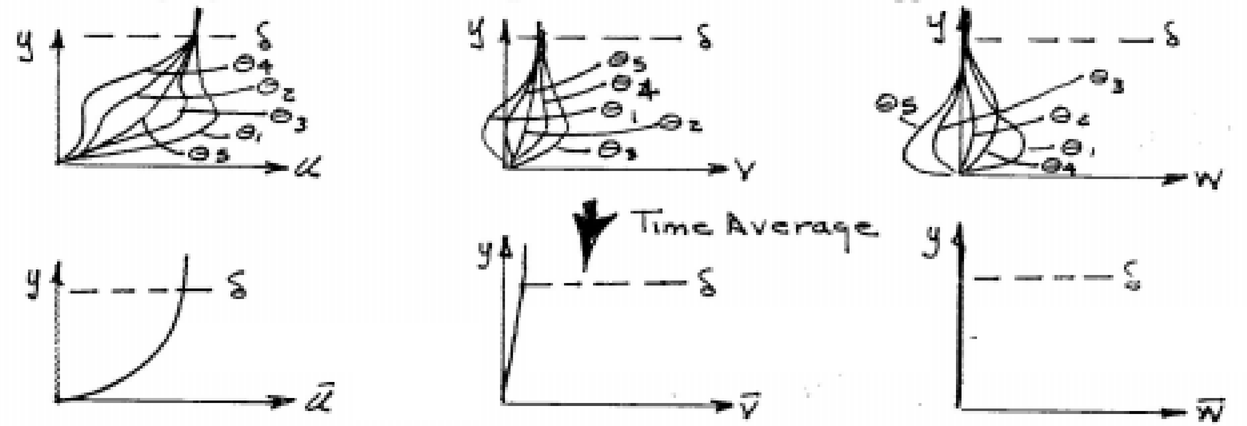
Turbulent Flows: ME8343

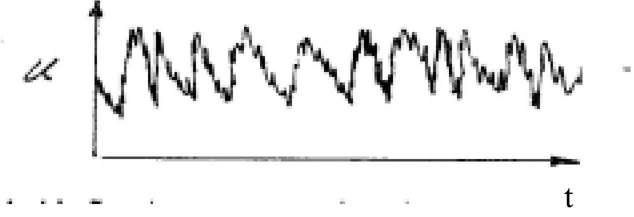
When the laminar boundary layer has become sufficiently mature, instabilities are amplified which rapidly lead to a state of turbulent boundary layer flow. In a turbulent state, there is a mechanism for converting the energy associated with the linear momentum of the flow into energy associated with eddies of a multitude of scales within the flow. The eddies receiving this energy can, by way of another instability mechanism, break up to smaller eddies, and so forth until the eddy scales become so small (and, therefore, the velocity gradients become so steep) that viscosity is able to convert that eddy energy to molecular kinetic energy (thermal energy).

The figure below shows the effect of the onset of these mechanisms.

If one were to be able to see the velocity gradients in the flow u(y), w(y) or v(y) in a 2-D turbulent boundary layer, one would observe just how "random" this flow is.



Velocity readings taken at a single point in the flow would show the waveform $u(t)$ below. Note that there are many frequencies represented in this flow. A boundary layer over a 2m-long flat plate cooled with a flow velocity of 15 m/sec would have, typically, a boundary layer thickness of 25 mm and would have frequencies represented in the velocity waveform of 500 Hz to 20,000 Hz.



We typically deal with this flow in two ways,
(1) writing and solving equations for zones that are so small that we can compute the fluctuations with unsteady (on the eddy scale) momentum equations (unsteady Navier-Stokes Eqns.) or
(2) writing the time-averaged (eddy scale averaged) equations with the hope that the information we had "integrated out" can be replaced, somehow.

The first is called Direct Numerical Simulation (DNS) or, sometimes called Direct Navier-Stokes (DNS) analysis for the unsteady equations must be solved computationally. With the capacity of present computers, this type of solution is limited tosimple geometries and flow conditions and, even then, it is very computationally intensive (and expensive).

The second, solution of the "Reynolds-averaged" equations, is the most common approach, the one we will discuss next.

Reynolds Decomposition:
With Reynolds decomposition, we divide the fluctuating quantities (velocities, temperature, density, enthalpy, mass concentration, shear stresses, etc.) into a time-averaged (on the turbulent eddy time scales) and the fluctuations about that average, e.g. for the velocity: $u = \bar{u} + u'$.

By this definition, we can say: $\bar{u} = \ell im_{\theta \to \infty} \frac{1}{\theta} \int_0^\theta u d\varsigma$, and can note that:

$\overline{u'} = \ell im_{\theta \to \infty} \frac{1}{\theta} \int_0^\theta u' d\varsigma = 0$. The unsteadiness is often expressed in terms of the "turbulence intensity" (Tu) given as the rms values of the fluctuation, normalized by the velocity external to the boundary layer, $\sqrt{u'^2} \Big/ u_\infty$. Plots of profiles of Tu, below, show the streamwise development of u' profiles and profiles of the different fluctuations, u', v' and w'.

One can note:

1. Though the boundary layer thickness, δ_{99}, is the thickness where the velocity has reached 99% of the freestream velocity and, of course, the velocity is essentially the freestream velocity beyond δ_{99}, the turbulence of the boundary layer extends to $1.5\,\delta_{99}$.
2. The rms fluctuation within a fully-developed turbulent boundary layer is as large as 10% of the freestream velocity.
3. The rms levels within the boundary layer for u', v' and w' are essentially the same, differing by no more that half, while u and v (≈ 0) and w (≈ 0) are much different.
4. The turbulence is anisotropic, especially near the wall.
5. All turbulence quantities, and the anisotropy, approach 0 as $y \to \infty$.
6. All components $\to 0$ as $y \to 0$.

Reynolds decomposition is applied to the continuity, momentum, species diffusion and energy equations, giving them in terms of mean and time-averaged fluctuation quantities. Note that in doing so we lose all the time-related information and reduce the problem to statistical terms which we must model before we can solve to get even the time-averaged flow, temperature and mass concentration fields (the need for a model to relate the statistical terms to the mean flow, mass concentration and temperature fields is called the "closure problem").

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

applying Reynolds decomposition; $\frac{\partial(\bar{u}+u')}{\partial x} + \frac{\partial(\bar{v}+v')}{\partial y} + \frac{\partial(\bar{w}+w')}{\partial z} = 0 \cdot$

After time averaging: $\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \cdot$

So, the time-averaged, Reynolds-decomposed equation is the same as the unsteady, continuity equation because, e.g. $\overline{\frac{\partial \bar{u}}{\partial x}} = \frac{\partial \bar{u}}{\partial x}$ and $\overline{\frac{\partial u'}{\partial x}} = \ell\text{im}_{\theta\to\infty} \frac{1}{\theta}\int_0^\theta \frac{\partial u'}{\partial x} d\varsigma = \frac{\partial}{\partial x}\left\{\ell\text{im}_{\theta\to\infty} \frac{1}{\theta}\int_0^\theta u' d\varsigma\right\} = 0 \cdot$

Subtracting the time-averaged, Reynolds-decomposed equation from the Reynolds-decomposed equation gives: $\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \cdot$

The unsteadiness (separate from the steady flow behavior) of the flow follows the continuity relationship.

Now, applying the same to the x-momentum equation for a steady (other than turbulence), incompressible, 2-D, turbulent boundary layer flow:

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \frac{1}{\rho}\frac{d\bar{P}}{dx} = \frac{\partial}{\partial y}\left(\upsilon\frac{\partial \bar{u}}{\partial y} - \overline{u'v'}\right) + \frac{\partial}{\partial x}\left(-\overline{u'^2}\right)$$

The overbar represents time-average. This equation can be derived from the momentum equation. It requires using the continuity equation as well as the momentum equation. The $-\overline{u'v'}$ and $-\overline{u'^2}$ terms are the Reynolds shear and normal stresses, respectively. They really represent convective terms on the eddy scales; but, because it was convenient to move them to the right hand side and because they have a form which is similar to that of the viscous shear stresses, we assign the name "turbulence stresses." We can continue the analogy:

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

molecular stress tensor, depends on the fluid state

- thermodynamically irreversible

$$\begin{bmatrix} -\overline{u'^2} & -\overline{u'v'} & -\overline{u'w'} \\ -\overline{v'u'} & -\overline{v'^2} & -\overline{v'w'} \\ -\overline{w'u'} & -\overline{w'v'} & -\overline{w'^2} \end{bmatrix}$$

Reynolds stresses tensor, turbulence stresses, depends on fluid flow

- thermodynamically reversible in the large scales where they have their greatest contribution.

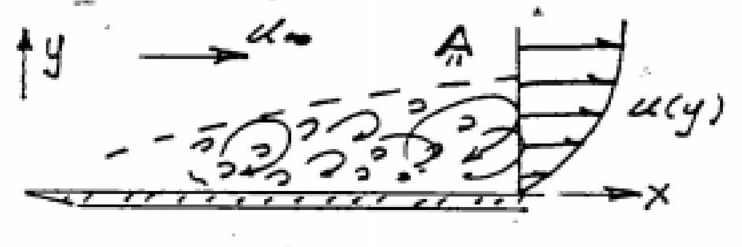
Now apply Reynolds decomposition to the 2-D species concentration boundary layer equation

$\bar{u}\frac{\partial \bar{\omega}_j}{\partial x} + \bar{v}\frac{\partial \bar{\omega}_j}{\partial y} = \frac{\partial}{\partial y}\left(D_j\frac{\partial \bar{\omega}_j}{\partial y} - \overline{\omega_j' v'}\right)$ The last term is the turbulent mass transport term.

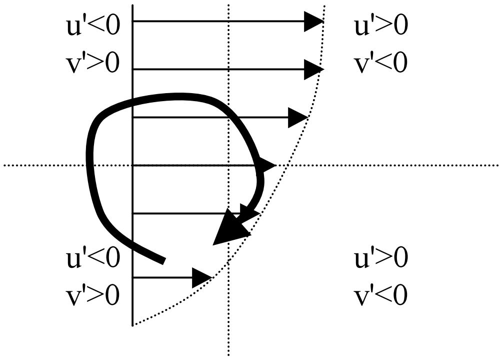
Now apply Reynolds decomposition to the thermal-plus-kinetic energy transport equation:

$\bar{u}\frac{\partial \bar{t}}{\partial x} + \bar{v}\frac{\partial \bar{t}}{\partial y} = \frac{\partial}{\partial y}\left(\alpha\frac{\partial \bar{t}}{\partial y} - \overline{t'v'}\right)$ The last term is the turbulent heat transport term.

Let's look more at the turbulent transport:
Picture the flow as a parade of eddies;



Look at one eddy as a simple rolling log:



In each quadrant, $\overline{u'v'} < 0$

Note that when $\overline{u'v'} < 0$, momentum transport is down the velocity gradient. We can see the comparison with the molecular transport term: $\nu \frac{\partial \bar{u}}{\partial y} - \overline{u'v'}$.

Note that there are no second-law constraints -- $u'v'$ can reverse sign without violation.