocity fields specifies the best motion direction given the current position.
flywheel dynamics:
\[
\begin{bmatrix}
M(q) & 0 \\
0 & M_F
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_p \\
\dot{q}_p
\end{bmatrix}
+ 
\begin{bmatrix}
C_L(q, \dot{q}) & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q}_p \\
\dot{q}_p
\end{bmatrix}
= 
\begin{bmatrix}
F_{PVFC} \\
\tau_F
\end{bmatrix}
\]
\[
+ 
\begin{bmatrix}
F_{OA} + (\rho + 1)F_{human} + F_{env}
\end{bmatrix}
\]
(42)
where \(M_F\) is the fictitious flywheel inertia, scalars \(q_F, \dot{q}_F\) and \(\tau_F\) are its angle, speed and input torque, \(\ddot{q} = [q^T, q_F]^T \in \mathbb{R}^3\), \(\ddot{M}(\ddot{q}) \in \mathbb{R}^{3 \times 3}\) and \(\ddot{C}(\dot{q}, \ddot{q}) \in \mathbb{R}^{3 \times 3}\) are the configuration, inertia and Coriolis matrix of the augmented system, \(\ddot{T}\) is the control that couples the fictitious flywheel with the locked system.

In order to control and utilize the fictitious flywheel, the desired velocity field \(V : q \in \mathbb{R}^2 \rightarrow V(q) \in \mathbb{R}^2\) needs to be augmented as:
\[
\ddot{V}(q) = [V(q)^T, V_F(q)]^T \in \mathbb{R}^3
\]
such that the kinetic energy of the augmented system is constant when the augmented field is tracked. This can be accomplished by ensuring that for all \(q \in \mathbb{R}^2\),
\[
\dot{E} = \frac{1}{2} \ddot{V}(q)\dddot{M}(\ddot{q})\ddot{V}(\ddot{q})
\]
where \(\dot{E}\) is a sufficiently large constant. In other words, the desired flywheel velocity field is given by:
\[
V_F(q) = \sqrt{\frac{2}{M_F}} \left(\dot{E} - \frac{1}{2} V(q)^T M_L(q)V(q)\right)
\]
(44)

2) Coupling control: With the augmented locked system and velocity field, the coupling control \(\ddot{T}\) in (42) is designed as
\[
\ddot{T} = \ddot{\Omega}(\dot{q}, \ddot{q})\dddot{q}
\]
(45)
where \(\ddot{\Omega}(\dot{q}, \ddot{q}) \in \mathbb{R}^{3 \times 3}\) is skew-symmetric and defined as:
\[
\ddot{\Omega}(\ddot{q}, \dddot{q}) = \frac{1}{2E} (\dddot{P}^T - \dddot{\bar{P}}^T) + \gamma (\dddot{P}^o - \dddot{\bar{P}}^o^T)
\]
(46)
where
\[
\dddot{P} := \dddot{M}(\dddot{q})\ddot{V}(\ddot{q}); \quad \dddot{\bar{P}} := \dddot{M}(\dddot{q})\dddot{q}
\]
\[
\dddot{\bar{P}} := \dddot{M}(\dddot{q})\dddot{V}(\ddot{q}) + \dot{C}(\dot{q}, \ddot{q})\dddot{V}(\ddot{q})
\]
Here, \(\dddot{P}(\dddot{q})\) is the desired momentum field, \(\dddot{\bar{P}}(\dddot{q}, \dddot{q})\) is the actual momentum, \(\dddot{\bar{P}}(\dddot{q}, \dddot{q})\) is the covariant derivative of the desired momentum field and \(\gamma\) is a feedback gain.

Roughly speaking, the first skew-symmetric term in (46) generates the feedforward coupling for tracking a scaled copy of \(\dddot{V}(q)\), and the second skew-symmetric term in (46) generates the error feedback. The fact that \(\dddot{\Omega}(\dddot{q}, \dddot{q})\) is skew symmetric means that the power generated by the coupling torque is 0 since \(\dddot{q}^T T = \dddot{q}_3^T \dddot{\Omega}(\dddot{q}, \dddot{q})\dddot{q} = 0\). Please see [10] for a fuller interpretation of how PVFC is defined.

With the PVFC controller in (42)-(46), the augmented locked system dynamics become:
\[
\dddot{M}(\dddot{q})\dddot{q} + \dddot{V}(\dddot{q}, \dddot{q})\dddot{q} = \begin{bmatrix}
F_{OA} + (\rho + 1)F_{human} + F_{env} \\
0
\end{bmatrix}
\]
(47)

![Potential Field](image)

Fig. 6. An example potential field for a rectangular obstacle in Cartesian (workspace) coordinates.

where \(\dddot{V}(\dddot{q}, \dddot{q}) = \dddot{C}(\dot{q}, \dddot{q}) - \Omega(\dot{q}, \dddot{q})\). It has the following properties.

**Theorem 2** \(\dddot{M}(\dddot{q})\) and \(\dddot{Y}(\dddot{q}, \dddot{q})\) in (47) satisfy:
\[
\dddot{M}(\dddot{q}) - 2\dddot{Y}(\dddot{q}, \dddot{q}) \text{ is skew-symmetric.}
\]

Eq.(47) has the passivity property that: there exists \(c_{PVFC}\) such that for all inputs, and any \(t > 0\),
\[
\int_0^t V_F^T[(\rho + 1)F_{human} + F_{env} + F_{OA}]d\tau \geq -c_{PVFC}^2
\]
Furthermore, when \((\rho + 1)F_{human} + F_{env} + F_{OA} = 0\), let \(\delta := \text{sign}(\gamma) \sqrt{\frac{\dddot{M}(\dddot{q})\dddot{q}}{2E}}\). Then, \(\delta\) is a constant and \(\dddot{q} \rightarrow \delta V(\dddot{q}(t))\) (and therefore \(\dddot{q} \rightarrow \delta V(\dddot{q}(t))\)) exponentially except for the initial condition of \(\dddot{q}(t = 0) = -\delta V(\dddot{q}(t = 0))\).

See Appendix B for a proof.

This theorem means the locked system remains energetically passive after applying PVFC, and that its velocity converges exponentially to a scaled copy of the desired velocity field. Moreover, as the kinetic energy of the system increases (such as with input by the human operator or the environment), the speed at which the desired velocity is tracked will also increase. Hence, the PVFC controls the direction of motion while the available kinetic energy in the HHPV determines the speed.

**B. Obstacle avoidance**

To prevent the machine from entering prohibited area in the workspace to protect itself or other objects, artificial potential fields [11] are used to provide the operator a tactile feedback to repel the machine from running into the obstacle.

The potential field is designed to be a non-negative continuous and differentiable function that increases towards the obstacle. Its effect should be limited to the obstacle’s vicinity to avoid undesirable effects elsewhere.
For a point obstacle at \( m \) in the Cartesian workspace of the tip of HHPA, the field is defined as:

\[
U_p(q, m) = \begin{cases}
U_d e^{-k_{oa} \Lambda(q, m)} & \Lambda(q, m) \leq \Lambda_0 \\
0 & \Lambda(q, m) > \Lambda_0
\end{cases}
\]  

where \( \Lambda(q, m) \) is the Cartesian distance of the tip of HHPA with generalized coordinate \( q \) to an obstacle at \( m \). \( k_{oa}, U_d \) and \( \Lambda_0 \) are parameters that define the decay rate, magnitude, and the domain size of the field. The discontinuity of (48) is negligible if \( k_{oa} \) is sufficiently large compared to \( \Lambda_0 \). It is extended to a region \( B \) with boundaries by integrating individual \( U_p \)'s at points along the boundaries: i.e.

\[
U_{oa}(q) = \int_{m \in \partial B} U_p(q, m) \, ds
\]

Fig. 6 shows an example for a rectangular obstacle region.

The obstacle avoidance guidance force is the negative derivative of \( U_{oa}(q) \):

\[
F_{OA} = - \left( \frac{\partial}{\partial q} U_{oa}(q) \right)^T
\]

In summary, the guidance control \( F_{guide} \in \mathbb{R}^2 \) is the combination of PVFC in (45) and obstacle avoidance in (49):

\[
\begin{bmatrix}
F_{guide} \\
M_{F} \dot{q}_p
\end{bmatrix} = \Omega(\dot{q}, \dot{\dot{q}}) \dot{q} + \begin{bmatrix}
F_{OA} \\
0
\end{bmatrix}.
\]

VI. CLOSED LOOP PASSIVITY PROPERTY

In this section, we study the energetic passivity property of the HHPA under the proposed control. Assuming that the human and physical environments are strictly energetically passive with respect to supply rates \( -q^T F_{human} \) and \( -\dot{q}^T F_{env} \) which are physical power inputs from the HHPA, then coupling stability with these environments will be guaranteed if the HHPA satisfies the energetic passivity property defined in (9), i.e. for some \( c^2 \),

\[
\int_0^t q^T \left[ (\rho + 1) F_{human} + F_{env} \right] \, d\tau \geq c^2
\]

Note that the human and physical environment interact via the physical system’s velocity \( \dot{q} \), not the locked system velocity \( V_L \). Hence the locked system passivity property in Theorem 2 is not sufficient.

For the locked system (41) with \( F_{guide} \) in (50), using the storage function,

\[
W_{lock} := \frac{1}{2} \dot{q}^T M(\dot{q}) \dot{q} + U_{oa}(q)
\]

we have:

\[
\dot{W}_{lock} = V_L^T \left[ (\rho + 1) F_{human} + F_{env} \right] + \left( \frac{\partial}{\partial q} U_{oa} \right) V_E
\]

Thus, after \( V_E \) has converged to 0 so that \( V_L = \dot{q} \), we have the desired energetic passivity property (9).

In the transient, if for some finite bound \( M < \infty \),

\[
- \int_0^t {\dot{V}_E} \left[ (\rho + 1) F_{human} + F_{env} + F_{OA} \right] \, d\tau < M
\]

then, (9) is also achieved. Condition (51) is satisfied if \( V_E(\cdot) \in L_1 \) and \( F_{ext} \) is bounded.

Unfortunately, in the shape control system, because \( F_{E1} \) in (31) has to be estimated, Theorem 1 only guarantees that \( V_E \to 0 \) but not necessarily exponentially, hence, theoretically, \( V_E \) may not be \( L_1 \). In other words, the closed loop system may not remain energetically passive during the transient before \( V_E \) has converged to 0.

To strengthen the enforcement of the passivity property (9), either the shape system control or the locked system control can be modified as follows.

**Theorem 3** The closed loop passivity property (9) is observed with either of the following control modifications:

1. \( F_{guide} \) for the locked system in (41) is modified with additional damping:

\[
F_{guide} = F_{PVFC} + F_{OA} - \lambda_L V_L
\]

such that: there exists \( M < \infty \) s.t. for all \( t \geq 0 \),

\[
- \int_0^t \{ V_E^T \dot{F}_{ext} + \lambda_L \| V_L \| \} \, d\tau < M
\]

2. Let \( F_{ext} = [F_{ext,\theta}, F_{ext,x}]^T \). \( F_{a,d} \) in (31) for the shape system is modified with a robust feedback term:

\[
F_{a,d} = C_{EL}(\dot{q}, q)V_L - \Lambda V_E - \hat{F}_{E1} - F_{E2} - F_{pass,\theta} \cdot \text{sgn}(V_{E,\theta}) - F_{pass,x} \cdot \text{sgn}(V_{E,x})
\]

where \( V_E = (V_{E,\theta}, V_{E,x})^T \) and \( F_{pass,\theta,x}(\tau) \) is chosen s.t. \( F_{pass,\theta,x}(\tau) > |F_{ext,\theta,x}(\tau)| \).

**Proof:** With the locked system modification (52),

\[
W_{lock} = (\dot{q} - V_E)^T [(\rho + 1) F_{human} + F_{env} - \lambda_L V_L] - F_{OA} V_E
\]

On integration and using the condition (53), (9) is obtained.

With the shape system modification (54), notice first that convergence of \( V_E \) is not compromised, since the modification introduces the negative term, \( -F_{pass,\theta} |V_{E,\theta}| - F_{pass,x} |V_{E,x}| \) to the derivative of Lyapunov function \( W_1 \) in (36).

Using the total system energy \( W_{total} = W_{lock} + W_2 \) where \( W_2 \) is defined in (37), \( W_2 \) is modified to be:

\[
W_2 \leq \dot{q}^T ([(\rho + 1) F_{human} + F_{env}] + F_{ext,\theta} V_{E,\theta} + F_{ext,x} V_{E,x}) - F_{pass,\theta} |V_{E,\theta}| - F_{pass,x} |V_{E,x}|
\]

\[
\leq \dot{q}^T [(\rho + 1) F_{human} + F_{env}]
\]

so that the desired passivity property is preserved.

The integral (51) can be thought of as extraneous energy generation that theoretically may cause passivity to be violated. Both control modifications in Theorem 3 dissipate energy either in the locked system or in the shape system. Passivity is preserved if the the extraneous energy generation does not exceed the extra dissipation by a finite amount. In this regard, the locked and shape system modifications can also be combined with less stringent gains to achieve the needed net dissipation. Similarly, imperfect convergence of \( V_E \) can also be compensated this way. Furthermore, friction, which is inevitable in physical systems, may provide sufficient dissipation already that additional damping is not necessary.
TABLE I
PHYSICAL PARAMETERS OF THE HHPA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia Matrix</td>
<td>$M_p$</td>
<td>diag((2.5 kg · m² + 7 kg · x_p, 7 kg))</td>
</tr>
<tr>
<td>Virtual inertia</td>
<td>$M_v$</td>
<td>0.1M_p</td>
</tr>
<tr>
<td>Piston area</td>
<td>$A_3$</td>
<td>1.187·10^{-3} m²</td>
</tr>
<tr>
<td>Motor Displacement</td>
<td>$D_{m}$</td>
<td>12.9 cc/rev</td>
</tr>
<tr>
<td>Transformer Displacement</td>
<td>$D_1,D_2$</td>
<td>3.15 cc/rev</td>
</tr>
<tr>
<td>Transformer Inertia</td>
<td>$J$</td>
<td>2 × 10^{-5} kg·m²</td>
</tr>
</tbody>
</table>

TABLE II
SHAPE SYSTEM CONTROL PARAMETERS

<table>
<thead>
<tr>
<th>Gain</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_E$ gain</td>
<td>$\Lambda$</td>
<td>diag([110, 110])</td>
</tr>
<tr>
<td>Pressure feedback gain - pitch</td>
<td>$\lambda_{p,\theta}$</td>
<td>1 · 10^{-10}</td>
</tr>
<tr>
<td>Pressure feedback gain - reach</td>
<td>$\lambda_{p,x}$</td>
<td>1 · 10^{-10}</td>
</tr>
<tr>
<td>Estimation</td>
<td>$\sigma$</td>
<td>5000</td>
</tr>
</tbody>
</table>

VII. EXPERIMENTAL RESULTS

The controller proposed in this paper has been experimentally implemented on a 2-DoF HHPA. The physical parameters are shown in Table I. For the reach axis, the servo-valve used (MTS series 252) is rated at 9.5 L/min (2.5 gal/min) and has a bandwidth of (250 Hz). For the pitch axis, an energy efficient custom developed hydraulic transformer consisting of two 3.15cc micro-piston pump/motors (Takako TFH-315) connected mechanically on a common shaft is used. Since stepper motors and lead-screws are used to rotate the swash-plate to adjust displacements, it bandwidth is more limited than a servo-valve. The controller is implemented on a PC running Matlab/Simulink Realtime Desktop with a sampling time of 1.0 ms.

The shape system coordination control parameters are summarized in Table II.

A. Force amplification

We first examine the force amplification aspect with amplification factor $\rho = 7$ and with guidance control turned off ($F_{\text{guide}} = 0$). This value of $\rho$ is a compromise between being powerful and being sensitive to the environment loads.

Two cases are considered. Fig. 7 shows the case when the HHPA is largely unconstrained (43-48s). In this and following figures, only a portion of the results that pertain to the particular scenario is displayed so as to preserve a sufficiently detailed time resolution in the figures. Fig. 8 shows the case when the operator used the HHPA to wrestle with another person who tried periodically to disturb or oppose the machine.

Overall, the measured actuator force $F_a$ and the desired force ($pF_{\text{human}}, \rho = 7$), as well as the actual velocity $\dot{q}$ and the velocity of the virtual mass $\dot{x}_v$ match quite well for both the pitch and reach axes. The absolute torque/force errors are of the order of 8Nm and 2N for the pitch and reach degrees of freedom; and the coordination errors are of the order of 0.1 rad/s and 1 m/s. At $\rho = 7$ and moment arm of $\sim 1m$, the torque/force errors correspond to approximately 1.1N and 0.3N of human force input for the pitch and reach DoF respectively. The levels of performance are adequate for the application for both axes. However, as percentages of absolute torque/force or velocities, the servo-valve controlled reach axis, performs better than the hydraulic transformer controlled pitch axis. This is especially prominent in Fig. 7 where the desired force/torque and velocities are smaller.

Fig. 9 shows, for the wrestling case in Fig. 8, that the hydraulic transformer control was able to track the desired pressure $P_d$ and the transformer shaft speed $\omega_d$.

B. Guidance

Assistive guidance dynamics are examined next. $F_{\text{guide}}$ as described in section V is turned on. Parameters used are summarized in Table III.

The velocity field in Fig. 5 for guiding the HHPA so that its tip converges to and moves around a circle was used for testing. In this test, the human operator simply pushed the HHPA in its natural movement without paying much attention to tracing the circle or even looking at the HHPA. Fig. 10 shows the resultant movements of the HHPA tip, superimposed...
Fig. 8. Wrestling task (amplification factor $\rho = 7$). Top: pitch torque and angle; bottom: reach force and displacement. The RMS pitch torque error is 6.9 Nm; RMS pitch coordination error is 0.09 rad/s. The RMS reach force error is 1.8 N; RMS reach coordination error is 0.1 m/s.

Fig. 9. Hydraulic transformer control performance (amplification factor $\rho = 7$): Top: Actual ($\omega$) vs desired ($\omega_d$) transformer shaft speed; bottom: Pressure tracking ($P_d$ vs. $P_{d,\theta}$).

Table III

<table>
<thead>
<tr>
<th>PVFC AND OBSTACLE AVOIDANCE (OA) PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVFC Parameters</td>
</tr>
<tr>
<td>Virtual Flywheel</td>
</tr>
<tr>
<td>Augmented Energy</td>
</tr>
<tr>
<td>Obs. Avoidance Parameters</td>
</tr>
<tr>
<td>Obstacle Gain</td>
</tr>
<tr>
<td>Field scaling</td>
</tr>
</tbody>
</table>

Fig. 10. Movements of the tip of the HHPA in Cartesian workspace coordinates superimposed with the desired velocity field. The magenta sector prescribes the allowable workspace.

with the desired velocity field $\dot{V}(q)$ from two initial conditions. The tip of the HHPA moved towards the circle directly (a characteristic of velocity field encoding compared to timed trajectory planning) and around the circle in the anti-clockwise direction. Fig. 11 shows that the augmented velocity field $\bar{\dot{V}}(\bar{q})$ was indeed tracked up to the scaling according to the square root of the instantaneous kinetic energy.

Finally, Fig. 12 shows the results for the obstacle avoidance control. A potential field was generated to create a virtual wall for the rectangular region in the top right corner of the Cartesian workspace. The figure shows that the obstacle avoidance control was successful in prohibiting the HHPA
from entering the rectangle despite the effort by the human operator.

VIII. CONCLUSIONS

A control approach has been developed for a fully coupled multi-DoF hydraulic human power amplifier device that allows the human to operate the machine as a passive mechanical tool such that the machine’s inertia and any external loads are perceived to be smaller. In addition, the control incorporates passive guidance dynamics in the form of passive velocity field control (PVFC) and artificial potential fields to perform specific tasks while avoiding obstacles. The control law structure observes and makes use of the intrinsic passivity properties of both the mechanical and hydraulic components. The force control requirement to amplify human force and apply guidance dynamics is converted into one of coordinating the velocities of a virtual inertia and of the actual system. This results in a closed loop system that is passive with respect to a scaled human and environment power input. Experimental results demonstrate good force tracking and velocity coordination performance with either servo valve or the more efficient hydraulic transformer, as well as effective task guidance. Although the control is developed in the context of a machine operated via a handle, similar approach should be adaptable to wearable exoskeletons if the interaction force can be measured or estimated.

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APPENDIX A

PROOF OF PROPOSITION 1

Proof: $M_L(q)$ and $M_E(q)$ are positive definite is a direct consequence of (24) and $S(q)$ being non-singular.

From direct computation and $\dot{M}_p - 2C_p(q, \dot{q})$ being skew-symmetric, it can be shown that:

$$
\begin{bmatrix}
M_L(q) & 0 \\
0 & M_E(q)
\end{bmatrix} - 2
\begin{bmatrix}
C_L(q, \dot{q}) & C_{LE}(q, \dot{q}) \\
C_{EL}(q, \dot{q}) & C_E(q, \dot{q})
\end{bmatrix}
$$

is skew-symmetric. The skew-symmetry of the matrices in (25), and $C_{LE}(q, \dot{q}) = -C_{EL}^T(q, \dot{q})$ are direct consequences. The latter shows that decoupling requires no energy.

By differentiating the locked and shape system energies

$$
W_L = \frac{1}{2} V_L^T M_L(q) V_L; \quad W_E = \frac{1}{2} V_E^T M_E(q) V_E
$$

and making use of (25), we have

$$
\dot{W}_L = V_L^T [F_d + F_{env} + F_{human}].
$$

Hence, on integration,

$$
W_L(t) - W_L(0) = \int_0^t V_L^T [F_d + F_{env} + F_{human}] \cdot d\tau.
$$

Making use of $W_L(t) > 0$, the required passivity property is obtained with $c_L^2 = W_L(t = 0)$. Similar results for the shape system are obtained by differentiating $W_E$. ■
**APPENDIX B**

**PROOF OF THEOREM 2.**

Proof: Since $\Omega(\dot{q}, \ddot{q})$ and $\ddot{M} - 2CL$ are skew-symmetric, $\ddot{M} - 2Y$ is skew-symmetric.

The passivity property can be obtained using the kinetic energy of the augmented system as a storage function and the fact that $\ddot{M} - 2Y$ is skew-symmetric:

$$W_{PVFC} = \frac{1}{2} \dot{\bar{q}}^T \dot{M}(\bar{q}) \dot{\bar{q}}$$

$$W_{PVFC} = V_L[(\rho + 1) F_{human} + F_{env} + F_{OA}]$$

so that the desired passivity property is obtained on integration and with $W_{PVFC} = W_{PVFC}(t = 0)$.

When $(\rho + 1) F_{human} + F_{env} + F_{OA} = 0$, then $W_{PVFC}$ is constant so that $\delta$ is also a constant. Let $e_\delta := \dot{\bar{q}} - \delta \dot{V}(\bar{q})$.

Using the Lyapunov function,

$$W_\delta := \frac{1}{2} e_\delta^T \dot{\bar{M}}(\bar{q}) e_\delta$$

it can be shown (see [10] for details) that

$$\dot{W}_\delta(t) \leq -\lambda \delta \dot{\bar{E}} \cdot \mu(t) W_\delta(t)$$

where

$$\mu(t) = \frac{1}{2} \left[ 1 + \dot{\bar{V}}(\bar{q}(t)) \dot{M}(\bar{q}(t)) \dot{\bar{q}}(t) \right] / 2 \delta \dot{E}$$

Hence, $e_\delta \to 0$ exponentially from any initial condition except when $\mu(t \to 0) = 0$ or $\dot{\bar{q}}(0) = -\delta \dot{V}(\bar{q}(0))$.  □

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