Jet Attachment Behavior using Counterflow Thrust Vectoring

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A modeling and experimental study was performed to examine the jet attachment process, which can develop during the fluidic thrust vector control of subsonic and supersonic jets. Experiments performed in rectangular jets up to Mach 1.4 revealed that continuous control of jet exhaust is possible with secondary mass flow rates of the order of 1% of the primary jet mass. Under certain operating conditions, the primary jet attaches to the adjacent collar wall; model predictions agree well with measurements. The study suggests that the jet attachment process can be avoided by two principal means, namely collar truncation and pressure release.

Introduction

Recent studies have shown the applicability of vectoring subsonic and supersonic jets using asymmetrically applied counterflow in the presence of a short extension or collar. This concept has received considerable attention due to absence of mechanical actuation and the inherently fast time response of fluidics. However, implementation of fluidic thrust vector control schemes must first demonstrate robust operation in the absence of bistability or hysteresis, the most common form of which is due to jet attachment to the adjacent collar surface. Figure 1 illustrates the attachment of a slightly underexpanded jet to an adjacent curved surface. The entrainment of the primary jet is inhibited by the presence of the surface, giving rise to jet curvature and stable attachment. Counterflow thrust vectoring demands proportional control of the jet response which is lost during attachment. The goal of this work is to experimentally examine and model the jet attachment process to predict the conditions for which stable attachment occurs and in so doing, design systems where such effects can be avoided over the entire operating domain of the vehicle.

Geometry & Profile Assumptions

Previous counterflow thrust vectoring studies have utilized truncated circular-arc collars offset from the lip of the nozzle. Consequently, we have chosen to investigate this specific geometry for its attachment characteristics in subsonic and supersonic flow regimes. It should be noted that any mathematically descriptive geometry, such as a flat wall, elliptic or parabolic arcs, could be modeled in a similar manner. Upon deciding the geometry of the collar, a control volume is drawn from the exit plane of the nozzle to the point of attachment; see Fig. 2. The radial outward boundary extends sufficiently far to insure that there is no significant pressure gradient across it, and the momentum flow across the boundary is negligible.

We assume that the centerline of the jet arcs towards the wall at a constant radius, R'. The streamwise coordinate, x, originates at the exit plane of the jet, and designates the axial distance along the jet's centerline. The variable \( x_{att} \) denotes the streamwise position, as measured along the centerline, at which the attaching streamline intersects the wall. The transverse
coordinate, \( y \), is everywhere orthogonal to the streamwise coordinate. The following geometric relationships can be immediately written:

\[
R' = \frac{x_{\text{att}}}{\gamma} 
\]

(1)

\[
R \sin \alpha = \left[ R' - Y(x_{\text{att}}) \right] \sin \gamma
\]

(2)

\[
R(1 - \cos \alpha) + G + \frac{H}{2} = R'(1 - \cos \gamma) + Y(x_{\text{att}}) \cos \gamma
\]

(3)

By conserving axial momentum, Görtler's spreading rate parameter, \( \sigma \), can be written in terms of the core length.

\[
\frac{\rho u_1 H}{2} = \int_0^{\Delta y} \frac{\rho u_1^2 dy}{\Delta y} + \int_{\Delta y}^{\infty} \rho u_1^2 \text{sech}^2 \left( \frac{\sigma(y - \Delta y)}{x} \right) dy
\]

(5)

Since the exit plane momentum flux, \( \rho u_1^2 \), is a constant, it can be taken out of the integrand. The resulting integration, leaves:

\[
\frac{H}{2} = \left( \frac{2}{3(\sigma/\epsilon)} \right) \Delta y
\]

(6)

When combined with equation (4), this simplifies to \( \sigma = 4/3 \, c_L \), where \( c_L = x_c/H \). Substituting this expression into the shear layer velocity profile we get in the shear layer:

\[
u(x, y) = u_1 \text{sech}^2 \left( \frac{4c_L(y - \Delta y)}{3x} \right)
\]

when \(|y| \geq \Delta y\).

**Application of Conservative Equations**

We begin by applying the principle of mass conservation. This is done by postulating a mean-flow streamline which exits from the jet and intersects the wall; i.e. the attaching streamline. By definition, no mass can cross a streamline or the solid boundary of the collar. In the absence of counterflow, the attaching streamline must originate at the lip of the nozzle. The mass flow on one side of the streamline is from the primary jet. The flow on the bubble side is entrained from the surroundings, then as the jet impinges on the wall it is turned back and recirculated into the bubble region. Counterflow, when applied, is ultimately drawn from the primary jet during attachment. Thus the attaching streamline originates at some position within the exit plane of the jet. Accounting for the possibility of counterflow, \( Q_c \), an integral equation can be written to mathematically describe this attaching streamline \( Y(x) \):

\[
\frac{Q}{2} - Q_c = \int_0^{\Delta y} \rho u(x, y) \cdot dy
\]

(7)

Substituting our velocity profile assumption, we get:

\[
\frac{Q}{2} - Q_c = \int_0^{\Delta y} \rho u_1 dy + \int_{\Delta y}^{\infty} \rho u_1 \text{sech}^2 \left( \frac{4c_L(y - \Delta y)}{3x} \right) dy
\]

(8)

Integrating, and combining with equation (4):

\[
\frac{Q}{2} - Q_c = \frac{\rho u_1}{2} \left( H - \frac{x_c}{c_L} \right) + \frac{3\rho u_1 x}{4c_L} \tanh \left( \frac{4c_L(Y(x) - \Delta y)}{3x} \right)
\]

(9)

Dividing through by \( Q = \rho u_1 H \) and rearranging, we get an explicit form of the streamfunction \( Y(x) \), which is convenient to write in the following manner:

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**Fig 2: Control volume used to model jet attachment.**
t = \tanh\left(\frac{4c_1(L-x)-\Delta y}{3x}\right) = \frac{2}{3} \frac{4c_1HQ_c}{3x} \frac{Q}{Q_c} \tag{10}

Since the right side is known, this becomes the defining parametric equation for the attaching streamline \(Y(x)\). In the absence of counterflow, it diverges linearly away from the jet centerline, its slope dependent on the core length. With counterflow the streamfunction becomes a more complex hyperbolic function.

To apply conservation of momentum to the control volume shown in Fig. 2, we need some information about the pressure distributions on the boundaries. Our pressure field assumptions, are based on physical arguments, and were validated by experimental measurements. Once all the forces acting on the control volume are evaluated, the unknown variables appearing in Fig. 2 can be evaluated.

**Collar Pressure Profile**

The bubble region originates at the exit plane of the jet and continues to the angular location, \(\alpha\), where the attaching streamline intersects the wall. Due to the nature of a recirculatory region, our first approximation was that the pressure would be constant in the bubble region. This is true within the first half, however owing to fluid impingement on the wall, the pressure rises in the downstream half of the bubble until it reaches a local maximum at the streamline intersection point. We thus assume that the pressure is constant, and equal to \(P_B\), until it reaches an angular location of \(\alpha/2\), and then increases linearly to a stagnation pressure, \(P_I\), at the angular attachment location \(\alpha\).

The shape of the modeled collar pressure profile was evaluated through measurements. Figure 3 shows a typical collar pressure profile plotted against the model assumptions; note the relatively constant pressure in the near-field of the bubble, followed downstream by the local maximum, and subsequent flow acceleration.

A free jet, with stagnation pressure \(P_o\), exhausts into the atmosphere, at \(P_{\infty}\), with a more or less uniform velocity. However, for an attached jet, the pressure in the exit plane is non-uniform. In subsonic flow, this transverse pressure variation will lead to a skewed velocity distribution. Since the bubble pressure is sub-atmospheric, this effect actually augments the momentum flux entering the control volume. We clearly see that the effective momentum influx is considerably greater than the free jet momentum, \(J\).

\[
J_{in} = \rho u_{\infty}^2 H + (P_o - P_{\infty})H
\]

The degree of augmentation depends on the magnitude of the bubble pressure. For the collars used in this study for subsonic flow, augmented momentum accounts for up to 20% of the total jet momentum. In supersonic flow, there is no skewing of the velocity profile, nor is there any momentum augmentation. The momentum inflow to the control volume is identically equal to the free jet momentum.

**Momentum Streams**

As the jet attaches to the wall, a certain fraction of its momentum is turned back. The fluid that turns back due to the wall, recirculates into the bubble region, and is either re-entrained by the jet, or pumped out by the secondary flow system. The flow that continues on, leaves the control volume at an angle \(\psi\) (the computation of this angle will be discussed later). The attaching streamline \(Y(x)\), is the dividing line between the flow that gets turned back, and the flow that continues in a “forward” direction. Thus the magnitude of this forward momentum can be computed by integrating from \(-\infty\) to the location of the attaching streamline \(Y(x)\) at the streamwise attachment location, \(x_{att}\).

\[
J_{out} = \int_{-\infty}^{y_{y=0}} dy
\]

Substituting in the velocity profile assumption, \(J_{out}\) becomes:

\[
J_{out} = \frac{\rho u_{\infty}^2 H}{2} + \int_0^{\Delta y} dy + \int_0^{y_{y=0}} \rho u_{\infty}^2 \sech^2 \left(\frac{4c_1(y-\Delta y)}{3x_{att}}\right) dy
\]

The other momentum outflow from the control volume is from the counterflow being drawn through the gap region, which can typically be neglected as the secondary reverse flow is of the order of 1% of the primary jet mass.
Evaluation of Pressure Terms

In the bubble region, the flow is nominally parallel in the streamwise direction, meaning that radial velocity is negligible, as are the gradients in the streamwise direction. Thus the flow can be approximately by:

\[
\frac{\partial P}{\partial r} = \frac{\mu u_r^2}{r}
\]

This differential equation can be solved by separating both parts and integrating. On the radially outward shear layer, the pressure is equal to that of the ambient, \(P_\infty\), on the inner shear layer, the pressure is equal to that of the bubble, \(P_B\). The integral on the right hand side is just the streamwise momentum of the jet, \(J\). This analysis breaks down as the flow approaches the wall, but is valid in the first half of the bubble region (where it is being applied).

In the downstream half of the bubble region, the pressure increases to its local maximum value, \(P_l\), where the attaching streamline stagnates on the wall. The velocity of a fluid particle traveling on this streamline, \(u_y\), can be computed using the velocity profile assumption presented earlier.

Summation of Forces

Thus far, each individual component of force that acts on the fluid control volume has been evaluated, with the exception of \(F_{out}\), which is the pressure force acting over the outflow control volume boundary. Short of solving the field using CFD, we have no reliable source of information about the pressure field in the exit plane, and the momentum equations provide little useful insight, due to their complexity in the vicinity of the wall. Nevertheless, we can combine all of the forces acting on the control volume into two scalar equations:

\[
\sum F_x = F_{jet} + F_{gap} + F_{c_x} - F_{out} \cos \psi = J_{out} \cos \psi - J_1 - J_{in}
\]

\[
\sum F_y = F_{cy} + F_{out} \sin \psi = -J_{out} \sin \psi
\]

Combining these equations, we obtain the following expressions:

\[
\tan \psi = \frac{-F_{cy}}{J_{in} + J_c + F_{jet} + F_{gap} + F_{c_x}}
\]

\[
F_{out} + J_{out} = \sqrt{(F_{jet} + F_{gap} + F_{c_x} + J_{in} + J_c)^2 + F_{c_y}^2}
\]

The angle \(\psi\) measures the direction in which the flow leaves the control volume. It can be predicted empirically in terms of the attachment angle, \(\alpha\), and the turning angle of the jet, \(\gamma\).

\[
\psi = \phi \alpha + (1 - \phi) \gamma
\]

Where \(\phi\) is an empirically determined weighting factor. A value of \(\phi = 0\) would correspond to the flow leaving the control volume at an angle \(\gamma\), which is the turning angle of the jet centerline. A value of \(\phi = 1\) would correspond to the flow leaving the control volume at an angle \(\alpha\), which is the angle tangent to the wall. However due to the interaction with the wall, and high pressure recovery on the surface, the flow is deflected and, on average, exits the control volume at an angle smaller than both \(\alpha\) and \(\gamma\). This deflection effect has been widely observed in previous wall attachment studies.\(^1\,^3\,^5\) Using a value of \(\phi = 1.3\) gives the best agreement with experimental attachment data, so that is the value chosen throughout. With that, the momentum balance is complete. The only remaining unknown is the non-dimensional potential core length, \(c_L\), which is used to characterize the mixing characteristics of the jet.

Shear Layer Dynamics

The mixing characteristics of a shear layer may be characterized by its spatial growth rate. An advantage of this model is that the details of the velocity field can be manipulated to relate the shear layer growth rate to the potential core length, \(c_L\). This is needed to predict how \(c_L\) changes with changing flow variables; such as Mach number, density ratio, and velocity ratio, which is necessary for extending the attachment model to jets operating at high temperature and high Mach numbers.

We define the shear layer thickness based on uniform shear (vorticity thickness). For the assumed velocity profile, the expression for the vorticity thickness becomes:

\[
\delta_\omega = 0.9743 \frac{X}{c_L}
\]

It is convenient to compute the shear layer growth rate, \(\delta_\omega/\delta x\) and employ the relationship developed by Papamoschou & Roshko,\(^6\) to relate the growth rate to the velocity ratio, density ratio, and convective Mach number, namely:

\[
\frac{\delta_\omega}{\delta x} = \frac{c(M_1)}{2} \left( \frac{u_2}{u_1} \right) \left( \frac{u_2}{u_1} \right) \left( \frac{u_2}{u_1} \right)
\]

The subscripts 1 and 2 refer to the primary and secondary flow streams, respectively. The secondary velocity \(u_2\), is in the opposite direction to the primary
flow velocity $u_1$. The empirically determined constant, $c$, is a function of convective Mach number. It is constant for convective Mach numbers up to about 0.4, then drops by roughly a factor of three at convective Mach numbers greater than 0.9. There is considerable scatter in the data throughout the transonic regime, though it is found to drop in a more or less linear fashion from $M_c = 0.4$ to $M_c = 0.9$. The expression above allows us to predict how the core length, and thus the predicted velocity field, is affected by different jet operating conditions.

**Supersonic Attachment Results**

Experiments were conducted in a Mach 1.4 rectangular jet having an aspect ratio of 4:1 (short dimension of 1.05 cm). The primary nozzle was operated at its design pressure ratio at a stagnation temperature of 300 K; total mass flow of the primary jet was approximately 0.3 kg/sec. Various collar designs were examined, with a circular arc having $R'/H \approx 15$ and a streamwise extent of ~5H used for the results discussed below.

The collar pressure distributions provided in Fig. 4 indicate the basic operating conditions during thrust vector control in the continuous and attached regimes. For modest levels of counterflow, the vacuum pressure on the collar is relatively constant, rising to atmospheric pressure at the streamwise termination of the collar (0.05 m). As vacuum pressure is increased, the basic shape of the profile is unchanged rising monotonically, except for the three profiles indicated in Fig. 4, displaying the local maximum indicative of jet attachment.

Fig. 4 Collar pressure profiles at $M=1.4$, $G/H =0.38$.

The corresponding thrust vector response of the supersonic jet is shown in Fig. 5; jet response is quantified for the purposes of this discussion as the jet vector angle $\delta_v$. The thrust vector angle is proportionally and linearly related to vacuum pressure up to angles of approximately 12°, after which the attachment of the jet to the collar leads to saturation, and insensitivity of jet response to secondary flow.

Fig. 5: Thrust vector angle versus vacuum pressure supplied to the secondary plenum chamber ($M=1.4$, $G/H =0.38$).

Secondary mass flow measurements indicate the balance which must be established between the pumping characteristics of the primary jet (ejector effect) and the vacuum pump performance curve. The mass flow rates provided in Fig. 6 indicate the presence of both coflow and counterflow in the secondary flow path during thrust vector control.

Fig. 6: Secondary mass flow requirements versus jet deflection for $M = 1.4$, $G/H = 0.38$.

When the vacuum system is deactivated, the pumping action of the primary jet leads to coflow through the secondary plumbing system. Due to the restrictions imposed by the collar and hardware, the coflow produces to slightly reduced pressure in the neighborhood of the collar and concomitant flow vectoring; approximately 1% coflow gives rise to jet deflections of ~ 5°. As the
vacuum system is activated, the pumping action of the primary jet works against the secondary vacuum, creating an equilibrium which provides continuous response of jet deflection (linear region in Fig. 5). Eventually a point is reached where no net mass is drawn through the secondary system, corresponding to ~11˚ of vectoring. A similar jet response can be obtained by simply closing a valve to the secondary flow path. This is the classic Coanda "passive" vectoring which leads to a constant jet deflection; the image of Fig. 1 shows the jet responding under these conditions. The disadvantage of passive vectoring is that continuous jet response is not possible; bang-bang control could, however, be exploited under these conditions.

Eventually the vacuum pump wins the "tug-of-war" between the balanced systems and leads to true counterflow drawn in the secondary flow path. Notice that counterflow mass flow rates of ~1% achieve jet deflections of 12˚, after which saturation due to jet attachment is observed. After attachment, modest increases in secondary reverse flow can be expected to originate from the primary jet itself. Experimental results agree well with model predictions as will be discussed in detail at the conference.

References