1. **(Robust Performance)**. The transfer of an uncertain system is of the form:

\[
G(s) = \frac{Y(s)}{U(s)} = \frac{5p}{s + p} e^{-\tau s},
\]

where the time delay \(\tau \in [0, 0.05]\) and the (negative of the) pole \(p \in [1, 5]\). Use a nominal model of the form:

\[
G_o(s) = \frac{5\bar{p}}{s + \bar{p}} \left( \frac{-\bar{\tau}s + 2k}{\bar{\tau}s + 2k} \right)^k
\]

where \(\bar{p}\) is a nominal pole location, \(\bar{\tau}\) is a nominal delay, and \(k\) is the Pade approximation of the time delay (see P100 of Goodwin), all of your choice.

(a) Design an uncertainty filter \(W_u(s)\) to capture the uncertainty. (Show that it works in a Matlab plot).

(b) Let \(W_p(s) = 100p^3/(s + p)^3\) where \(p \in \mathbb{R}\) determines the desired bandwidth. Suppose that the proportional controller is:

\[
u = -k_p \cdot y
\]

With different \(k_p\)'s, if possible solve the robust performance problem for the largest \(p\) you can find.

If Robust Performance cannot be satisfied for any \(p\), then please find a) the minimum \(k_p\) to solve the nominal performance problem; b) the maximum \(k_p\) to solve the robust stability problem.

Plot \(|W_pS_o|, |W_uT_o|\), and the Robust Performance criteria to illustrate your solutions.

2. **(Affine (Q-) parameterization)** In this problem you will use the Q-parameterization technique to solve problem 1.

(a) Design an observer of the system and placed the closed loop poles at \(-5\). The system may the the nominal system in Problem 1 or the nominal problem with a proportional control you found from Problem 1 - your choice.

(b) Compute the transfer functions from \(U(s)\) and \(Y(s)\) to the innovation \(\nu(s) := C\hat{X}(s) - Y(s)\) in the expression:

\[
u(s) = \frac{\alpha(s)}{E(s)} U(s) - \frac{\beta(s)}{E(s)} Y(s)
\]
Suppose that we apply the innovation feedback, \( U(s) = Q_u(s)\nu(s) \). Determine in terms of \( Q_u(s) \) and your answers above, a) the sensitivity function \( S(s) \); b) the complementary sensitivity function \( T(s) \); c) the input disturbance sensitivity \( S_i(s) \); d) the control sensitivity \( S_u(s) \). Simplify as much as possible.

(c) By using approximate inversion + filtering, design \( Q_u(s) \) (innovation feedback) so that the objective in problem 1 is satisfied with as large a \( p \) as possible.

(d) Write down the controller in the usual form (i.e. \( U(s) = -C(s)Y(s) \)) in terms of output feedback). Compute the closed loop pole locations to ensure that the system is stable.

(e) Plot \( |W_p S_o| \), \( |W_u T_o| \) and the Robust Performance criteria to show that your design is successful.