Control design via innovation feedback

Recall that using observer state feedback,

\[ \dot{x} = A\hat{x} + Bu - L(C\hat{x} - y) \]
\[ u = -K\hat{x} \]

the controller itself satisfies [See Goodwin pp. 512 for proof]:

\[ \frac{L(s)}{E(s)}U(s) = -\frac{P(s)}{E(s)}Y(s) + V(s) \quad (27) \]

where

\[ \frac{L(s)}{E(s)} = 1 + KT_1(s) = \frac{\text{det}(sI - A + LC + BK)}{E(s)} \]
\[ \frac{P(s)}{E(s)} = KT_2(s) = \frac{K\text{Adj}(sI - A)J}{E(s)} \]
\[ \frac{P(s)}{L(s)} = K(sI - A + LC + BK)^{-1}L \]
Controller can be written as a two degree of freedom controller form:

\[ U(s) = \frac{E(s)}{L(s)} \left( V(s) - \frac{P(s)}{E(s)} Y(s) \right) \]

Or as a 1 degree of freedom controller form:

\[ U(s) = \frac{P(s)}{E(s)} (R(s) - Y(s)) \]

where \( V(s) = \frac{P(s)}{E(s)} R(s) \).

The innovation is the output prediction error:

\[ \nu := y - C\hat{x} = -Ce \]

Therefore,

\[ \nu(s) = Y(s) - C\hat{X}(s) \]
\[ = Y(s) - CT_1(s)U(s) - CT_2(s)Y(s) \]
\[ = (1 - CT_s(s))Y(s) - CT_1(s)U(s) \]

where

\[ T_1(s) = (sI - A + LC)^{-1}B \]
\[ T_2(s) = (sI - A + LC)^{-1}L \]
In transfer function form:

\[
\frac{L(s)}{E(s)} U(s) = -\frac{P(s)}{E(s)} Y(s)
\]

\[
G(s) = \frac{B_o(s)}{A_o(s)} = \frac{C \text{Adj}(sI - A)B}{\text{det}(sI - A)}
\]

\[
E(s) = \text{det}(sI - A + LC)
\]

\[
F(s) = \text{det}(sI - A + BK)
\]

\[
L(s) = \text{det}(sI - A + LC + BK)
\]

\[
P(s) = K \text{Adj}(sI - A)L
\]

\[
\frac{P(s)}{L(s)} = K [sI - A + LC + BK]^{-1} L
\]

Then, it can be shown (see Goodwin P545) that the innovation

\[
\nu(s) = \frac{A_o(s)}{E(s)} Y(s) - \frac{B_o(s)}{E(s)} U(s)
\]

Consider now that observer state feedback augmented
with innovation feedback,

\[ u = v - K\hat{x} - Q_u(s)\nu \]

where \( Q_u(s)\nu \) is \( \nu \) filtered by the stable filter \( Q_u(s) \) (to be designed). Then,

\[
\frac{L(s)}{E(s)}U(s) = -\frac{P(s)}{E(s)}Y(s) + Q_u(s) \left[ \frac{B(s)}{E(s)}U(s) - \frac{A(s)}{E(s)}Y(s) \right]
\]

The controller transfer function becomes then:

\[
C(s) = \frac{\frac{P(s)}{E(s)} + Q_u(s)\frac{A(s)}{E(s)}}{\frac{L(s)}{E(s)} - Q_u(s)\frac{B(s)}{E(s)}}
\]

The nominal sensitivity functions, which define the robustness and performance criteria, are modified affinely by \( Q_u(s) \):

\[
S_o(s) = \frac{A_o(s)L(s)}{E(s)F(s)} - Q_u(s)\frac{B_o(s)A_o(s)}{E(s)F(s)} \tag{28}
\]

\[
T_o(s) = \frac{B_o(s)P(s)}{E(s)F(s)} + Q_u(s)\frac{B_o(s)A_o(s)}{E(s)F(s)} \tag{29}
\]
For plants that are open-loop stable with tolerable pole locations, we can set $K = 0$ so that

$$F(s) = A_o(s)$$
$$L(s) = E(s)$$
$$P(s) = 0$$

so that

$$S_o(s) = 1 - Q_u(s)\frac{B_o(s)}{E(s)}$$
$$T_o(s) = Q_u(s)\frac{B_o(s)}{E(s)}$$

In this case, it is common to use $Q(s) := Q_u(s)\frac{A_o(s)}{E(s)}$ to get the formulae:

$$S_o(s) = 1 - Q(s)G_o(s) \quad (30)$$
$$T_o(s) = Q(s)G_o(s) \quad (31)$$

Thus the design of $Q_u(s)$ (or $Q(s)$) can be used to directly influence the sensitivity functions.
For instance, using Eqs.(28)-(29):

Minimize sensitivity $\|W_p(s)S(s)\|$ for nominal performance:

$$Q_u(s) = \frac{L(s)}{B_o(s)}F_1(s)$$

Minimize complementary sensitivity $\|W_u(s)T(s)\|$:

$$Q_u(s) = -\frac{P(s)}{A_o(s)}F_2(s)$$

where $F_1(s), F_2(s)$ are close to 1 at frequencies where $\|S(s)\|$ and $\|T(s)\|$ need to be decreased.

Similarly, using Eqs.(30)-(31) for stable open loop systems:

Minimize sensitivity $\|W_p(s)S(s)\|$ for nominal performance:

$$Q(s) = G_o^{-1}(s)F_1(s)$$

Minimize complementary sensitivity $\|W_u(s)T(s)\|$:

$$Q_u(s) = -G_o^{-1}F_2(s)$$
where $F_1(s)$, $F_2(s)$ are close to 1 at frequencies where $\|S(s)\|$ and $\|T(s)\|$ need to be decreased. Notice that it is not possible to do both at the same time.

**Remarks:**

- Generally, if $F_1(s)$ and $F_2(s)$ need to be active in overlapping ranges, then the control design will not be feasible.

- The internal stability of the system is ensured if $Q_u(s)$ is stable.

- How $Q_u(s)$ should be designed need to be modified in case when $B_o(s)$ or $A_o(s)$ are non-minimum phase.
Electrohydraulic Actuator Example using $Q_u(s)$ feedback

Simplified model for the EH system,

$$G(s) = \frac{B_o(s)}{A_o(s)} = \frac{1}{s}$$

States space model:

$$\dot{x} = u + d_i$$
$$y = x + d_o$$

where $d_i$ and $d_o$ are input and output disturbances.

The observer is:

$$\dot{\hat{x}} = u - L(\hat{x} - y)$$

$$(s + L)\hat{x}(s) = u(s) + Ly(s)$$
The innovation $\nu$ is:

$$
\nu = y - \hat{y} = y - \hat{x}
$$

$$
\nu(s) = y(s) - \frac{u + Ly}{s + L} = \frac{s \cdot y(s)}{s + L} - \frac{u(s)}{s + L}
$$

Let the observer state-feedback with innovation feedback be:

$$
u(s) = -K\hat{x}(s) - Q_u(s)\nu(s)
$$

$$
(s + K + L) - Q_u(s) \cdot u(s) = -(KL + Q_u(s)s)\cdot y(s)
$$

$$
S_o(s) = \frac{y(s)}{d_o(s)} = \frac{s(s + K + L)}{(s + L)(s + K)} - Q_u(s)\frac{s}{(s + L)(s + K)}
$$

$$
T_o(s) = \frac{u(s)}{d_i(s)} = \frac{KL}{(s + K)(s + L)} + Q_u(s)\frac{s}{(s + L)(s + K)}
$$

This is consistent with the formulae (28)-(29) with
these definitions:

\[
E(s) = s + L \\
F(s) = s + K \\
L(s) = (s + L + K) \\
P(s) = LK
\]

First we consider \( K = 180 \). Without \( Q_u(s) \) feedback, this is robustly stable, but does not have the required performance.

To improve nominal performance, we focus on frequency below 200 rad/s:
>> wc1=200;
>> [B1,A1]=butter(2,wc1,'s');

\[ Q(s) = \frac{(s + L + K) \times B1(s)}{A1(s)} \]

This satisfies the robust performance criteria easily.

Let’s try starting out with performance. Set \( K = 1200 \). Without \( Q_u(s) \), do not satisfy robust stability.
Clearly, this satisfies nominal performance but violates robust stability in the region of $[400, 4000]$ rad/s.

$$\gg \quad [B_2, A_2] = \text{butter}(2, [200, 5000], 's');$$

$$Q(s) = -\frac{K L B_2(s)}{s A_2(s)}$$
This also satisfies robust performance!