

HOMEWORK 6 Solution

HOMEWORK PROBLEM 1

This problem is related to a Type (a) shock such as that is described on pages 100 - 102 of the notes. On those pages, the velocity of the V of the air is given, and the unknown is the shock velocity V_s . In problem 8, the shock velocity is given ($V_s = (100\text{ft})/(0.1036\text{sec}) = 965.25 \text{ ft/sec}$), and the desired result is the air velocity V . Pre-capped pressure and temperature are 12.31psia and 67.3°F, respectively.

(a) $V_s = (100)/(0.1036) = 965.25 \text{ ft/sec}$ and the desired result is the air velocity V .

(b) $V_y = 965.25 \text{ ft/sec}$

(c) $M_x = V_x/[49.02*(67.3 + 459.7)^{0.5}] = V_x/(1125.3)$

Solution strategy:

Guess $M_x = 1.400 \rightarrow V_x/V_y = 1.6896$

$V_x = (V_x/V_y)*V_y = 1.6896*965.25 = 1631\text{ft/s}$

$M_x = V_x/1125.3 = 1.449$

Guess $M_x = 1.500 \rightarrow V_x/V_y = 1.8621$

$V_x = (V_x/V_y)*V_y = 1.8621*965.25 = 1797\text{ft/s}$

$M_x = V_x/1125.3 = 1.597 \rightarrow$ our answer is further from the previous guess, go in the other direction

Guess $M_x = 1.300 \rightarrow V_x/V_y = 1.5157$

$V_x = (V_x/V_y)*V_y = 1.5157*965.25 = 1463\text{ft/s}$

$M_x = V_x/1125.3 = \mathbf{1.300}$ -> our guess checks out

$V_f = V_x - V_y = 1463 - 965.3 = \mathbf{497.8 \text{ ft/sec}}$

HOMEWORK PROBLEM 2

For a Type (a) moving shock, the temperature and pressure in the pre-capped flow are 8.3F and 15.1 psi, respectively. In the air that is brought to rest due to the capping of the end of the pipe, the temperature is 200.5F and the pressure is 29.6 psi.

We know that there is an error because the temperature and pressure ratios are fixed. I sent out an email stating the temperature as 83F, which is actually incorrect anyways as the temperature is 80.3F I will solve the problem using each of the three temperatures.

Using 8.3F:

(a) What is the velocity of the pre-capped flow?

$$V_x = V_s + V_f, V_y = V_s$$

$$V_x = M_x * 49.02(T_f + 459.7)^{0.5}$$

$$P_x = 15.1 \text{ psi}, P_y = 29.6 \text{ psi} \rightarrow P_y/P_x = 1.9603 \rightarrow T_y/T_x = 1.2226$$

$$P_y/P_x \rightarrow M_x = 1.350, M_y = 0.7616$$

$$\text{However, } T_y/T_x = (200.5 + 459.7) / (8.3 + 459.7) = 1.4107 \rightarrow \text{which is a conflict with above}$$

$$V_x = 1.350 * 49.02(468)^{0.5} = 1431.6 \text{ ft/s}$$

$$V_y = 0.7615 * 49.02(660.2)^{0.5} = 959.1 \text{ ft/s}$$

$$V_y = V_x - V_f; V_f = \mathbf{472.5 \text{ ft/s}}$$

(b) What is the velocity of the shock that is caused by the capping?

$$V_s = V_y = \mathbf{959.1 \text{ ft/s}}$$

Using 83F:

$$P_x = 15.1 \text{ psi}, P_y = 29.6 \text{ psi} \rightarrow P_y/P_x = 1.9603 \rightarrow T_y/T_x = 1.2226$$

$$P_y/P_x \rightarrow M_x = 1.350, M_y = 0.7616$$

$$\text{However, } T_y/T_x = (200.5 + 459.7) / (83 + 459.7) = 1.2165 \rightarrow \text{which is a conflict with above}$$

$$V_x = 1.350 * 49.02(542.7)^{0.5} = 1541.7 \text{ ft/s}$$

$$V_y = 0.7615 * 49.02(660.2)^{0.5} = 959.1 \text{ ft/s}$$

$$V_y = V_x - V_f; V_f = \mathbf{582.6 \text{ ft/s}}$$

$$V_s = V_y = \mathbf{959.1 \text{ ft/s}}$$

Using 80.3F:

$$P_x = 15.1 \text{ psi}, P_y = 29.6 \text{ psi} \rightarrow P_y/P_x = 1.9603 \rightarrow T_y/T_x = 1.2226$$

$$P_y/P_x \rightarrow M_x = 1.350, M_y = 0.7616$$

$$T_y/T_x = (200.5 + 459.7) / (80.3 + 459.7) = 1.2226 \rightarrow \text{which is a match!}$$

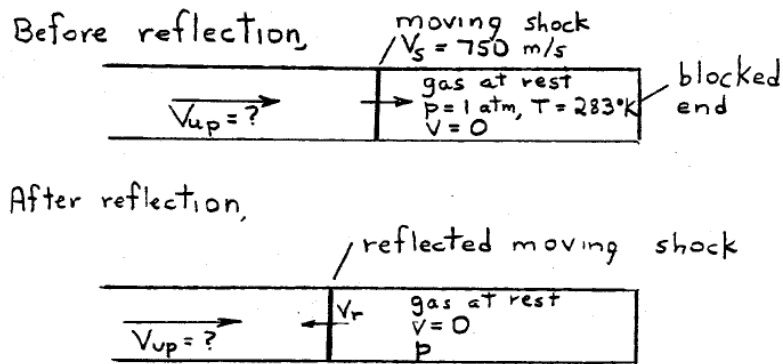
$$V_x = 1.350 * 49.02(540)^{0.5} = 1537.8 \text{ ft/s}$$

$$V_y = 0.7615 * 49.02(660.2)^{0.5} = 959.1 \text{ ft/s}$$

$$V_y = V_x - V_f; V_f = \mathbf{578.7 \text{ ft/s}}$$

$$V_s = V_y = \mathbf{959.1 \text{ ft/s}}$$

HOMEWORK PROBLEM 3



- (a) What is the speed V_r of the reflected shock?
 (b) What is the pressure p in the rest gas behind the reflected shock?

(a) What is the speed V_r of the reflected shock?

Part 1

$$M_x = V_x / (20.05 * T_x^{0.5}) = 750 / (20.05 * (283)^{0.5}) = 2.22$$

$$P_y/P_x = 5.5831 \rightarrow P_y = 565.6 \text{ kPa}$$

$$T_y/T_x = 1.8746 \rightarrow T_y = 530.5 \text{ K}$$

$$V_x/V_y = 2.9783 \rightarrow V_y = 251.8 \text{ m/s}$$

$$V_{up} = V_x - V_y = 498.2 \text{ m/s}$$

Part 2

$$P_x = 565.5 \text{ kPa}, T_x = 530.5 \text{ K}$$

$$\text{Guess } M_x = 1.5 \rightarrow V_x/V_y = 1.8621 \rightarrow T_y/T_x = 1.3202$$

$$V_x = M_x * 20.05 * (530.5)^{0.5} = 692.7 \text{ m/s}$$

$$V_y = V_x - 498.2 \text{ m/s} = 194.5 \text{ m/s} \rightarrow V_x/V_y = 1.9746 \rightarrow \text{answer doesn't check, guess again}$$

$$\text{Guess } M_x = 1.85, V_x/V_y = 2.4381 \text{ (B3)}$$

$$V_x = M_x * 20.05 * T_x^{0.5} = 854.3 \text{ m/s}$$

$$V_r = V_x - 498.2 \text{ m/s} = V_y = 356.1 \text{ m/s} \rightarrow V_x/V_y = 2.3990 \rightarrow \text{answer doesn't check, guess again}$$

$$\text{Guess } M_x = 1.8385, V_x/V_y = \mathbf{2.4200} \text{ (B3)}$$

$$V_x = M_x * 20.05 * T_x^{0.5} = 849.0 \text{ m/s}$$

$$V_r = V_x - 498.2 \text{ m/s} = V_y = 350.8 \text{ m/s} \rightarrow V_x/V_y = \mathbf{2.4200} \rightarrow \text{answer matches!}$$

$$\mathbf{V_y = V_r = 350.8 \text{ m/s}}$$

(b) What is the pressure p in the rest gas behind the reflected shock

$$M_x = 1.8385, P_y/P_x = 3.7768$$

$$P_y = (P_y/P_x) * P_x = \mathbf{2,136 \text{ kPa (21.1 atm)}}$$