PASSIVE FEEDFORWARD APPROACH TO BILATERAL TELEOPERATED MANIPULATORS *

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ABSTRACT

A control scheme for linear dynamically similar bilateral teleoperated manipulator system which ensures that the closed loop system is energetically passive is proposed. Energetic passivity implies that the teleoperated manipulator system is safe to interact with, and that the coupling between the system and any strictly passive environment is stable. The control objective is for the two manipulators in the system to behave in unison under the influence of both the operator and the work environment, while maintaining energetic passivity. The dynamics of the system in unison and its response to the operator and work environment can be specified as kinematic and power scalings. To maintain energetic passivity with feedback and feedforward actions, the proposed control makes use of two fictitious internal energy storages. The result is that when the internal state of the storage elements are suitably initialized, the teleoperated manipulator system achieves asymptotic locking even in the presence of external (bounded) forcing from the operator and work environment.

1 INTRODUCTION

A bilateral teleoperated manipulator system interacts with both the work environment and the human operator. To be successful, the motions of the slave and master manipulators must mimic each other in a manner determined by the kinematic scaling. Thus, their motions must be coordinated. The coordination control for a bilateral teleoperator must be designed not to inhibit the effect of the work environment and operator's forces on the movement of the overall system to obtain kinesthetic coupling in a natural manner.

Since a teleoperated manipulator simultaneously interacts with two environments (the human operator, and the work environment), an a-priori requirement must be that a broad class of environments with which the system interacts do not destabilize the system. It is particularly important when the teleoperator is required to provide force or power amplification / attenuation. If the system is rendered passive with respect to a supply function related to the mechanical power input, the interaction stability with *any* strictly passive work environment and human operator can be guaranteed using the familiar passivity theorem [1], i.e. the interconnection between a passive and a strictly passive system is necessarily stable.

Our control philosophy for bilateral teleoperator is that the teleoperated system should present itself as a passive mechanical tool to both the human operator and to the work environment. We refer to the tool as the *locked* mechanical system since it is the system that one gets when the master robot and the slave robot are perfectly coordinated (locked).

In [2], a passive control methodology, using kinematic (position and velocity) feedback only, was proposed for linear, dynamically similar bilateral teleoperated manipulator systems. The development of the control law makes use of the desirable property that a pair of $n-$ DOF dynamically similar master and slave manipulators (total 2n-DOF) can be decomposed, both dynamically and energetically, into two $n-$DOF mechanical systems. The decomposed systems represent the coordination and the gross motion aspects respectively. Comparing with other approaches [3, 4], the main contribution of this control law is in that it controls the two aspects of teleoperation separately and achieves passivity of the whole system by passifying the decomposed systems individually. The passivity of this control law is also based on time domain analysis extensible to general nonlinear cases. This control law, which is based on the passive velocity field control (PVFC) technique [5, 6, 7], has the following features: 1) It enables the teleoperator to be coordinated according to a prescribed linear kinematic scaling; 2) It provides for bilateral power amplification / attenuation; 3) It ensures that the closed loop system is energetically passive; 4) The dynamics of

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the locked system in response to the operator and the environment forces is as prescribed.

Unfortunately, the control scheme in [2] suffers from the fact that the coordination performance degrades in the presence of unmatched operator and environment forcing. In this paper, we remedy this drawback. Force sensors are used to measure the human operator and the work environment forces, and then a feedforward action is designed to compensate for any mismatched operator / environment force. The challenge of feedforward compensation is that it can destroy the passivity property of the system. This difficulty is overcome by the use of fictitious energy storages. The kinematic feedback portion of the controller has also been redesigned to achieve a better convergence rate.

The paper is organized as follows. In section 2, the control problem is formulated. In section 3, we review results from [2] that for linear dynamically similar teleoperated manipulators, the system dynamics can be decomposed into two parts. The control of the gross motion (*locked* system) is discussed in section 4 and the coordination control is discussed in section 5. For the coordination control, novel passive feedback stabilization and passive external force compensation schemes are utilized. Experiment results are presented in Section 6. Section 7 contains some concluding remarks.

2 PROBLEM FORMULATION

2.1 PLANT

Consider a *linear* teleoperated manipulator system consisting of two $n-$degree of freedom manipulators with dynamics given by:

$$\mathbf{M}_1\ddot{\mathbf{q}}_1 = \mathbf{T}_1 + \mathbf{F}_1, \qquad \mathbf{M}_2\ddot{\mathbf{q}}_2 = \mathbf{T}_2 + \mathbf{F}_2 \tag{1}$$

where $\mathbf{T}_1, \mathbf{T}_2 \in \Re^n$ are the control forces, $\mathbf{F}_1, \mathbf{F}_2 \in \Re^n$ are the environment forces that the master and slave systems encounter, and $\mathbf{M}_1$ and $\mathbf{M}_2 \in \Re^{n\times n}$ are the inertia matrices for the manipulators.

Let $\alpha \in \Re^{n\times n}$ be the desired bijective linear kinematic scaling so that ideally, we would like

$$\alpha\mathbf{q}_1(t) = \mathbf{q}_2(t)$$

so that the system achieves the perfect coordination (locking).

As in [8], we assume that the slave and master manipulators are **dynamically similar with respect to** α in the sense that there exists a scalar $\zeta > 0$, s.t.

$$\zeta\mathbf{M}_1 = \alpha^T\mathbf{M}_2\alpha.$$

Any one DOF linear mechanical system, such as the experiment setup in Figure 1, satisfies this property.

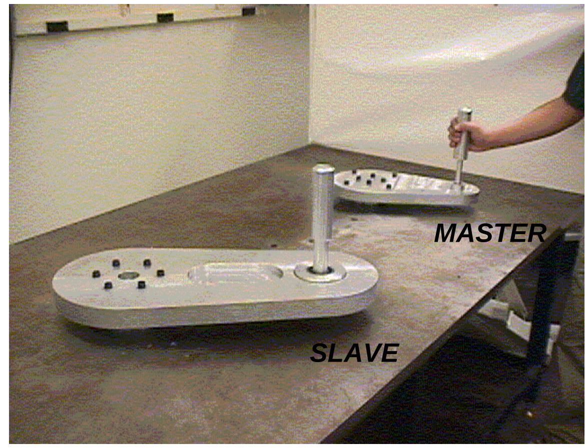

Figure 1. Teleoperator with Two Single-DOF Planar Manipulators in MIML. The master manipulator is equipped with force sensor to measure human force.

2.2 PASSIVITY WITH POWER SCALING

To ensure that both the human and the work environment can interact with the teleoperated manipulator in stable fashions, we require that the two port teleoperator system be energetically passive. In addition, we are also interested in the teleoperator system providing power scaling ρ. An appropriate definition of the supply rate is:

$$s_\rho(\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2, \mathbf{F}_1, \mathbf{F}_2) = \rho\mathbf{F}_1^T\dot{\mathbf{q}}_1 + \mathbf{F}_2^T\dot{\mathbf{q}}_2 \tag{2}$$

which is the sum of the power exerted by the force $\mathbf{F}_2$ and ρ times of the power exerted by the force $\mathbf{F}_1$. We require that the closed loop controlled teleoperated system be passive with respect to this supply rate, i.e.

$$\int_0^t s_\rho(\dot{\mathbf{q}}_1(\tau), \dot{\mathbf{q}}_2(\tau), \mathbf{F}_1(\tau), \mathbf{F}_2(\tau))d\tau \geq -c^2 \tag{3}$$

for some $c \in \Re$ which would depend on the initial condition at $t = 0$. Notice that since $\rho > 0$, if the two port system is energetically passive with respect to the supply rate in (2), it is also energetically passive as one port systems (i.e. when one of the ports is open with $\mathbf{F}_2 = 0$ or $\mathbf{F}_1 = 0$) in the usual sense:

$$\int_0^t \mathbf{F}_1^T\dot{\mathbf{q}}_1 dt \geq -c_1^2; \qquad \text{when } \mathbf{F}_2 = 0$$
$$\int_0^t \mathbf{F}_2^T\dot{\mathbf{q}}_2 dt \geq -c_2^2; \qquad \text{when } \mathbf{F}_1 = 0.$$

The supply rate defined in (2) which incorporates power scaling inspires the following definition of scaled kinetic energy for the teleoperated manipulator system:

$$\kappa_\rho(\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2) = \frac{\rho}{2}\dot{\mathbf{q}}_1^T \mathbf{M}_1 \dot{\mathbf{q}}_1 + \frac{1}{2}\dot{\mathbf{q}}_2^T \mathbf{M}_2 \dot{\mathbf{q}}_2 \geq 0. \tag{4}$$

The objective of this work is to design the control action $\mathbf{T}_1$ and $\mathbf{T}_2$ so that the teleoperator system is coordinated, the gross motion mimics some desired dynamics, and at the same time the energetic passivity property towards the human operator and the environment is preserved.

3 SHAPE AND LOCKED SYSTEM DECOMPOSITION

In this section, we decompose the dynamics of the teleoperator system into two systems concerned with the coordination of the two manipulators and the gross motion of the system, respectively. It turns out that for linear dynamically similar teleoperated manipulators, the two resulting systems can be individually controlled, and as long as the individual controller is energetically passive, the combined teleoperator system is also energetically passive.

We begin by defining

$$\mathbf{E} = \frac{\zeta}{\rho+\zeta}[\alpha \mathbf{q}_1 - \mathbf{q}_2]$$

to be the coordination error. Now consider the constant coordinate transformation of the velocity space (i.e. tangent bundle) to get both dynamical and energetic decomposition:

$$\begin{pmatrix} \mathbf{V}_L \\ \dot{\mathbf{E}} \end{pmatrix} = \underbrace{\frac{\zeta}{\rho+\zeta}\begin{pmatrix} \frac{\rho}{\zeta}\alpha & \mathbf{I} \\ \alpha\mathbf{I} & -\mathbf{I} \end{pmatrix}}_{\mathbf{S}} \begin{pmatrix} \dot{\mathbf{q}}_1 \\ \dot{\mathbf{q}}_2 \end{pmatrix}. \tag{5}$$

Notice that when the teleoperator is perfectly coordinated, i.e. $\alpha\dot{\mathbf{q}}_1 = \dot{\mathbf{q}}_2$, then $\mathbf{V}_L = \dot{\mathbf{q}}_2 = \alpha\dot{\mathbf{q}}_1$. Similarly, consider the compatible transformation of the forces:

$$\begin{pmatrix} \mathbf{T}_L \\ \mathbf{T}_E \end{pmatrix} = \mathbf{S}^{-T}\begin{pmatrix} \rho\mathbf{T}_1 \\ \mathbf{T}_2 \end{pmatrix}, \qquad \begin{pmatrix} \mathbf{F}_L \\ \mathbf{F}_E \end{pmatrix} = \mathbf{S}^{-T}\begin{pmatrix} \rho\mathbf{F}_1 \\ \mathbf{F}_2 \end{pmatrix}. \tag{6}$$

Under these transformations, the dynamics of the teleoperator system (1) block diagonalizes into:

$$\mathbf{M}_L \dot{\mathbf{V}}_L = \mathbf{T}_L + \mathbf{F}_L, \qquad \mathbf{M}_L = \frac{\zeta+\rho}{\zeta}\mathbf{M}_2 \tag{7}$$

and

$$\mathbf{M}_E \ddot{\mathbf{E}} = \mathbf{T}_E + \mathbf{F}_E, \qquad \mathbf{M}_E = \frac{\rho(\zeta+\rho)}{\zeta^2}\mathbf{M}_2. \tag{8}$$

We may consider (8) as a n-DOF mechanical system with configuration coordinates $\mathbf{E}$, inertia $\mathbf{M}_E$, control input $\mathbf{T}_E$ and environment force $\mathbf{F}_E$; and (7) as a separate $n-$ DOF mechanical system with velocity $\mathbf{V}_L$, inertia $\mathbf{M}_L$, control $\mathbf{T}_L$ and environment force $\mathbf{F}_L$. We shall refer to (8) as the *shape* system since it determines the relative configuration (coordination) of the two manipulators. Similarly, (7) will be referred to as the *locked* system since it determines $\mathbf{V}_L$, which is the average velocity (gross motion) of the two manipulators.

The surprising aspect of the coordinate transformation (5) is that the scaled kinetic energy of $2n$-DOF teleoperator system in Eq.(4) can be written as a sum of the kinetic energies of the *shape* and of the *locked* systems.:

$$\kappa_\rho(\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2) = \frac{1}{2}\mathbf{V}_L^T \mathbf{M}_L \mathbf{V}_L + \frac{1}{2}\dot{\mathbf{E}}^T \mathbf{M}_E \dot{\mathbf{E}}. \tag{9}$$

The following proposition is the consequence of this observation.

Proposition 1 *If the* locked *system in* (7) *and the* shape *system in* (8) *are individually controlled using* $\mathbf{T}_L$ *and* $\mathbf{T}_E$ *respectively, such that each system is individually passive: i.e.* $\exists\ c_L, c_s$ *s.t.* $\forall\ \mathbf{F}_L, \mathbf{F}_E$, *and* $\forall t \geq 0$,

$$\int_0^t \mathbf{F}_L^T \mathbf{V}_L d\tau \geq -c_L^2, \quad \int_0^t \mathbf{F}_E^T \dot{\mathbf{E}} d\tau \geq -c_s^2, \tag{10}$$

then, the teleoperator system (1) *is passive with respect to the supply rate* $s_\rho(\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2, \mathbf{F}_1, \mathbf{F}_2)$ *in* (2) *(i.e.* (3) *is satisfied).*

Proof: Notice that

$$s_\rho(\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2, \mathbf{F}_1, \mathbf{F}_2) = \mathbf{F}_L^T \mathbf{V}_L + \mathbf{F}_E^T \dot{\mathbf{E}}$$

where $s(\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2, \mathbf{F}_1, \mathbf{F}_2)$ is the scaled supply rate defined in (2), Thus, we have

$$\frac{d}{dt}\kappa_\rho(\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2) = s_\rho(\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2, \mathbf{F}_1, \mathbf{F}_2)$$

which gives rise to the passivity inequality in (3) on integration.

■

Thus, the control of the $2n-$DOF system (1) reduces to the independent control of the two n-DOF *locked* and *shape* systems, (7)-(8), while maintaining their individual passivity properties. The objective for the *shape* system is to regulate $(\mathbf{E}, \dot{\mathbf{E}})$ at $(0,0)$; while the objective for the *locked* system is to mimic the desired *target locked system dynamics* given as below.

4 LOCKED SYSTEM CONTROL

As mentioned earlier, our control philosophy is that the teleoperated system should appear to be a $n-$DOF passive mechanical tool with which both the work environment and the human operator commonly interact. The mechanical tool, which is achieved when the master and slave robots are perfectly coordinated, is referred to as the *locked* system.

Suppose that the two manipulators in the system are perfectly coordinated, we can take the coordinate of the *locked* system, $\mathbf{q}_L \in \Re^n$ to be either $\mathbf{q}_1$ or $\mathbf{q}_2$. If we take the coordinate to be $\mathbf{q}_L = \mathbf{q}_2$, we would like the resulting locked system (expressed in the units and dimension of robot 2) to behave according to *the target locked system dynamics* as given by:

$$\mathbf{M}_L \ddot{\mathbf{q}}_L + \mathbf{C}_L(\mathbf{q}_L, \dot{\mathbf{q}}_L)\dot{\mathbf{q}}_L = \rho\alpha^{-T}\mathbf{F}_1 + \mathbf{F}_2 \qquad (11)$$

where $\rho > 0$ is the desired power scaling to amplify / attenuate the operator's force and power, and $\mathbf{M}_L$, as defined in (7), is the apparent inertia that appears to the environment of robot 2. The $n \times n$ skew symmetric matrix

$$\mathbf{C}_L(\mathbf{q}_L, \gamma\dot{\mathbf{q}}_L) = -\mathbf{C}_L(\mathbf{q}_L, \gamma\dot{\mathbf{q}}_L)^T = \gamma\mathbf{C}_L(\mathbf{q}_L, \dot{\mathbf{q}}_L),$$

specifies the unforced dynamics of the target locked system by formally defining a connection [9] of the target system. For example, the geodesics of the connection can be used to prescribe the preferred direction of travel for the target system. The requirement that $\mathbf{C}_L(\mathbf{q}_L, \dot{\mathbf{q}}_L)$ should be skew symmetric is to ensure that the target locked system is a passive system with respect to a supply rate that respects the desired power scaling. For path guidance applications, $\mathbf{C}_L(\mathbf{q}_L, \dot{\mathbf{q}}_L)$ may be designed using passive velocity field control technique as applied to contour following problems [6].

Notice from (6) that $\mathbf{F}_L = \rho\alpha^{-T}\mathbf{F}_1 + \mathbf{F}_2$ which is exactly the second term on the RHS of (11), the target system dynamics. Moreover, when $(\mathbf{E}, \dot{\mathbf{E}}) = (0,0)$, then $\mathbf{V}_L = \alpha\dot{\mathbf{q}}_1 = \dot{\mathbf{q}}_2$. Therefore, if we define the *locked* configuration of the two manipulators (in the robot 2 units and dimensions) to be

$$\mathbf{q}_L = \frac{\zeta}{\zeta+\rho}\left[\frac{\rho}{\zeta}\alpha\mathbf{q}_1 + \mathbf{q}_2\right]$$

then a possible control for the *locked* system is:

$$\mathbf{T}_L = -\mathbf{C}_L(\mathbf{q}_L, \mathbf{V}_L)\mathbf{V}_L, \qquad (12)$$

which duplicates the target locked system dynamics (11). In addition, if indeed $\mathbf{E}(t) \to 0$, then the dynamics of robot 2 also converges to (11).

Since $\mathbf{C}_L(\mathbf{q}_2, \dot{\mathbf{q}}_2)$ is skew-symmetric, by considering $\frac{1}{2}\mathbf{V}_L^T(t)\mathbf{M}_L\mathbf{V}_L(t)$ to be a storage function candidate, the locked system (7) under the control (12) can easily be shown to be passive in the sense of (10).

5 SHAPE SYSTEM CONTROL

The shape system control should make the master and slave manipulators to be perfectly coordinated with each other. Recall that the shape system dynamics are given by:

$$\mathbf{M}_E \ddot{\mathbf{E}} = \mathbf{T}_E + \mathbf{F}_E \qquad (8)$$

where $\mathbf{T}_E$ is the control torque and $\mathbf{F}_E$ is the transformed disturbance forces from the human operator and the work environment. We wish to design a control law for $\mathbf{T}_E$ so that the shape system dynamics is passive with respect to the supply rate $s_E(\mathbf{F}_E, \dot{\mathbf{E}}) = \mathbf{F}_E^T\dot{\mathbf{E}}$, and also that the coordination error $\mathbf{E} \to 0$.

The controller structure below ensures that passivity is enforced. First, we augment the system with two fictitious energy storages which are to be implemented within the controller. One is associated with the potential energy in a fictitious spring with a positive definite spring constant $\mathbf{K} \in \Re^{n\times n}$. The other is associated with the kinetic energy of a fictitious flywheel with dynamics

$$M_f \ddot{x}_f = T_f$$

where $M_f > 0$ and $x_f \in S^1$ are the inertia and the configuration of the flywheel respectively. The *augmented system* becomes:

$$\begin{pmatrix} \mathbf{M}_E & 0 & 0 \\ 0 & \mathbf{K} & 0 \\ 0 & 0 & M_f \end{pmatrix}\begin{pmatrix} \ddot{\mathbf{E}} \\ \dot{\mathbf{E}} \\ \ddot{x}_f \end{pmatrix} = \begin{pmatrix} \mathbf{T}_E + \mathbf{F}_E \\ \mathbf{K}\dot{\mathbf{E}} \\ T_f \end{pmatrix}$$

and the total energy of the *augmented system* is given as:

$$\kappa_E(\mathbf{E}, \dot{\mathbf{E}}, \dot{x}_f) = \frac{1}{2}\dot{\mathbf{E}}^T\mathbf{M}_E\dot{\mathbf{E}} + \frac{1}{2}\mathbf{E}^T\mathbf{K}\mathbf{E} + \frac{1}{2}M_f\dot{x}_f^2. \qquad (13)$$

Consider the control law of the form:

$$\begin{bmatrix} \mathbf{T}_E \\ \mathbf{T}_f \end{bmatrix} = \begin{bmatrix} -\mathbf{S}_{fb}(t) & -\mathbf{K} & -\mathbf{U}_{fb}(t) \\ \mathbf{U}_{fb}^T(t) & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\mathbf{E}} \\ \mathbf{E} \\ \dot{x}_f \end{bmatrix}$$
$$+\, p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}})\begin{bmatrix} -\mathbf{S}_{ff}(t) & 0 & -\mathbf{U}_{ff}(t) \\ \mathbf{U}_{ff}^T(t) & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\mathbf{E}} \\ \mathbf{E} \\ \dot{x}_f \end{bmatrix} \qquad (14)$$

where $p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}})$ is a switching function to turn on/off the feedforward action, $\mathbf{S}_{fb}(t)$ and $\mathbf{U}_{fb}(t)$ are positive semi-definite positive matrices to give damping effect in feedback control, and $\mathbf{S}_{ff}(t) = -\mathbf{S}_{ff}^T(t)$ and $\mathbf{U}_{ff}(t)$ are responsible for generating feedforward cancellation of mismatched human and environment based on force measurement. These matrices are defined below.

If $\mathbf{S}_{fb}(t)$ and $\mathbf{U}_{fb}(t)$ are defined s.t.

$$\mathbf{S}_{fb}(t)\dot{\mathbf{E}} + \mathbf{U}_{fb}(t)\dot{x}_f = \mathbf{B}\dot{\mathbf{E}},$$

then a constant damping effect is achieved in feedback, where $\mathbf{B} \in \Re^{n\times n}$ is a strictly positive definite damping matrix. One possibility is to define:

$$\mathbf{U}_{fb}(t) = g(\dot{x}_f(t))\mathbf{B}\dot{\mathbf{E}}(t), \quad \mathbf{S}_{fb}(t) = (1 - g(\dot{x}_f(t))\dot{x}_f(t))\mathbf{B} \tag{15}$$

where

$$g(\dot{x}_f) = \begin{cases} \frac{1}{\dot{x}_f} & |\dot{x}_f| > f_0 \\ \frac{1}{f_0}\mathrm{sign}(\dot{x}_f) & 0 \neq |\dot{x}_f| \leq f_0 \\ \frac{1}{f_0} & |\dot{x}_f| = 0 \end{cases} \tag{16}$$

and $f_0 > 0$ is a threshold on the flywheel speed $\dot{x}_f$ to ensure that $\mathbf{U}_{fb}(t)$ is bounded.

Using only the feedback action cannot ensure that $\mathbf{E} \to 0$ in the presence of mismatched operator / environment force $\mathbf{F}_E$. If $\mathbf{F}_E$ is completely canceled, then the damping effect will cause $\mathbf{E} \to 0$ exponentially, so we assume that the teleoperator is equipped with force sensors so that $\mathbf{F}_E$ can be computed from (6) and canceled out with a feedforward action. However, an ordinary feedforward action can lead to destroying the passivity.

To achieve *passive feedforward*, the matrices $\mathbf{S}_{ff}(t)$ and $\mathbf{U}_{ff}(t)$ are designed so that,

$$\begin{pmatrix} -\mathbf{S}_{ff}(t) & -\mathbf{U}_{ff}(t) \\ \mathbf{U}_{ff}^T(t) & 0 \end{pmatrix} \begin{pmatrix} \dot{\mathbf{E}} \\ \dot{x}_f \end{pmatrix} = p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) \begin{pmatrix} -\mathbf{F}_E \\ \frac{1}{\dot{x}_f}\mathbf{F}_E^T\dot{\mathbf{E}} \end{pmatrix} \tag{17}$$

where $\mathbf{S}_{ff}(t) \in \Re^{n\times n}$ is a skew symmetric matrix to ensure that the feedforward action preserves the passivity, and $p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}})$ is a switching function:

$$p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) = \begin{cases} 1 & \text{if } (\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) \in \mathcal{C} \\ 0 & \text{otherwise} \end{cases}$$

where $\mathcal{C} \subset \{(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) \in \Re^{2n+1}\}$,as in Figure 3, is an invariant region having some "good" convergence properties with feedforward action. The region $\mathcal{C}$ will be determined later. Typically $(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}})$ would belong to $\mathcal{C}$ if $|\dot{x}_f|$ is sufficiently large, and / or the error $(\mathbf{E}, \dot{\mathbf{E}})$ are sufficiently small. This prevents the feedforward portion of the control law (14) from becoming unbounded. When $(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}})$ is indeed in $\mathcal{C}$, then the feedforward action is activated and the environment / operator force $\mathbf{F}_E$ is exactly canceled out.

With both the feedback and feedforward control (14), the coordination error (shape system) dynamics (8) become:

$$\begin{bmatrix} \mathbf{M}_E & 0 & 0 \\ 0 & \mathbf{K} & 0 \\ 0 & 0 & M_f \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{E}} \\ \dot{\mathbf{E}} \\ \ddot{x}_f \end{bmatrix} = \begin{bmatrix} -\mathbf{S}_{fb}(t) & -\mathbf{K} & -\mathbf{U}_{fb}(t) \\ \mathbf{K} & 0 & 0 \\ \mathbf{U}_{fb}(t)^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{E}} \\ \mathbf{E} \\ \dot{x}_f \end{bmatrix}$$
$$+ p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) \begin{bmatrix} -\mathbf{S}_{ff}(t) & 0 & -\mathbf{U}_{ff}(t) \\ 0 & 0 & 0 \\ \mathbf{U}_{ff}^T(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{E}} \\ \mathbf{E} \\ \dot{x}_f \end{bmatrix} + \begin{bmatrix} \mathbf{F}_E \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

This controller does indeed preserve passivity of the shape system even when it uses both feedback and feedforward control (14) because,

$$\frac{d}{dt}\kappa_E(t) = \mathbf{F}_E^T \cdot \dot{\mathbf{E}} + \left(\dot{\mathbf{E}}^T \ \mathbf{E}^T \ \dot{x}_f\right) \begin{pmatrix} -\mathbf{S}_{fb} - \mathbf{S}_{ff} & -\mathbf{K} & -\mathbf{U}_{fb} - \mathbf{U}_{ff} \\ \mathbf{K} & 0 & 0 \\ \mathbf{U}_{fb}^T + \mathbf{U}_{ff}^T & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\mathbf{E}} \\ \mathbf{E} \\ \dot{x}_f \end{pmatrix}$$
$$= \mathbf{F}_E^T \cdot \dot{\mathbf{E}} - \dot{\mathbf{E}}^T \mathbf{S}_{fb} \dot{\mathbf{E}} \leq s(\mathbf{F}_E, \dot{\mathbf{E}})$$

where $\kappa_E(\mathbf{E}(t), \dot{\mathbf{E}}(t), \dot{x}_f(t))$ denotes the total energy function defined in (13) evaluated at time t. Utilizing the fact that $\kappa_E(t) \geq 0$, the passivity property of the shape system under the control (14) is obtained on integration.

We now analyze the shape system control presented above, and determine the invariant region $\mathcal{C}$.

Proposition 2 *Without feedforward cancellation, the shape system variables* $\mathbf{E}(t)$ *and* $\dot{\mathbf{E}}(t)$ *are ultimately bounded with bounded environment / operator force* $\mathbf{F}_E$*. Moreover, the energy in the fictitious flywheel,* $\frac{1}{2}M_f\dot{x}_f^2$ *is non-decreasing.*

Proof: Under the feedback control alone,

$$\mathbf{M}_E\ddot{\mathbf{E}} + \mathbf{B}\dot{\mathbf{E}} + \mathbf{K}\mathbf{E} = \mathbf{F}_E \tag{19}$$

where $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{B}$ are positive definite. The ultimate boundedness of $\mathbf{E}$ and $\dot{\mathbf{E}}$ are established using the fact that the linear system is exponentially stable.

To see that the kinetic energy of the flywheel is non-decreasing, observe from the shape system dynamics with the control law that

$$\frac{d}{dt}\left[\frac{1}{2}M_f\dot{x}_f^2\right] = \dot{x}_f g(\dot{x}_f)\dot{\mathbf{E}}^T\mathbf{B}\dot{\mathbf{E}} \geq 0.$$

■

Unlike regular damping, the damping effect in the controller is at least partially implemented using a flywheel. The extent of this depends on the current value $\dot{x}_f$. Thus, instead of a damper

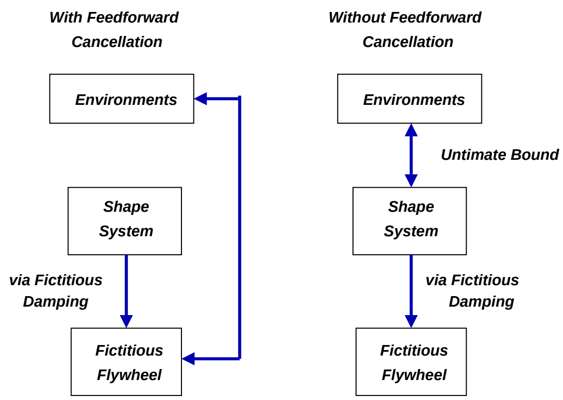

Figure 2. Energy flow with or without feedforward cancellation in the augmented shape system. With feedforward cancellation, energy flows between the environments and the fictitious flywheel without affecting the real shape system. Without feedforward cancellation, energy flows only into the fictitious flywheel to increase flywheel energy monotonically so that the system enters into the invariant region $\mathcal{C}$.

simply dissipating the energy to the environment, the energy developed in the damper is stored in the flywheel. The stored flywheel energy will be used to provide the feedforward cancellation of the mismatched operator / environment force $\mathbf{F}_E$ without violating energetic passivity. Energy flow in the augmented shape system with or without feedforward cancellation is illustrated in Figure 2.

Remark 5.1 *We compute the ultimate bound using a Lyapunov function. For the exponentially stable system* (19)*, we can define positive define matrices* $\mathbf{P} \in \Re^{2n\times 2n}$ *and* $\mathbf{Q} \in \Re^{2n\times 2n}$*, and a Lyapunov function,*

$$V(t) = \frac{1}{2}(\mathbf{E}^T\ \dot{\mathbf{E}}^T)\mathbf{P}\begin{pmatrix}\mathbf{E}\\ \dot{\mathbf{E}}\end{pmatrix} \tag{20}$$

so that for some $\gamma > 0$ *and* $\zeta > 0$,

$$\begin{aligned}\dot{V}(t) &= -(\mathbf{E}^T\ \dot{\mathbf{E}}^T)\mathbf{Q}\begin{pmatrix}\mathbf{E}\\ \dot{\mathbf{E}}\end{pmatrix} + (\mathbf{E}^T\ \dot{\mathbf{E}}^T)\mathbf{P}\begin{pmatrix}0\\ \mathbf{I}_{n\times n}\end{pmatrix}\mathbf{F}_E \\ &\le -\gamma V(t) + \zeta V^{\frac{1}{2}}(t)\|\mathbf{F}_E\| \end{aligned} \tag{21}$$

where γ *is the exponential convergence rate (which may be estimated from* $\gamma \ge \frac{\underline{\sigma}[\mathbf{Q}]}{\bar{\sigma}[\mathbf{P}]} > 0$*, where* $\underline{\sigma}[\cdot]$ *and* $\bar{\sigma}[\cdot]$ *denote the minimum and maximum singular values of their arguments respectively). From* (21)*, the ultimate bound for the Lyapunov function is found to be*

$$\bar{V} = \left[\frac{\zeta}{\gamma}F_{max}\right]^2 \tag{22}$$

where $\|\mathbf{F}_E(t)\| \le F_{max}$*. Therefore, if* $V^{\frac{1}{2}}(0) \le \frac{\zeta}{\gamma}F_{max}$*, then* $V^{\frac{1}{2}}(t) \le \frac{\zeta}{\gamma}F_{max}$ *for all* $t \ge 0$*. Otherwise, for any* $V^{\frac{1}{2}}(0)$*, and for any* $\varepsilon > 0$*, there exists* $T > 0$*, so that* $V^{\frac{1}{2}}(t) \le \frac{\zeta}{\gamma}F_{max} + \varepsilon$ *whenever* $t \ge T$*.*

Define first a subregion $\mathcal{C}_1 \subset \mathcal{C} \subset \Re^{2n+1}$ in Figure 3 to be:

$$\mathcal{C}_1 := \{(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) | V^{\frac{1}{2}} \le \frac{\gamma\delta}{2F_{max}}\left[\frac{1}{2}M_f[\dot{x}_f^2 - f_0^2]\right]\}, \tag{23}$$

where $\mathbf{V}(t)$ is the Lyapunov function (20) and $f_0 > 0$ is the threshold in (16).

Proposition 3 *Let* $(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) \in \mathcal{C}_1$ *at* $t = t_1$*, then we can ensure that* $(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) \in \mathcal{C}_1$ $\forall t \ge t_1$ *with the feedforward cancellation* i.e. $p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) = 1$ *in* (14).

Proof: Suppose for a moment that $(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) \in \mathcal{C}_1$ and $p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) = 1$ at $t = t_1$, then the shape system dynamics is given by:

$$\dot{\mathbf{M}}_E\ddot{\mathbf{E}} + \mathbf{B}\dot{\mathbf{E}} + \mathbf{K}\mathbf{E} = 0$$

because the feedforward term (17) is able to exactly cancel out $\mathbf{F}_E$. Using the same Lyapunov function in Remark 5.1,

$$V(t) = \frac{1}{2}(\mathbf{E}^T\ \dot{\mathbf{E}}^T)\mathbf{P}\begin{pmatrix}\mathbf{E}\\ \dot{\mathbf{E}}\end{pmatrix} \ge \delta^2\|\dot{\mathbf{E}}\|^2$$

for some $\delta > 0$, so that

$$\dot{V}(t) = -(\mathbf{E}^T\ \dot{\mathbf{E}}^T)\mathbf{Q}\begin{pmatrix}\mathbf{E}\\ \dot{\mathbf{E}}\end{pmatrix} \le -\gamma V(t).$$

Continuing with the supposition that $p(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) = 1$ and consider the energy stored in the flywheel, we have

$$\begin{aligned}\frac{d}{dt}\frac{M_f(\dot{x}_f)^2}{2} &= T_f\dot{x}_f = \mathbf{U}_{fb}^T\dot{\mathbf{E}}\dot{x}_f + \mathbf{F}_E^T\dot{\mathbf{E}} \\ &= \dot{x}_f g(\dot{x}_f)\dot{\mathbf{E}}^T\mathbf{B}\dot{\mathbf{E}} + \mathbf{F}_E^T\dot{\mathbf{E}} \ge -\|\mathbf{F}_E\|\,\|\dot{\mathbf{E}}\| \ge -\frac{F_{max}}{\delta}V^{\frac{1}{2}}(t)\end{aligned}$$

Therefore, by integrating the above relation, we can ensure that

$$\sqrt{(\mathbf{E}^T\ \dot{\mathbf{E}}^T)\frac{\mathbf{P}}{2}\begin{pmatrix}\mathbf{E}\\ \dot{\mathbf{E}}\end{pmatrix}} = V(t)^{\frac{1}{2}} \le \frac{\gamma\delta}{2F_{max}}\left[\frac{1}{2}M_f[\dot{x}_f(t)^2 - f_0^2]\right],$$

$\forall t \ge t_1$, since $V(t) \le e^{-\gamma t}V(0)$.

■

Define f_1 to be the threshold for the flywheel energy s.t.:

$$\frac{\gamma\delta}{2F_{max}}\left[\frac{1}{2}M_f[f_1^2 - f_0^2]\right] = \bar{V}^{\frac{1}{2}} + \varepsilon = \frac{\zeta}{\gamma}F_{max} + \varepsilon$$

where $\varepsilon > 0$ is a small number. The rationale for the definition of f_1 is that the boundary of $\mathcal{C}_1$ intersects with the ultimate bound

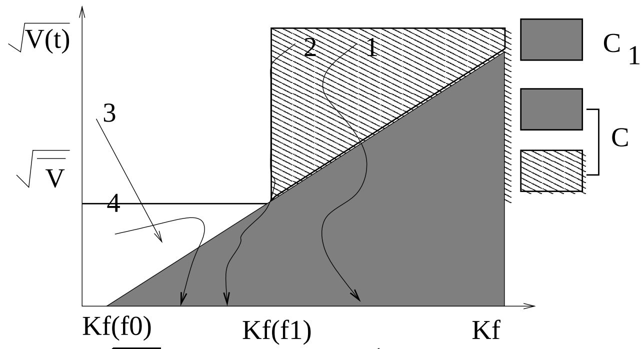

Figure 3. $\sqrt{V(t)}$ versus $K_f(\dot{x}_f(t)) = \frac{1}{2}M_f\dot{x}_f^2$ where $V(t)$ is the Lyapunov function in (20), $K_f(\dot{x}_f)$ is the kinetic energy in the flywheel. Also shown are 4 sample paths of $(K_f(\dot{x}_f(t)), \sqrt{V(t)})$. In Paths 1 and 2, $V(t) \to 0$, and in Path 3, $V(t)$ becomes ultimately bounded by $\bar{V}$. In Path 4, $V(t) \to 0$ but it is not guaranteed.

in Remark 5.1 when $\dot{x}_f = f_1$. The regions $\mathcal{C}$, $\mathcal{C}_1$, and the relationships to the threshold f_1, and the ultimate bound $\bar{V}$ are illustrated in Figure 3.

We are now ready to define the invariant region $\mathcal{C}$, which is used to define (17), to be

$$\mathcal{C} := \mathcal{C}_1 \cup \{(\dot{x}_f, \mathbf{E}, \dot{\mathbf{E}}) | \dot{x}_f > f_1\}. \tag{24}$$

Theorem 1 *Consider the shape system* (8) *under the controller structure, the damping effects given by* (15),(16)*, and feedforward effects provided by* (17) *with the switching region $\mathcal{C}$ given in* (24)*.*

1. *The closed loop shape system is passive with respect to the supply rate $s_E(\mathbf{F}_E, \dot{\mathbf{E}}) = \mathbf{F}_E^T\dot{\mathbf{E}}$;*
2. *Suppose that $\|\mathbf{F}_E(t)\| \le F_{max}$ $\forall t \ge 0$. For any initial condition $(\dot{x}_f(0), \mathbf{E}(0), \dot{\mathbf{E}}(0))$, the Lyapunov function $V(t)$ in* (20) *will be ultimately bounded by $\bar{V}$ in* (22)*.*
3. *Suppose again that $\|\mathbf{F}_E(t)\| \le F_{max}$ for all $t \ge 0$. Then the region $\mathcal{C}$ defined in* (24) *is invariant.*
4. *If furthermore, $(\dot{x}_f(0), \mathbf{E}(0), \dot{\mathbf{E}}(0)) \in \mathcal{C}$, then $\mathbf{E}(t) \to 0$, $\dot{\mathbf{E}}(t) \to 0$, and $\dot{x}_f(t) \ge f_0$ $\forall t \ge 0$.*
5. *If $\dot{x}_f(0) \ne 0$ and $V(t) \not\to 0$, then $\dot{\mathbf{E}}(0) \to 0$.*

Proof:

1. Passivity has also already been demonstrated using the total energy $\kappa_E(t)$ in (13) as storage function candidate.
2. Notice that the feedforward action either cancels or leaves alone the environment / operator force $\mathbf{F}_E$. Thus the effective bound on $\mathbf{F}_E$ does not increase over the case when only damping effect is used. Thus Proposition 2 and Remark 5.1 apply.
3. Proposition in 3 shows that the subregion $\mathcal{C}_1$ is invariant. If the initial state belongs to $\mathcal{C}$ but does not belong to $\mathcal{C}_1$, then 1) $\dot{x}_f(t) \ge f_1$ for all $t \ge 0$; 2) eventually, $V(t) \le \bar{V} + \varepsilon$. The 1st result is due to the fact that the flywheel energy is non-decreasing when the feedforward term is not used (Prop. 2). The 2nd result is due to ultimate boundedness. Because of the way that f_1 is defined, the state must eventually enter $\mathcal{C}_1$ and remains there.
4. Since eventually, the state enters $\mathcal{C}_1$, $\mathbf{F}_E$ is canceled out, $\mathbf{E}(t), \dot{\mathbf{E}}(t) \to 0$ exponentially. The definition of $\mathcal{C}_1$ ensures that $\dot{x}_f(t)$ will not be depleted below f_1.
5. If $V(t) \not\to 0$, then the state must not have entered $\mathcal{C}$. From Proposition 2, this means that $V(t)$ would be ultimately bounded, and the energy in the flywheel would be non-decreasing. Since the state does not enter $\mathcal{C}$, $\dot{x}_f$ is bounded by f_1 and so $\dot{x}_f(t)$ must converge. This implies that $\dot{\mathbf{E}} \to 0$.

■

Roughly speaking, the shape system control presented above consists of spring / damper term for stabilization, and a feedforward term to cancel out the effect of $\mathbf{F}_E$. The innovation of the controller lies in that fact that it makes use of the energy stored in the fictitious flywheel to ensure that control system is energetically passive.

In Theorem 1, we can only show that $\mathbf{E} \to 0$ if the system state enters the region $\mathcal{C}$. Otherwise, $\mathbf{E}$ would only be ultimately bounded. In reality, however, the state is likely to eventually to enter $\mathcal{C}$ or $\mathbf{E} \to 0$ and $\dot{\mathbf{E}} \to 0$ if the shape system is constantly being excited by time varying mismatched environment / human force $\mathbf{F}_E(t)$. The reason is that if $\mathbf{F}_E(t)$ excites the system perpetually, it is not likely that $\mathbf{E} \to 0$ which by Theorem 1 item 5 is a necessity for $\mathbf{E} \not\to 0$ or $\mathbf{E} \not\to 0$. Thus, an operator can shake up the teleoperator first in order to cause the state to enter $\mathcal{C}$ before using the teleoperator for manipulation. Hereafter, if the measurement of $\mathbf{F}_E$ is accurate, then $\mathcal{C}$ will be invariant, and the teleoperator will behave exactly like a locked system.

6 EXPERIMENTAL RESULTS

Experiments are performed on the teleoperated system as shown in Figure 1. The master manipulator is equipped with a force sensor and excited by human and an external environment. Both kinematic and power scalings are set to be 1 for simplicity. With different power scaling, the human operator feels different inertia of the tool represented by the locked system. We consider two sets of initial conditions: 1) zero flywheel energy and coordination error (from origin in Figure 4); 2) outside of the invariant region $\mathcal{C}$ with nonzero flywheel energy and coordination error. The trajectories representing energy flow and $\mathcal{C}$ are shown in Figure 4 and the actual angle (position) trajectories are shown in Figure 5 for the two initial conditions respectively.

With the first initial condition, the human operator should excite the system to generate sufficient energy so that the system enters into $\mathcal{C}$, and feedforward cancellation is activated. Before the system enters into $\mathcal{C}$, the feedback part is used alone, so the coordination performance is poor (with error around 5 *deg*)

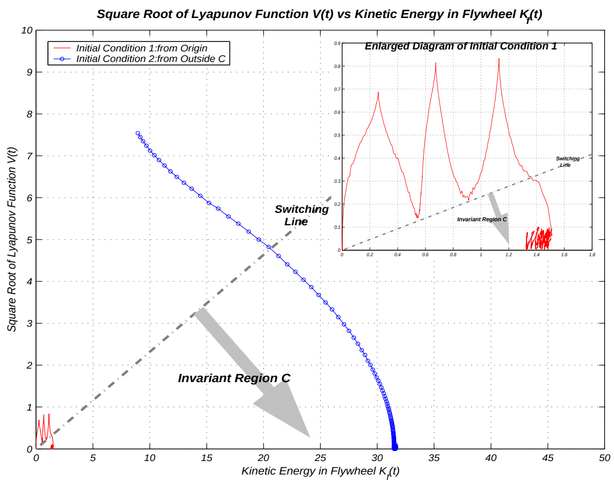

Figure 4. Trajectories for two sets of initial conditions traced out in the $\sqrt{V(t)}$ - flywheel energy diagram and the invariant region $\mathcal{C}$. See figure 3 for interpretation. Result of the first initial condition is enlarged and shown separately. Note the ripples after entering $\mathcal{C}$ with the first condition due to real implementation.

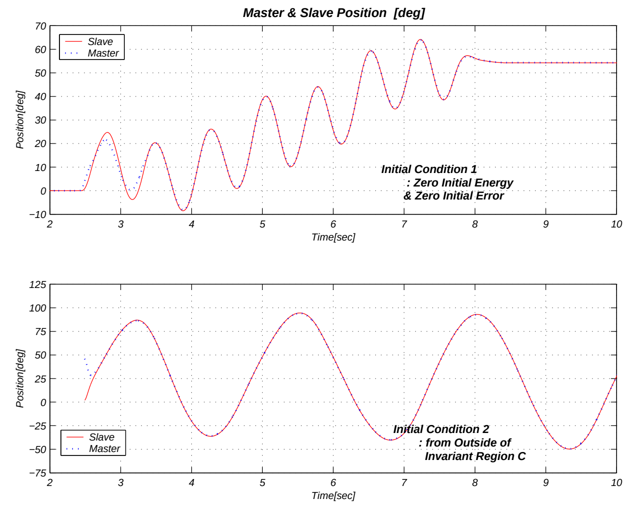

Figure 5. Real angle trajectories of master and slave manipulators with two initial conditions. Note that the system achieves locking after getting energy from environments (human operator) in the first initial condition.

due to mismatched operator/environment force. When the system gets enough energy from human operator, the feedforward cancellation is turn on and near perfect coordination (with error less than 0.5 *deg*) is achieved.

The second initial condition has nonzero initial flywheel energy and coordination error. The system starts outside of $\mathcal{C}$, but it enters into the invariant region $\mathcal{C}$ quickly (after 0.2*second*) and achieves the near perfect coordination after that. This quick locking is mainly caused by the energy flow via the fictitious damping as in Figure 2.

The bandwidth of the feedforward cancellation degrades in real implementation. The ripples in $\mathcal{C}$ in Figure 4 with the fist initial condition shows the effects of noise, discretization and actuator limitation. The ultimate bound is too large to be shown in Figure 4. This large ultimate bound, which represents the guaranteed coordination performance of feedback control alone, highlights the importance of feedforward cancellation.

7 CONCLUSIONS

A passive control approach has been proposed for linear dynamically similar teleoperator systems. The control law coordinates the two robots in the system and ensures that the closed loop system is energetically passive. The control law makes use of fictitious energy storages to generate both feedback and feedforward control based upon both kinematic and force measurements. By the use of feedforward compensation, the teleoperated system becomes perfectly coordinated despite the presence of mismatch in environment / operator force. Experimental implementation validates the proposed control scheme. Although the idea of decomposition into shape and locked system does not apply to generic nonlinear teleoperator systems, the main concept of passive feedback and feedforward control should be applicable.

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