Hint for homework

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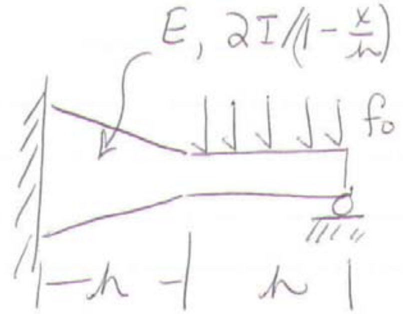

$$\int_{x_A}^{x_B} EI \frac{d^2\phi_i}{dx^2}\frac{d^2\phi_j}{dx^2}\,dx$$

switch to local coords, since $x_A' = 0$; $x = \bar{x}$

$$2EI\int_0^{h_e} \frac{1}{\left(1-\frac{\bar{x}}{h}\right)} \frac{d^2\phi_i}{d\bar{x}^2}\frac{d^2\phi_j}{d\bar{x}^2}\,d\bar{x}$$

define a new variable $y = \frac{\bar{x}}{h_e}$

$y h_e = \bar{x}$

$h_e\,dy = d\bar{x}$

$$\frac{d^2\phi_1}{d\bar{x}^2} = -\frac{6}{h_e^2}\left(1-2\frac{\bar{x}}{h_e}\right) = -\frac{6}{h_e^2}(1-2y)$$

$$\frac{d^2\phi_2}{d\bar{x}^2} = -\frac{2}{h_e}\left(3\frac{\bar{x}}{h_e}-2\right) = -\frac{2}{h_e}(3y-2)$$

$$\frac{d^2\phi_3}{d\bar{x}^2} = -\frac{d^2\phi_1}{d\bar{x}^2} \qquad \frac{d^2\phi_4}{d\bar{x}^2} = -\frac{2}{h_e}(3y-1)$$

so at $\bar{x} = h_e$ $\qquad y = 1$

and

$$K_{11} = 2EI \int_0^1 \frac{1}{1-y} \left(\frac{36}{h_e^4}\right)(1-2y)^2 \overbrace{h_e\, dy}^{d\bar{x}}$$

$$= \frac{2EI}{h_e^3}\, 36 \underbrace{\int_0^1 \frac{(1-2y)^2}{1-y}\, dy}$$

integrate this numerically -
use matlab, mathematica, excel ...

You can make this substitution for other entries and ~~find~~ evaluate the resulting integral numerically since you no longer have h_e in the integral