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Convection

Whenever heat transfer takes place in the presence of fluid motion, the mode of heat transfer is called convection. The majority of heat transfer problems encountered in practice involve convection. Currently, an area of intense activity in convective heat transfer is the cooling of electronic equipment.

Insights into convective heat transfer phenomena can be obtained by grouping and contrasting various types of fluid flows. Three such groupings will be looked at here.

A. Forced convection vs. natural convection
The distinction between these two types of fluid flows is the cause of the fluid flow. If the flow is caused by a fan, blower, pump, or other source of pressuriza-
tion, it is said to be a **forced convection** flow. Representative forced convection flows include air or water flowing in a pipe or duct, air passing through the fins of an automobile radiator, and a cool wind passing over a person. On the other hand, when the flow is caused by buoyancy, it is said to be a **natural convection** flow. Buoyancy is created when the density of a fluid varies from place to place. At places where the density is relatively high, the fluid moves downward; on the other hand, the fluid moves upward at places where the density is relatively low.

The photo at the right is a heated horizontal cylinder (black circle) situated in air. The air adjacent to the cylinder is heated by the cylinder, and it rises.
B. Laminar vs. turbulent flow

In laminar flow, fluid particles glide along smooth paths, i.e.,

On the other hand, in a turbulent flow, particles of fluid execute random motions as they proceed along their mainflow paths, i.e.,

By comparing the schematic illustrations for laminar and turbulent flow, it is clear that whereas in the former the respective particles "mind their own business," this is not true in the latter. Rather, the sidewise turbulent motions enable fluid particles
to interact and exchange energy. By this process, a turbulent flow is a better heat transfer mode than a laminar flow. On the other hand, a turbulent flow sustains higher losses than a laminar flow and is more costly to pump.

C. External vs. internal flow
An external flow passes over the outside of a body. In contrast, an internal flow is constrained within a confined space such as a tube or duct.

All of these various types of flow will be dealt with in the forthcoming unit on convection.
Boundary Layers and External Flows

An external flow occurs outside of bodies. Examples include a wind blowing over the roof of a house, the Space Shuttle reentering the earth's atmosphere, water passing over the hull of a submarine, and many others. Immediately adjacent to the body over which the flow is passing, there is a very thin layer where the velocity of the fluid particles changes very rapidly. At the very surface, the fluid particle in contact with the surface moves at a velocity exactly equal to the surface velocity. This is called the no-slip condition. The adhesion is caused by the viscosity of the fluid. Thus, if the surface is stationary (for example, the roof of a house), the surface-adjacent fluid particle is at zero velocity. On the other hand, if the surface is moving with a velocity $u_s$, then the velocity of the surface-adjacent fluid particle is $u_s$. 
To explore what is happening in the fluid at positions near but not on the surface, it is useful to report some experiments involving fluid velocity measurements. For concreteness, suppose that the surface is stationary and that the fluid is in motion. Figure BL-1 shows a fluid flow over a blunt body. As seen there, the streamlines approaching the body are deflected because the flow regards the body as an obstacle which is to be avoided. The deflection of the flow is not caused by the fluid's viscosity. In fact, an inviscid fluid (sometimes called a "perfect" fluid or an "ideal" fluid) would also be deflected. The viscous effects are concentrated near the surface. To examine this region more closely, an enlarged view of it is shown in Fig. BL-2.
The figure shows a velocity profile, which is a plot of the magnitude of the velocity at various distances from the surface. The velocity rises rapidly from its zero (no-slip) value at the surface and attains a uniform value which, to a first approximation, is the same as would be measured in an inviscid fluid. The region in which the fluid flow experiences its rapid change in velocity is a viscous-dominated region. It is called the boundary layer or, more precisely, the velocity boundary layer. The local thickness of the velocity boundary layer is denoted by \( \delta \). Generally, \( \delta \) is very small compared to the distance between the forward edge of the body and the surface location at which \( \delta \) is being examined. In general, \( \delta \) is not the same at each surface location. Normally, \( \delta \) increases with increasing downstream...
distance from the forward edge of the body.

A schematic representation of the velocity boundary layer on a blunt body, in which the boundary thickness is much enlarged, is shown in Fig. BL-3.

Fig. BL-3

A blunt body deflects an oncoming flow due to obstruction. On the other hand, a very thin flat plate aligned with the flow causes virtually no deflection due to obstruction. In fact, for an inviscid fluid passing over a flat plate, the streamlines are straight and parallel, as indicated in Fig. BL-4.

Inviscid-flow streamlines

Fig. BL-4.
In the presence of fluid viscosity, the no-slip condition applies, so that the fluid particles passing near the plate surface are slowed down. In order to escape the retarding action of the plate, the streamlines tend to bend away from the plate. The bending is very slight so that its portrayal generally involves a significant exaggeration; for instance, as in Fig. BL-5.

Fig. BL-5.

In general, highly deflected streamlines, such as those of Fig. BL-1 for flow over a blunt body, are associated with relatively large pressure gradients. On the other hand, the pressure gradients associated with slightly curved streamlines (e.g., Fig. BL-5), are very slight. As a first approximation, the flow over a flat plate is regarded as a constant pressure flow. This feature causes the solution for boundary layer flow over a flat plate
to be less difficult to obtain than that for a blunt body.

Boundary layer flow also occurs downstream of the inlet of a tube or duct, as illustrated in Fig. BL-6. The definitive difference between boundary layers in external flows and in internal flows (i.e., tubes and ducts) is that the former grow without interference whereas the latter interfere and merge, as shown in Fig. BL-6.

The simplest boundary layer is that for the flat plate, especially when the flow is steady and laminar. This is the problem to which attention will now be turned.
Velocity Boundary Layer for Laminar Flow Over a Flat Plate

Velocity profiles at a few representative positions on a flat plate for laminar flow are illustrated in Fig. BL-7.

Fig. BL-7

The boundary layer is shown speckled in the figure. It is the region where the streamwise velocity \( u \) rises from its no-slip value of zero at the plate surface to the value \( U_\infty \). The boundary layer thickness \( \delta \) is the distance from the wall at which \( u = U_\infty \). Note that \( \delta \) increases in the streamwise direction as the retarding effect diffuses outward toward the free stream.
A plot of the successive velocity profiles at $x = x_1$, $x = x_2$, and $x = x_3$ ($x_1 < x_2 < x_3$) has the form

![Graph showing velocity profiles](image)

**Fig. BL-8**

This figure confirms the information conveyed by the boundary layer schematic, **Fig. BL-7**.

If **Fig. BL-8** were to be replotted with $y/\delta(x)$ as the abscissa variable rather than $y$, it would be found that

![Graph showing replotted velocities](image)

**Fig. BL-9**
The fact that all the x-specific profiles could be mapped onto a single curve suggests that those profiles have some feature in common. It is customary to describe this commonality by the statement that the velocity profiles are similar.

To solve for the velocity profiles, the first step is to derive the mathematical statement of conservation of momentum. This derivation involves setting the sum of the forces in the x-direction equal to the net outflow of the flux of momentum. The process of performing this derivation is the same as that used to set up the conservation of energy equation. Also, the solution method for the momentum and energy equations is the same. Because of this, it is not truly necessary to go through both derivations and solutions in detail. Since the main focus of this presentation is temperature and heat transfer and the velocity is an input to the heat transfer-temperature analysis, it will be sufficient to state the velocity results and to do the heat transfer problem in detail.
An approximate but accurate algebraic representation of the velocity profile for laminar boundary layer flow over a flat plate is

$$\frac{u(x,y)}{U_\infty} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \quad (BL-1)$$

where $\delta = \delta(x) = 4.64 \sqrt{\frac{y x}{U_\infty}} \quad (BL-2)$

in which the kinematic viscosity $\nu$ is related to the viscosity $\mu$ and the density $\rho$ via the definition

$$\nu = \frac{\mu}{\rho} \quad (BL-3)$$

The velocity profile of equation (BL-1)

Fig. BL-10 \hspace{1cm} \frac{y}{\delta}
is plotted in Fig. BL-10. Note that at any axial station \(x\), equation (BL-1) applies within the boundary layer i.e., for

\[
0 < y \leq \delta(x) \quad (BL-4)
\]

Outside the boundary layer,

\[
u = U_\infty, \quad \text{for} \quad y > \delta(x) \quad (BL-5)
\]

The boundary layer thickness \(\delta\) grows larger in the streamwise direction in accordance with equation (BL-2). Figure BL-

\[
\delta = \sqrt{x}
\]

Fig. BL-11

11 illustrates the growth of the boundary layer thickness.

The velocity results conveyed in the foregoing will now be used in the heat transfer-
Temperature analysis.

Thermal Boundary Layer for Laminar Flow Over a Flat Plate

The thermal boundary layer is the region adjacent to a surface within which the thermal interactions between the flowing fluid and the surface are confined. For concreteness, suppose the surface temperature $T_w$ of a flat plate is uniform along the surface and greater than the free-stream temperature $T_0$. Then, heat diffuses from the surface toward the lower-temperature freestream. At the forward edge of the plate, the thermal interaction is confined to the fluid particle that is in direct contact with the surface. That particle attains an elevated temperature and, as a consequence, it transfers heat to the neighboring particle - causing a temperature rise of that particle. This temperature rise gives rise to heat flow to the next neighbor, and so on. This process can be
picted schematically as

Fluid particles with temperatures elevated above $T_0$

Fig. BL-12

Consistent with Fig. BL-12, the temperature profiles at axial stations $x_1, x_2, x_3$ ($x_1 < x_2 < x_3$) are

Fig. BL-13

The quantity $\delta_T(x)$ is the thickness of the thermal boundary layer. It is the y-coordinate at which the fluid temperature at a given axial station returns to the freestream
value $T_\infty$. It is evident that the thickness $\delta$ of the thermal boundary layer increases in the streamwise direction $x$.

It is, of course, proper to ask if the velocity boundary layer thickness $\delta$ and the thermal boundary thickness $\delta_T$ are the same. The relationship between $\delta$ and $\delta_T$ is one of the results of the forthcoming analysis, but, for now, it can be said that they are not the same except for a special situation to be identified shortly.

Experience with the velocity boundary layer, especially with Figs. BL-8 and BL-9, suggests that it is worthwhile to try to replot the temperature profiles in a manner analogous to the format used in Fig. BL-9. To proceed along these lines, a short table of analogous quantities can be prepared.

**Table BL-1**

<table>
<thead>
<tr>
<th>Velocity Problem</th>
<th>Temperature Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>$T - T_w$</td>
</tr>
<tr>
<td>$U_{\infty}$</td>
<td>$T_\infty - T_w$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$\delta_T$</td>
</tr>
</tbody>
</table>
Then, the sought-for representation of the temperature profiles can be portrayed as

\[
\frac{T - T_\infty}{T_w - T_\infty} \quad \text{all } x
\]

\[
0 \quad y / \delta_T(x) \quad 1
\]

Fig. BL-14

It still remains to show that a representation such as Fig. BL-14 is possible and to find its algebraic representation.

The x-specific temperature profiles of Fig. BL-13, as well as the provisional merged profile of Fig. BL-14, are smooth, well-behaved curves. This suggests that a polynomial representation would be suitable. Therefore, let

\[
T(x, t) = c_1(x) + c_2(x)y + c_3(x)y^2 + c_4(x)y^3 + \cdots \quad (BL-7)
\]

where the \( c_1, c_2, \ldots \) may be functions of \( x \). The number of terms in the polynomial is,
Seemingly, arbitrary. However, very high-order polynomials may be inappropriate since they tend to be wavy. On the other hand, very low-order polynomials do not have the flexibility to accommodate all the available information. It is worthwhile to list the most important information that is known about the temperature distribution. This is done in Table BL-2.

**Table BL-2**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 0 )</td>
<td>( T = T_w ) [ \frac{\partial^2 T}{\partial y^2} = 0 ]</td>
</tr>
<tr>
<td>( y = \delta_T )</td>
<td>( T = T_\infty ) [ \frac{\partial T}{\partial y} = 0 ]</td>
</tr>
</tbody>
</table>

At the plate surface, the fluid particle that is in contact with the surface is at the surface temperature \( T_w \); therefore, \( T = T_w \) at \( y = 0 \). At the edge of the thermal boundary layer \( y = \delta_T \), the temperature of the fluid is, by definition, \( T_\infty \). Also, by examining Fig. BL-13, it is evident that the slope of the temperature profile at \( y = \delta_T \) is perfectly flat, i.e., \( \frac{\partial T}{\partial y} = 0 \).
The boundary condition \( \frac{\partial^2 T}{\partial y^2} = 0 \) at \( y = 0 \) is shown in a box in Table BL-2 to indicate its non-obvious status. To derive this boundary condition, it is helpful to look at a small control volume in the fluid immediately adjacent to the plate surface, as depicted in Fig. BL-15.

Fig. BL-15

An enlarged view of the element is shown at the lower left of the figure. The possible transfers of energy at the control volume faces are indicated in the diagram using symbols which are defined at the lower right.
In the figure, the directions of the energy flows have been chosen so that they are positive in the positive coordinate directions. Note that no convective heat flow is shown at the base of the element. This is because the element rests on a solid surface through which fluid cannot pass. Since a moving fluid crossing a control volume boundary is a requisite for convective transfer across that boundary, it is proper to omit the convection term at the bottom of the element.

The magnitude of the energy convected into the control volume through its left-hand face and out of the control volume through its right-hand face depends on the rate of mass flow through those faces. In general, the rate of mass flow in passing through an area is given by

$$m = \text{density} \times \text{velocity} \times \text{area}$$

If the left- and right-hand faces are each of area $dA = Wdy$, as shown in Fig. BL-16, and $u$ is the axial velocity (normal to $dA$), also shown in the figure, then the mass
flow rate is

$$\mathrm{d}m = \rho u \, \mathrm{d}A = \rho u W \, \mathrm{d}y \quad \text{(BL-9)}$$

where $\mathrm{d}m$ is used instead of $m$ because the cross-sectional area for fluid flow is $\mathrm{d}A$.

Fig. BL-16

Next, it is worthwhile to estimate the magnitude of the velocity $u$. The control volume faces under consideration are very close to the surface of the plate, where the no-slip condition requires that $u = 0$. Intuition suggests that because the control volume faces are so close to the surface, the velocity should be very small for instance, $u \sim \mathrm{d}y$. This expectation can be verified in a formal way by writing a Taylor's series for the velocity at $y = \frac{1}{2} \, \mathrm{d}y$ (half-way up the face of the...
control volume),

\[ u(x, \frac{1}{2} dy) = u(x, 0) + \frac{du}{dy}\bigg|_{x,0} \cdot \frac{1}{2} dy + \ldots \]  

so that

\[ u(x, \frac{1}{2} dy) \sim dy \]  

If this information is substituted into equation (BL-9), it follows that

\[ dm \sim (dy)^2 \]  

which means that the convective terms shown at the left- and right-hand faces of the control volume (Fig. BL-15) are proportional to \((dy)^2\). By similar arguments, it is easily shown that the convective transport out of the top face of the control volume is proportional to the product \(dxdy\).

The heat conducted into and out of the left- and right-hand faces will now be examined. From prior encounters with heat conduction into and out of control volumes, it follows that
\[ Q_x = -kA \frac{dT}{dx} \]  
(BL-13)

The area \( A \) appearing in this equation is actually \( dA \) as depicted in Fig. BL-16, so that \( A = W \text{dy} \) and

\[ Q_x = -kW \text{dy} \frac{dT}{dx} \]  
(BL-14)

Note that at the very surface of the plate, \( \frac{dT}{dx} = 0 \) since the plate surface temperature \( T_w \) is uniform. Consequently, since the midpoint of the height of the element is at \( y = \frac{1}{2} \text{dy} \)

\[ \frac{dT}{dx}(x, \frac{1}{2} \text{dy}) = \frac{dT}{dx}(x, 0) + \frac{d}{dy}\left(\frac{dT}{dx}\right)|_{x, 0} \cdot \frac{1}{2} \text{dy} \]

which gives

\[ \frac{dT}{dx}(x, \frac{1}{2} \text{dy}) \sim \text{dy} \]  
(BL-15)

With this and with equation (BL-14), it follows that

\[ Q_x \sim (\text{dy})^2 \]  
(BL-16)

Next, the heat transfer due to conduction in the y-direction at the top and
bottom of the control volume will be considered. From Fourier's law in analogy with equation (BL-13),

$$Q_y = -kA \frac{dT}{dy} \quad \text{(BL-18)}$$

Here, $A$ is the area normal to the $y$-direction heat flow, so that $A = Wdx$, and

$$Q_y = -kWdx \frac{dT}{dy} \quad \text{(BL-19)}$$

The value of $\frac{dT}{dy}$ in this equation is very nearly equal to the value $\frac{dT}{dy}$ at the plate surface. The latter is the slope of the temperature profile at the surface. Inspection of temperature profiles such as those of Fig. BL-13 indicates that $\frac{dT}{dy}$ is not an infinitesimal quantity, surely not on the order of $dx$ or $dy$. Therefore,

$$Q_y \sim dx \quad \text{(BL-20)}$$

Table BL-3 summarizes the findings of the preceding paragraphs with respect to the orders of magnitude of the various energy transfers that are indicated in Fig. BL-15.
Table BL-3

<table>
<thead>
<tr>
<th>Energy Transfer</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-direction convection</td>
<td>$(dy)^2$</td>
</tr>
<tr>
<td>y-direction convection</td>
<td>$dxdy$</td>
</tr>
<tr>
<td>x-direction conduction</td>
<td>$(dy)^2$</td>
</tr>
<tr>
<td>y-direction conduction</td>
<td>$dx$</td>
</tr>
</tbody>
</table>

This table shows that the convection and x-conduction terms can be neglected in the energy balance for the surface-adjacent element pictured in Fig. BL-15. Therefore, in the steady state,

$$(Q_y)_{\text{top}} - (Q_y)_{\text{bottom}} = 0 \quad (BL-21)$$

or

$$\frac{dQ_y}{dy} = 0 \quad (BL-22)$$

When $Q_y$ from (BL-19) is substituted into (BL-22), there emerges

$$\frac{d^2T}{dy^2} = 0 \quad (BL-23)$$

which verifies the boxed equation in Table BL-2.
The four entries in Table BL-2 will now be used to determine the coefficients \(c_1, \ldots, c_4\) in the polynomial (BL-7). Application of the two conditions at \(y = 0\) yields

\[
T_w = c_1, \quad \text{(BL-24a)}
\]

\[
o = 2c_3 \quad \text{(BL-24b)}
\]

and those at \(y = \delta_T\) yield

\[
T_0 = c_1 + c_2 \delta_T + c_3 \delta_T^2 + c_4 \delta_T^3 \quad \text{(BL-24c)}
\]

\[
o = c_2 + 2c_3 \delta_T + 3c_4 \delta_T^2 \quad \text{(BL-24d)}
\]

When these equations are solved, and the \(c_i\)'s are substituted into equation (BL-7), there is obtained

\[
\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2} \frac{y}{\delta_T} - \frac{1}{2} \frac{y^3}{\delta_T^3} \quad \text{(BL-25)}
\]

This is the sought-for algebraic equation for the illustrative curve of Fig. BL-14. Note that the temperature profile is a function of \(y/\delta_T\) alone. It is also noteworthy that the temperature profile of equation (BL-25)
has the same form as the velocity boundary layer profile, equation (BL-1). In fact, Fig. BL-10 can be used to represent the temperature profile by replacing \( u/U_\infty \) with \( (T-T_w)/(T_\infty - T_w) \) and \( y/\delta \) with \( y/\delta_T \).

Careful inspection of equation (BL-25) suggests that the description of the temperature cannot be regarded as being complete until the thickness \( \delta_T(x) \) of the thermal boundary has been determined. In general, the thicknesses \( \delta \) and \( \delta_T \) of the velocity and thermal boundary layers are not the same, so that equation (BL-2) cannot be used to represent \( \delta_T \).

The way to determine \( \delta_T \) is to write and solve the boundary layer energy balance.

**Boundary Layer Energy Balance**
The energy balance is facilitated by constructing the control volume shown in Fig. BL-17.

Fig. BL-17

The control volume spans the thickness of the thermal boundary and has a streamwise length dx. The base of the control volume (i.e., ab) rests on the surface of the plate, while the top cd of the control volume coincides with the $\delta_t$ curve.

In the steady state, the energy conservation principle requires that

\[
\text{Rate of energy inflow} - \text{Rate of energy outflow} = 0
\]

(BL-26)
or, alternatively,

Net rate of energy inflow = 0 \quad (BL-27)

If, as usual, all energy flows are depicted as positive (i.e., positive in the positive coordinate direction), the application of equation (BL-27) yields

(\text{convective inflow across ad} \\
- \text{convective outflow across bc})

+ (\text{conductive inflow across ad} \\
- \text{conductive outflow across bc})

+ \text{conductive inflow across ab} \quad (BL-28)

+ \text{convective outflow across cd} = 0

There appear, at first glance, to be two terms missing from equation (BL-28). One of these is convection into the element across ab. Since ab rests directly on the impermeable surface of the plate, the convection across ab is zero, since convection across a control volume face requires that a fluid flows across the face. The second
term in question is conduction out of the element across cd. However, for conduction to occur requires that the temperature gradient be non-zero; but $\partial T/\partial y = 0$ at the edge of the thermal boundary layer as indicated in Table BL-2 (page BL16). Therefore, the conduction across cd is zero.

The difference between the conductive inflow across ad and the conductive outflow across bc is normally omitted from the boundary layer energy equation. This is because the temperature along a line $y = \text{constant}$ (parallel to the isothermal plate surface) is nearly constant, as suggested

![Typical Isotherms](image)

Fig. BL-18

In Fig. BL-18, so that both $\partial T/\partial x$ and $\partial (\partial T/\partial x)/\partial x$ are very small. The difference between the inflow across ad and the outflow across bc is proportional to $\partial (\partial T/\partial x)/\partial x$. 
Next, attention will be turned to deriving the mathematical representations of the terms in equation (BL-29). The first term to be considered is that involving the convective flows in the x-direction (i.e., across ad and bc.). At ad, the analysis is facilitated by reference to Fig. BL-19. It is useful to begin by working with the element dy. A pictorial diagram of such an element has been presented in Fig. BL-16 (page BL-19), and the rate of mass flow dm through the element is

\[ dm = \rho u W dy \]  \hspace{1cm} (BL-9)

If \( h \) is the enthalpy per unit mass, the rate of energy flow through the small element is

\[ hdm = \rho u h W dy \]  \hspace{1cm} (BL-29)
\[ \overline{Nu} = \frac{\text{average } h \times \text{Representative length}}{\text{Thermal conductivity of fluid}} \]  

(BL-71)

Consequently,

\[ Nu_x = \frac{h_x}{k}, \quad \overline{Nu}_L = \frac{hL}{k} \]  

(BL-72)

Note that a subscript has been appended to the Nusselt number to indicate the specific characteristic dimension.

With the definitions of the Nusselt number, the final forms of the dimensionless heat transfer results for the flat plate, laminar boundary layer are

\[ Nu_x = 0.332 \, Re_x^{1/2} \, Pr^{1/3} \]  

(BL-73)

\[ Nu_L = 0.664 \, Re_L^{1/2} \, Pr^{1/3} \]  

(BL-74)

In order to actually calculate numerical results for \( q \) or \( Q \), there is need for the thermophysical properties \( k, \nu, \) and \( Pr \). These properties are listed in tables at the ends of the notes. To use the tables, the fluid temperature is needed.
as an input. The common practice is to evaluate the properties at a mean temperature \( \bar{T} \) defined as

\[
\bar{T} = \frac{1}{2}(T_w + T_0) \quad \text{(BL-75)}
\]

The properties \( k \) and \( Pr \) do not depend on pressure, and similarly for \( \nu \) for liquids. However, for gases, \( \nu \) varies inversely with pressure. To verify this, recall that

\[
\nu = \frac{\mu}{\rho} \quad \text{(BL-73)}
\]

The viscosity \( \mu \) is independent of pressure, but, according to the perfect gas law, \( \rho \sim p \). Therefore,

\[
\nu \sim \frac{1}{p} \quad \text{(BL-76)}
\]

In the tabulation of \( \nu \) for gases, the values correspond to \( p = 1 \) atmosphere. Therefore, if the actual pressure exceeds 1 atmosphere, the tabulated \( \nu \) values have to be corrected by a factor less than one, and vice-versa when the actual pressure is lower than one atmosphere.
Turbulent Boundary Layer Heat Transfer

The transition from laminar to turbulent flow in a boundary layer is not an abrupt event, although it can be made so by placing rods or wires in the layer to disturb it. The transition and where it occurs along the length of surface is characterized by a critical Reynolds number $Re_{x, crit}$ defined as

$$Re_{x, crit} = \frac{U_\infty x_{crit}}{\nu} \quad (BL-77)$$

![Diagram](image)

Fig. BL-26

Although $x_{crit}$ is supposed to represent the distance from the leading edge to
The beginning of turbulent flow, Fig. BL-26 shows that $x_{\text{crit}}$ truly denotes a range of distances. While such a range is taken into account in sophisticated analysis of boundary layers, it is more common to approximate $x_{\text{crit}}$ as a single value.

An accepted value of $Re_{x,\text{crit}}$ is

$$Re_{x,\text{crit}} = 5 \times 10^5 \quad \text{(BL-78)}$$

This appears to be quite a large value compared with the transition value of $Re$ for a pipe flow ($\sim 2300$). This great difference occurs because $Re_{x,\text{crit}}$ is based on a distance along the flow while the $Re$ for a pipe is based on a cross-sectional distance - the pipe diameter.

From equation (BL-78)

$$x_{\text{crit}} = \frac{5 \times 10^5 \nu}{U_\infty} \quad \text{(BL-79)}$$

To decide whether the boundary layer is laminar or turbulent, the first step
is to calculate \(x_{\text{crit}}\) from equation (BL-79). If \(L\) denotes the length of the plate in the flow direction, then

(a) If \(L \leq x_{\text{crit}}\), the flow is laminar over the entire plate.

(b) If \(L > x_{\text{crit}}\), the flow is laminar when \(0 \leq x \leq x_{\text{crit}}\) and turbulent when \(x_{\text{crit}} < x < L\).

The exception to this rule occurs when the boundary layer is purposefully disturbed. For instance, Fig. BL-27 shows a rod-like transverse disturbance (often called a trip rod) situated at the forward edge of a flat plate. Due to this disturbance, the boundary layer is turbulent for all \(x > 0\).

Fig. BL-27
The local heat transfer rate per unit area $q_f$ for turbulent boundary layer flow is based on experimental data.

$$q_f = 0.0296 \left( \frac{T_w - T_0}{x} \right) \frac{K}{x} \left( \frac{U_0 x}{v} \right)^{0.8} Pr^{1/3}$$  
(BL-80)

Note that

$$q_f \sim \frac{1}{x^{0.2}}$$  
(BL-81)

Other forms of equation (BL-80) are

$$\frac{h x}{K} = Nu_x = 0.0296 \left( \frac{Re_x}{Pr} \right)^{0.8} Pr^{1/3}$$  
(BL-82)

Suppose that $Q$ (overall rate of heat transfer) is needed for a flat plate whose length $L > x_{crit}$, as indicated in Fig. BL-28.
From the figure, it is evident that

\[ Q = Q_{\text{lam}} + Q_{\text{turb}} \quad \text{(BL-83)} \]

and

\[ Q_{\text{lam}} = \int_{x_{\text{crit}}}^{x_{\text{crit} +}} Q_{\text{lam}} \, W \, dx \quad \text{(BL-84)} \]

\[ Q_{\text{turb}} = \int_{x_{\text{crit}}}^{L} Q_{\text{turb}} \, W \, dx \quad \text{(BL-85)} \]

The \( Q_{\text{lam}} \) and \( Q_{\text{turb}} \) needed to integrate equations (BL-84) and (BL-85) are indicated there. The only other ingredient needed in the integration is \( x_{\text{crit}} \), and that is specified by equation (BL-79).

The end result of the integration is

\[ Q = Q_{\text{lam}} + Q_{\text{turb}} \]

\[ Q = (WK)(T_w - T_0)(0.037 \, Re_L^{0.8} - 871) \, Pr^{y_3} \]

\[ Re_L = \frac{U_0 L}{\nu} \quad \text{(BL-86)} \]
or, alternatively,

\[
\frac{T_h}{k} = \overline{\text{Nu}_L} = (0.037 \text{Re}_L^{0.8} - 871) \text{Pr}^{\frac{1}{3}}
\]

\[
\overline{h} = \frac{Q}{(WL)(T_w - T_\infty)} \quad (\text{BL-87})
\]

Equations (BL-86) and (BL-87) apply when transition occurs at \( x_{crrt} \) as given by equation (BL-79). When there is a boundary-layer trip at \( x = 0 \), the flow is turbulent for all \( 0 \leq x \leq L \). In that case,

\[
Q = Q_{turb} = \int_0^L q_{turb} W \, dx \quad (\text{BL-88})
\]

which gives

\[
Q = (WL)(T_w - T_\infty) \times 0.037 \text{Re}_L^{0.8} \text{Pr}^{\frac{1}{3}} \quad (\text{BL-89})
\]

or

\[
\frac{T_h}{k} = \overline{\text{Nu}_L} = 0.037 \text{Re}_L^{0.8} \text{Pr}^{\frac{1}{3}} \quad (\text{BL-90})
\]
EXAMPLE

Consider two cases of boundary layer flow along a flat plate.

In case I, an artificial disturbance at the leading edge causes the flow to be turbulent all the way from \( x = 0 \). In case II, if there is laminar-turbulent transition, it occurs naturally.

In both cases, the flowing fluid is air at one atmosphere and, for both, \( T_w = 126.85^\circ C \) and \( T_a = 26.85^\circ C \). Also, for both, \( U_\infty = 30 \text{ m/s} \) and \( L = 2 \text{ m} \).

(a) At \( x = 0.1 \text{ m} \), what is \( \frac{h_{II}}{h_I} \)?

(b) At \( x = 1 \text{ m} \), what is \( \frac{h_{II}}{h_I} \)?

(c) For the entire plate, what is \( \frac{\bar{h}_{II}}{\bar{h}_I} \)?

Case I - trip wire so entire plate is turbulent
Case II - natural transition

Find properties at \( T = \frac{T_w + T_a}{2} = \frac{(400 \text{ K} + 300 \text{ K})}{2} = 350 \text{ K} \)

Where is the transition point for case II?

\[
\frac{x_{crit}}{U_\infty} = \frac{5 \times 10^{-5} \mu}{\nu} \Rightarrow \frac{x_{crit}}{U_\infty} = 2.062 \times 10^{-5} \text{ m}^2 / \text{s}, \quad x_{crit} = \frac{5 \times 10^{-5} (2.062 \times 10^{-5})}{30} = 0.34 \text{ m}
\]

a) At \( x = 0.1 \text{ m} \), case I is turbulent but case II is still laminar

- turbulent \( h = 0.0296 \frac{k_x}{T_x} \frac{Re_x^{0.8}}{Pr_x^{1/3}} \)
- laminar \( h = 0.332 \frac{k_x}{T_x} \frac{Re_x^{1/2}}{Pr_x^{1/3}} \)

\[
Re_{x,0.1} = \frac{30 \text{ m/s} (0.1 \text{ m})}{2.062 \times 10^{-5} \text{ m}^2 / \text{s}} = 1454.90
\]

\[
\frac{h_{II}}{h_I} = \frac{0.332 \frac{Re_x^{1/2}}{k_x}}{0.0296 \frac{Re_x^{0.8}}{Pr_x^{1/3}}} = 0.317
\]

b) At \( x = 1 \text{ m} \), both cases I & II are turbulent, therefore \( \frac{h_{II}}{h_I} = 1 \)

c) Find \( \frac{\bar{h}_{II}}{\bar{h}_I} \) for entire plate

- case I : plate turbulent from \( \gamma = 0 \), \( \bar{h}_I = \frac{k_x}{L} (0.037 Re_x^{0.8} Pr_x^{1/3}) \)
- case II : plate undergoes natural transition, \( \bar{h}_II = \frac{k_x}{L} (0.037 Re_x^{0.8} - 850) Pr_x^{1/3} \)

\[
Re_x = \frac{50 \text{ (2)}}{2.062 \times 10^{-5}} = 2709796
\]

\[
\frac{\bar{h}_{II}}{\bar{h}_I} = \frac{(0.037 Re_x^{0.8} - 850)}{0.037 Re_x^{0.8}} = 0.845
\]
EXAMPLE

A thin flat plate is aligned parallel to a uniform free stream. The plate has a streamwise length $L = 30$ cm and a transverse width $W = 60$ cm. The temperatures are $T_w = 95^\circ$C and $T_\infty = 20^\circ$C. The boundary layer is laminar. At $x = 5$ cm, the local rate of heat transfer per unit area $q = 1,350$ W/m$^2$.

(a) What is $q$ at $x = 15$ cm and at $x = 25$ cm?

(b) What is the overall rate of heat transfer $Q$ for one side of the plate?

(c) What is the average heat transfer coefficient $h$?

The flow is laminar, so that the equation from page 46 applies

$$q = 0.332 \frac{k}{U_\infty} \sqrt{\frac{T_w - T_\infty}{\nu x \Pr^{1/3}}}$$

At $x = 5$ cm = 0.05 m

$$1350 = 0.332 \frac{k}{U_\infty} \sqrt{\frac{T_w - T_\infty}{\nu x \Pr^{1/3}}} \frac{1}{\sqrt{0.05}}$$

$$C = \frac{301.87}{C}$$

(a) At $x = 15$ cm, $q = \frac{301.87}{0.15} = 779.4$ W/m$^2$

At $x = 25$ cm, $q = \frac{301.87}{0.25} = 603.7$ W/m$^2$
(b) From page 48

\[ Q = 0.664 K (T_w - T_\infty) \sqrt{\frac{u_\infty L}{v}} \Pr^{3/4} W \]

or \[ Q = 2 \sqrt{L} W \times \xi^{301.87} \]

\[ L = 30 \text{ cm} = 0.3 \text{ m}, \quad W = 60 \text{ cm} = 0.6 \text{ m} \]

\[ Q = 2 \sqrt{0.3} (0.6)(301.87) = 198.4 \text{ W} \]

(c) \[ \bar{h} = \frac{Q}{LW(T_w - T_\infty)}, \text{ from page 4} \]

\[ \bar{h} = \frac{198.4}{(0.3)(0.6)(95-20)} = 14.7 \frac{W}{m^2\cdot ^\circ C} \]
Cylinder in Crossflow

Up to now, the discussion and analysis of external flows have been focused on flat plates which are so thin that they do not block and deflect the flow. Blunt bodies which do block and deflect the flow were mentioned on pages 6-10 but not treated in any detail. Such bodies are encountered quite often in convection heat transfer problems. The most commonly encountered blunt body is the cylinder in crossflow, shown schematically at the right.

\[ U_\infty \]
The flow around a cylinder in cross-flow and, in fact, around any blunt body, is greatly affected by a process called flow separation. This process is well known to stream fisherpersons and outdoors persons in general. Envision a shallow stream in which rocks protrude above the water surface. An observer looking down on such a stream would see whirls, eddies, and foam protruding from the surface of the rock.

If the protruding rock situation
were to be examined at different seasons when the rates of water flow were different, interesting differences would be noted in the pattern of flow behind the rock. For example, at very slow flowrates, the fluid actually comes together behind the rock. This can happen because the forward inertia of the slow-moving fluid is very low. At larger flowrates, the separation process discussed on the preceding page is activated, but the pattern of flow behind the obstacle changes in response to changes in the flowrate.

The most-studied blunt-body flow patterns are those for the cylinder in crossflow. The parameter used to convey the magnitude of the velocity and its variations is the Reynolds number

$$Re_D = \frac{U_0 \cdot D}{\nu}$$

where $U_0$ is the free stream velocity pictured in the figure on page 64, $D$ is the cylinder diameter, and $\nu$ is the kinematic viscosity. $Re_D$ is dimensionless.
In the diagram, sketches show the pattern of fluid flow around a cylinder in crossflow for different values of the Reynolds number $Re_D$. Only for $Re_D < 5$ does the flow move slowly enough.

<table>
<thead>
<tr>
<th>Reynolds number</th>
<th>Flow pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_D &lt; 5$</td>
<td>Unseparated streaming flow</td>
</tr>
<tr>
<td>$5 &lt; Re_D &lt; 40$</td>
<td>A pair of vortices fixed in the wake</td>
</tr>
<tr>
<td>$40 &lt; Re_D &lt; 150$</td>
<td>A laminar vortex street</td>
</tr>
<tr>
<td>$150 &lt; Re_D &lt; 3 \times 10^5$</td>
<td>The boundary layer is laminar up to the separation point; the vortex street is turbulent, and the wake flow field is increasingly three-dimensional</td>
</tr>
<tr>
<td>$3 \times 10^5 &lt; Re_D &lt; 3.5 \times 10^6$</td>
<td>The laminar boundary layer undergoes transition to a turbulent boundary layer before separation; the wake becomes narrower and disorganized</td>
</tr>
<tr>
<td>$3.5 \times 10^6 &lt; Re_D$</td>
<td>A turbulent vortex street is reestablished, but it is narrower than was the case for $150 &lt; Re_D &lt; 3 \times 10^5$</td>
</tr>
</tbody>
</table>

The main flow regimes for flow across a cylinder.
To be able to close up behind the cylinder. For all higher values of \( Re_0 \), flow separation is in evidence.

The two main features of the flow around a cylinder are the boundary layer on the forward face of the cylinder and the wake behind the cylinder. The boundary layer on the forward face is kept thin because it is pressed against the surface by the oncoming freestream flow. However, when the boundary layer flow reaches the widest part of the cylinder, it is no longer pressed against the surface. There is a great difference in the capability of the boundary layer to turn the corner when it reaches the widest part of the cylinder when it is laminar or when it
is turbulent. As the diagram at the bottom of the last page shows, the turbulent layer is able to ward off flow separation even beyond the widest part of the cylinder, i.e., beyond $\Theta = 90^\circ$. On the other hand, the laminar boundary layer separates before $\Theta = 90^\circ$.

In view of the very complicated fluid flow patterns around a cylinder in crossflow, it can be expected that the heat transfer results would be expressed in a somewhat more complex form than for the flat plate. Here are the $Nu_D$ formulas for the cylinder in crossflow:

$$\overline{Nu}_D = \frac{1}{0.8237 - \ln (Re_D Pr)^{1/2}}; \quad Re_D Pr < 0.2$$

$$\overline{Nu}_D = 0.3 + \frac{0.62Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{23}]^{1/4}}; \quad Re_D < 10^4$$

$$\overline{Nu}_D = 0.3 + \frac{0.62Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{23}]^{1/4}} \left[ 1 + \left( \frac{Re_D}{282,000} \right)^{1/2} \right]; \quad 2 \times 10^4 < Re_D < 4 \times 10^5$$

$$\overline{Nu}_D = 0.3 + \frac{0.62Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{23}]^{1/4}} \left[ 1 + \left( \frac{Re_D}{282,000} \right)^{5/8} \right]^{4/5}; \quad 4 \times 10^5 < Re_D < 5 \times 10^6$$
In these formulas, the fluid properties are to be evaluated at

\[ T = \frac{1}{2} (T_w + T_0) \]

The foregoing equations are used by introducing the values of Re_D and Pr into the right-hand side of the equation selected. Then, after the Nu_D is found, it is applied as follows:

\[ \overline{Nu}_D = \frac{hD}{k} \rightarrow \bar{h} = \frac{k}{D \overline{Nu}_D} \]

\[ Q = \bar{h}A \Delta T, \quad A = \pi DL \]

Note that D is the outside diameter of the cylinder.

Other Bodies in Crossflow

Another standard external flow is the sphere in crossflow. In this case, as with the cylinder, there is flow...
separation behind the sphere. The most-used Nusselt number equation for the sphere is

$$\overline{Nu_D} = 2 + (0.4 \, Re_D^{1/2} + 0.06 \, Re_D^{2/3}) \left( \frac{\mu_D}{\mu_w} \right)^{1/3} \Pr^{1/3}$$

In this equation, all the fluid properties are evaluated at $T_D$, except for $\mu_w$ which is evaluated at the temperature $T_w$ of the surface of the sphere. The equation has an interesting structure. When there is no flow, i.e., $Re_D \to 0$, then $\overline{Nu_D} = 2$. This is the case of pure conduction (i.e., no convection). The convection is represented by the sum of two terms. The first of these, $0.4 \, Re_D^{1/2}$, is the contribution due to the boundary layer on the front-facing part of the sphere. The second term, $0.06 \, Re_D^{2/3}$, is due to the wake region behind the sphere.

The $\overline{Nu_D}$ equation for the sphere is supposed to be applicable for

$$3.5 < Re_D < 7.6 \times 10^4$$
$$0.71 < Pr < 380$$
$$1 < \frac{\mu_D}{\mu_w} < 3.2$$
To compute the rate of heat transfer to or from a sphere,

\[ Q = \bar{h}A \Delta T, \quad A = \pi D^2 \]

where \( D \) is the outer diameter of the sphere.

Many decades ago, Jakob put together a table giving results for a variety of bodies in crossflow. His

<table>
<thead>
<tr>
<th>Body shape</th>
<th>Re range</th>
<th>( n )</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rightarrow \diamond )</td>
<td>5,000</td>
<td>100,000</td>
<td>0.588</td>
</tr>
<tr>
<td>( \rightarrow \circ )</td>
<td>2,500</td>
<td>15,000</td>
<td>0.612</td>
</tr>
<tr>
<td>( \rightarrow \diamond )</td>
<td>2,500</td>
<td>7,500</td>
<td>0.624</td>
</tr>
<tr>
<td>( \rightarrow \hexagon )</td>
<td>5,000</td>
<td>100,000</td>
<td>0.638</td>
</tr>
<tr>
<td>( \rightarrow \octagon )</td>
<td>5,000</td>
<td>19,500</td>
<td>0.638</td>
</tr>
<tr>
<td>( \rightarrow \square )</td>
<td>5,000</td>
<td>100,000</td>
<td>0.675</td>
</tr>
<tr>
<td>( \rightarrow \square )</td>
<td>2,500</td>
<td>8,000</td>
<td>0.699</td>
</tr>
<tr>
<td>( \rightarrow | )</td>
<td>4,000</td>
<td>15,000</td>
<td>0.731</td>
</tr>
<tr>
<td>( \rightarrow \octagon )</td>
<td>19,500</td>
<td>100,000</td>
<td>0.782</td>
</tr>
<tr>
<td>( \rightarrow | )</td>
<td>3,000</td>
<td>15,000</td>
<td>0.804</td>
</tr>
</tbody>
</table>
results are of the form

$$\overline{Nu} = C Re^n$$

with $C$ and $n$ provided by the table. The Nusselt and Reynolds in this equation are

$$\overline{Nu} = \frac{h f}{k}, \quad Re = \frac{U_\infty \delta}{v}.$$ 

$U_\infty$ is the freestream velocity, upstream of the body, and $\delta$ is the height of the body as seen by an upstream observer, e.g.,

$$U_\infty \rightarrow \bigtriangleup \bigtriangleup \bigtriangleup \bigtriangleup$$

These results are for air and for other gases having values of the Prandtl number in the neighborhood of 0.7. The fluid properties are to be looked up at a temperature $\mathcal{T} = \frac{1}{2} (T_m + T_\infty)$. 
EXAMPLE

The $\overline{\text{Nu}}$ formulas of page 69 are the best currently available for the cylinder in crossflow. These equations are easily evaluated using a pocket calculator. On the other hand, they do not yield a quick answer to questions such as:

How does $\overline{h}$ respond to a doubling of $U_\infty$, or

How does $\overline{h}$ respond to a tripling of $D$?

For this purpose, some other less-current formulas are available, i.e.,

$\overline{\text{Nu}}_D = 0.148 \text{Re}_D^{0.633}, \quad 5 \times 10^3 < \text{Re}_D < 5 \times 10^4$

$\overline{\text{Nu}}_D = 0.0208 \text{Re}_D^{0.814}, \quad 5 \times 10^4 < \text{Re}_D < 2 \times 10^5$

These are for air ($\text{Pr} \approx 0.7$).

(a) A cylinder in crossflow is operating at $\text{Re}_D = 2.13 \times 10^4$ (call this operating mode I). In operating mode II, the freestream velocity $U_{\infty \Pi} = 2 U_{\infty I}$. What is $\overline{h}_{\Pi} / \overline{h}_{I}$? Use both the approximate
equations for $\overline{Nu}_D$ on the preceding page and the more correct equations from page 69. Since $Re_{D_{II}} = 2.13 \times 10^4$ and $Re_{D_{II}} = 2Re_{D_{I}} = 4.26 \times 10^4$, both $Re_D$'s are in the range $5 \times 10^3 < Re_D < 5 \times 10^4$ and the first $\overline{Nu}_D$ equation on the prior page can be used. Then,

$$\frac{\overline{Nu}_{D_{II}}}{\overline{Nu}_{D_{I}}} = \frac{0.148 Re_{D_{II}}^{0.633}}{0.148 Re_{D_{I}}^{0.633}}$$

or

$$\frac{\overline{h}_{D_{II}/k}}{\overline{h}_{D_{I}/k}} = \left(\frac{U_{\infty_{II}} D/\nu}{U_{\infty_{I}} D/\nu}\right)^{0.633}$$

or

$$\frac{\overline{h}_{II}}{\overline{h}_{I}} = \left(\frac{U_{\infty_{II}}}{U_{\infty_{I}}}\right)^{0.633} = 1.55$$

Next, from the equations on page 69, the one to be used is that for $2 \times 10^4 < Re_D < 4 \times 10^5$. For $Re_{D_{II}} = 2.13 \times 10^4$ and $Pr = 0.7$, that equation gives

$$\overline{Nu}_{D_{II}} = 90.15$$

and for $Re_{D_{II}} = 4.16 \times 10^4$
\[ \overline{Nu}_{DII} = 136.4 \]

Then,
\[ \frac{\overline{h}_I}{\overline{h}_II} = \frac{\overline{Nu}_{DII}}{\overline{Nu}_{DI}} = 1.52 \]

which is comfortably close to the value 1.55 which was obtained on the preceding page.

(b) Next, attention is returned to operating mode I. Consider now an operating mode III in which \( D_{III} = 0.32 D_I \). Find \( \overline{h}_{III}/\overline{h}_I \) using the two approaches used in part (a) of the problem. (This is an assignable homework problem.)
The Bulk Temperature and the LMTD

The preceding discussion of convective heat transfer in boundary layers and external flows showed that because of heating or cooling at a bounding wall, the fluid temperature varies in the direction perpendicular to the wall. Outside the boundary layer, the fluid temperature is uniform.

When fluid flows in a duct or between clusters of tubes, temperature nonuniformities also exist within the fluid. It is useful to examine the fluid temperature distribution in a tube. Suppose, for concreteness, that fluid enters a tube with a fluid inlet temperature $T_{in}$ such that $T_{in}$ is lower than the tube wall temperature $T_w$. Under these conditions, heat from the tube wall flows into the fluid. The heat diffuses from the wall toward the centerline. This causes thermal bound-
Any layers to grow in the direction of fluid flow, as indicated in the

\[ \text{Thermal boundary layer} \]

\[ \begin{align*}
T_{in} & \rightarrow \\
T_w & \downarrow \\
X_1 & \rightarrow \\
\end{align*} \]

diagram. Outside the boundary layer, the fluid temperature is still \( T_{in} \). The growing boundary layers merge. After the merging, the boundary layer can no longer be identified, and the temperatures of all fluid particles exceed \( T_{in} \).

At a cross section such as \( X_1 \), the temperature profile has a generic shape which includes a value \( T_w \) at the tube wall and a dropoff in the direction from the wall to the centerline.

\[ \begin{align*}
& \text{T} \\
& \downarrow \\
& T_{in} \quad \text{---} \\
& O \quad R \quad R_{\text{wall}} \\
\end{align*} \]
The specifics of the profile shape depend on whether the flow is laminar or turbulent. If turbulent, the dropoff very near the wall is very rapid, but after the rapid dropoff, the temperature varies rather slowly over the rest of the cross section. On the other hand, for laminar flow, the "valley" in the centerline region is deeper than that illustrated in the diagram.

The important next step is to find the average fluid temperature at each cross section. There are two main reasons for needing the cross-sectional average temperature. The first is related to the definition of the local heat transfer coefficient $h$,

$$h = \frac{q}{T_w - T_{av}}$$
The second is related to the heat rate $Q$ which is transferred from the wall to the fluid between two axial stations $x = x_1$ and $x = x_2$.

In the steady state, the difference between the power carried out of the control volume through face 2 and the power carried into the control volume through face 1 is mostly due to the wall heat transfer $Q$. The energy carried in per unit mass of flowing fluid is the enthalpy $h_1$, and that carried out per unit mass is the enthalpy $h_2$. Note
that enthalpies are used rather than internal energies because the flow work (i.e., pu work) must be accounted for. If \( \dot{m} \) is the rate of mass flow, then

\[
\dot{m}(h_2 - h_1) = Q
\]

Furthermore, the enthalpy difference can be replaced by a temperature difference via the relation

\[
h_2 - h_1 = c_p (T_{av,2} - T_{av,1})
\]

so that

\[
\dot{m}c_p(T_{av,2} - T_{av,1}) = Q
\]

Students of thermodynamics will recognize this equation as being related to the steady-flow energy equation (also called the first law of thermodynamics for an open system). However, the steady flow energy equation contains more terms than appear above. The complete equation is
\[ \dot{m}c_p(T_{av,2} - T_{av,1}) + \dot{m}\left(\frac{U_2^2}{2} - \frac{U_1^2}{2}\right) \]

\[ + \dot{m}g(z_2 - z_1) = 0 \]

where \( U \) is the mean velocity, and \( z \) is the elevation. For most heat transfer applications, the kinetic energy change can be neglected - but not for high speed flows or for flows in which there are very large changes in cross-sectional area. Potential energy changes are always negligible.

The discussion presented in the preceding pages confirms the need to know the cross-sectional average temperature. A clue to the right way of determining the cross-sectional average is obtained by considering a simple experiment which may, at first glance, seem to be off the mark. In the experiment, a number of nominally identical beakers, each of known mass, are placed on a laboratory bench. Each
beaker is filled to an arbitrary depth with water from the cold tap. Then, different amounts of water from the hot tap are added to the respective beakers. Next, the contents of each beaker is well stirred and the respective temperatures are recorded. Also, the mass of each beaker (water plus beaker) is measured. Let $M$ denote the mass of water in a beaker and $T_1, T_2, T_3, T_4, T_5, \text{ etc.}$

The water temperature.

Next, suppose that the contents of each beaker is emptied into a single large beaker, and the combined mass of water is vigorously stirred, whereupon its temperature $\bar{T}$ is measured. It is readily apparent that $\bar{T}$ should be equal to

$$\bar{T} = T_{av} = \frac{M_1 T_1 + M_2 T_2 + \cdots}{M_1 + M_2 + \cdots} = \frac{\sum_{k=1}^{N} M_k T_k}{\sum_{k=1}^{N} M_k}.$$
Therefore, the temperature of the mixture is the mass-weighted average.

It will now be demonstrated that mass-weighting is also the proper averaging to obtain the cross-sectional averaged temperature $T_{av}$ in a duct flow. Suppose, for concreteness, that water is flowing in a round pipe. The water that enters the pipe is at a higher temperature than the ambient air in the room through which the pipe passes. Therefore, there is heat flow from the water out through the pipe wall and into the air. As a result, the fluid temperature in any cross section varies from the centerline to the wall, with the highest value at the center and the lowest at the wall. Next, suppose the pipe is cut open at some cross section $x = x_1$, and the water which discharges from the pipe is collected in a large tank. If the contents of the tank is well stirred, it will have a uniform temperature which is its average temperature. Since the
Water in the tank is the very water that has flowed through the pipe cross section, it has a unique average temperature, e.g., the temperature of the well-stirred water in the tank. That temperature will now be found.

The diagram shows the cut end of the tube and the collection tank. Attention may be focused on a small area $\Delta A$ in the cross section. The first step is to write an expression for the mass flow
rate which passes through $\Delta A$. In general, the mass flow is expressed by

$$\text{mass flow rate} = \text{cross sectional area} \times \text{fluid density at the selected area} \times \text{fluid velocity at the selected area}$$

If $u$ and $p$ are the local velocity and local density at the selected area $\Delta A$, then,

$$\Delta m = p u \Delta A$$

The $\Delta$ symbol which precedes $\Delta m$ is used to link the size of $\Delta m$ with the size of $\Delta A$.

The mass which crosses $\Delta A$ carries its temperature $T$. Therefore, the quantity $T \Delta m$ is deposited in the tank after crossing $\Delta A$. The entire exit cross section of the pipe can be regarded as being made up of an assembly of small area elements. Let these elements be called $\Delta A_1, \Delta A_2, \ldots$, and from each a
quantity $T_1 \Delta m_1$, $T_2 \Delta m_2$, ... is deposited in the tank. The total deposit is therefore,

$$\sum T_i \Delta m_i = \sum \frac{T_i \rho \cdot u_i \Delta A_i}{\Delta m_i}$$

The average temperature in the stirred tank is

$$T_{av} = \frac{\sum T_i \Delta m_i}{\sum \Delta m_i}$$

This is a mass-weighted average - a result similar to that found for the beaker experiment.

If $T$, $\rho$, and $u$ vary rapidly across the section, then, for better accuracy, it is appropriate to take $\Delta A$ very small, i.e., $\Delta A \rightarrow dA$ and replace the summation sign by an integral sign.

The "official" name for the mass-weighted average temperature is the bulk temperature $T_b$, so that $T_{av}$ is replaced by $T_b$. After all these changes
are made, the "official" definition of the bulk temperature emerges as

\[
T_b = \frac{\int_A T u \rho \, dA}{\int_A \dot{m} \, dA} = \frac{\int_A T u \rho \, dA}{\int_A u \rho \, dA}
\]

\[
= \frac{1}{\dot{m}} \int_A T u \rho \, dA
\]

It is useful to specialize the \(T_b\) definition to the case of axisymmetric flow in a circular tube. The idea of axisymmetry is illustrated in the figure. On a circle \(r = \text{constant}\), consider two points \(A\) and \(B\). The angular positions of \(A\) and \(B\) are arbitrary. If all aspects of the flowing fluid are identical at \(A\) and \(B\), then the flow is axisymmetric.

For an asymmetric flow in a round pipe, it is convenient to take \(dA\) as
an annulus i.e.,
\[ dA = 2\pi r dr \]
so that
\[ T_b = \frac{1}{m} \int_0^R 2\pi T_u r dr \]

**Equations Involving the Bulk Temperature**

From the discussion on pages 80 and 80a,

\[ Q = m c_p (T_{b2} - T_{b1}) \]
\[ T_{b2} = T_{b1} + \frac{Q}{m c_p} \]

The first of these equations is convenient for computing \( Q \) when \( T_{b1} \) and \( T_{b2} \) are
known. The second form is useful when $T_{b,1}$ and $Q$ are known, and $T_{b,2}$ is desired.

Suppose now that the distance between $x_1$ and $x_2$ (page 802) is reduced so that $(x_2 - x_1) \to dx$. Then, naturally,

$$(T_{b,2} - T_{b,1}) \to dT_b$$

$$Q \to dQ$$

so that

$$Q = m c_p (T_{b,2} - T_{b,1}) \to dQ = m c_p dT_b$$

This equation can be further rephrased with the help of the diagram. The

$$dQ = q dA_{wall} = q C dx$$

C = Circumference
surface of the short control volume segment shown shaded in the diagram is called \( dA_{\text{wall}} \) and is equal to the circumference \( C \) times the length \( dx \), so that \( dQ = q \cdot C \cdot dx \). With this substitution, the preceding equation (middle of page 80j) becomes

\[
\frac{dT_b}{dx} = \frac{q \cdot C}{m \cdot c_p}
\]

or

\[ q = \frac{m \cdot c_p}{C} \cdot \frac{dT_b}{dx} \]

In these equations, \( \frac{dT_b}{dx} \) is the local slope of the \( T_b \) versus \( x \) curve, i.e.,

![Diagram](image)

If the local slope is known, then the local \( q \) can be evaluated, and vice-versa. There is an interesting special case where \( q \) has the same...
value at all $x$. This means that the fluid is being uniformly heated or cooled all along the length of the tube. This case is called uniform heat flux (UHF). With $q = \text{constant}$, the first equation on page 80K yields

$$T_{b,x} = T_{b,in} + \frac{q C}{m c_p} x$$

The graph of this $T_b$ vs $x$ equation is

$$T_b$$

$q > 0$

$T_{b,in}$

$\text{heat added}$

$x$

$q < 0$

$\text{heat extracted}$

The Log-Mean Temperature Difference (LMTD)

If a fluid flowing in a duct is heated or cooled at the tube wall, the bulk temperature of the fluid
will vary in the x-direction. Except in special cases, the tube wall temperature will also vary in the x-direction. A typical graph showing $T_b$ and $T_w$ versus $x$ might be:

![Graph showing $T_w$ and $T_b$ versus $x$](image)

The feature to be noted is that the local temperature difference $(T_w - T_b)$ varies with $x$.

Suppose that attention is focused on the rate of heat transfer $Q$ passing from the wall to the fluid between $x_1$ and $x_2$. The average heat transfer coefficient $\bar{h}$ is

$$\bar{h} = \frac{Q}{A \Delta T_f}$$
The factor of uncertainty is the temperature difference $\Delta T_\theta$. The difficulty is that $\Delta T_\theta$ has to represent the varying values of $(T_W - T_b)$ between $x_1$ and $x_2$. A possible way of doing this is to define

$$\Delta T_\theta = \frac{1}{2} \left( (T_{W_1} - T_{b_1}) + (T_{W_2} - T_{b_2}) \right)$$

However, a more exact, albeit more complex, $\Delta T_\theta$ will now be derived.

This derivation can be performed for a somewhat restricted version of the situation depicted in the graph atop page 80m. This restriction is that the wall temperature be uniform all along $x$, i.e.,

$$T_W$$

This situation can be achieved in the laboratory and is sometimes encountered in practice, but it is a special case.
To begin the derivation, it may be assumed that a fluid enters a duct with a bulk temperature $T_{b,in}$ which is lower than the duct wall temperature $T_w$. Since $T_w = T_{b,in}$, heat flows from the wall to the fluid, causing the fluid temperature to rise. The situation is illustrated below. Notation for the derivation is presented in the sketch.
Let $dQ$ denote the rate of heat transfer from the wall to the fluid between $x$ and $x+dx$. Then, from the first law of thermodynamics (middle of page 80j),

$$dQ = mc_p d\Delta T_b$$

Alternatively, using the local heat transfer coefficient $h$ at $x$

$$dQ = h \frac{(C dx) (T_w - T_b)}{\text{surface area}}$$

These two representations for $dQ$ must be equal, so that

$$\frac{dT_b}{T_b - T_w} = - \frac{hC}{mc_p} dx$$

This equation, when solved, provides a way to get $T_b$ as a function of $x$. Also, it can be integrated from $x=0$, $T_b = T_{b,\text{in}}$ to $x = L$, $T = T_{b,\text{out}}$

$$\int_{T_{b,\text{in}}}^{T_{b,\text{out}}} \frac{dT_b}{T_w - T_b} = - \left( \frac{C}{mc_p} \right) \int_0^L h dx$$

constant
The integral at the left is of the standard form

\[ \int \frac{dz}{z+a} = \ln |z + a| \]

For the right-hand side, it is useful to define an average heat transfer coefficient \( \overline{h} \) as

\[ \overline{h} = \frac{1}{L} \int_{0}^{L} h \, dx \]

With these, the equation at the bottom of the preceding page becomes

\[ \ln \left[ \frac{T_{b,\text{out}} - T_w}{T_{b,\text{in}} - T_w} \right] = - \frac{\overline{h}(CL)A}{\dot{m}c_p} \]

Since \( C \) is the circumference around the inside wall of the tube, then \( CL \) is the surface area \( A \) of the inside wall.

Although the foregoing logarithmic equation is perfectly correct, it is more convenient to rephrase it by
Taking antilogs, which gives

\[ T_{b,\text{out}} = T_w + (T_{b,\text{in}} - T_w) e^{-\frac{hA}{mc_p}} \]

This is a very important result because it gives a convenient way of calculating the bulk temperature \( T_{b,\text{out}} \) at the end of the duct. In practice, \( T_{b,\text{in}} \) and \( T_w \) are usually known, but \( T_{b,\text{out}} \) is typically unknown.

With the results obtained so far, we can return to the question of what is \( \Delta T_\alpha \), which was introduced at the bottom of page 80m, i.e.,

\[ Q = hA \Delta T_\alpha \]

To pursue this question, note that

\[ Q = mc_p (T_{b,\text{out}} - T_{b,\text{in}}) \]

From the middle of page 80q, we can solve for \( mc_p \)

\[ mc_p = -\frac{hA}{\ln \left[ \frac{T_{b,\text{out}} - T_w}{T_{b,\text{in}} - T_w} \right]} \]
Then, \( mcp \) can be eliminated from the last two equations giving

\[
Q = \overline{h}A \frac{T_{b,\text{in}} - T_{b,\text{out}}}{\ln \left[ \frac{T_{b,\text{out}} - T_w}{T_{b,\text{in}} - T_w} \right]}
\]

Since \( T_w \) = constant for the derivation,

\[
T_{b,\text{in}} - T_{b,\text{out}} = (T_{b,\text{in}} - T_w) - (T_{b,\text{out}} - T_w)
\]

Also, note that

\[
\ln \frac{a}{b} = -\ln \frac{b}{a}
\]

These changes lead to

\[
Q = \overline{h}A \frac{(T_w - T_{b,\text{in}}) - (T_w - T_{b,\text{out}})}{\ln \left[ \frac{T_w - T_{b,\text{in}}}{T_w - T_{b,\text{out}}} \right]}
\]

By comparing with

\[
Q = \overline{h}A \Delta T
\]
it appears that this derivation leads to

\[ \Delta T_\theta = \underline{\quad} \]

This is a most unexpected result, but it is correct when the duct wall temperature is uniform.

It is useful to look into the content of the just-found \( \Delta T_\theta \). First, note that the quantity

\[
\frac{\alpha - \beta}{\ln \frac{\alpha}{\beta}} = \log \text{mean of } \alpha \text{ and } \beta
\]

so that

\[
\frac{(T_w - T_{b,in}) - (T_w - T_{b,out})}{\ln \left[ \frac{T_w - T_{b,in}}{T_w - T_{b,out}} \right]}
\]

\[= \log \text{mean of } (T_w - T_{b,in}) \text{ and } (T_w - T_{b,out})\]

or, for short

\[= \log \text{mean temperature difference}\]

\[= \text{LMTD}\]
With this definition
\[
Q = \text{ThA}(\text{LMTD})
\]

Although the uniform wall temperature case has been dealt with, it is necessary to face the fact that this is a special case. There is no way to get an exact expression for \( \Delta T \) when the wall temperature is not uniform. In practice, the LMTD is used and generalized for the non-uniform wall temperature case. That generalization is

\[
\text{LMTD} = \frac{(T_w - T_b)_\text{in} - (T_w - T_b)_\text{out}}{\ln \left[ \frac{(T_w - T_b)_\text{in}}{(T_w - T_b)_\text{out}} \right]}
\]

where

\( (T_w - T_b)_\text{in} = \) wall-to-bulk temperature at inlet

\( (T_w - T_b)_\text{out} = \) wall-to-bulk temperature at exit
Heat Transfer in Tubes and Ducts

Heat transfer applications involving fluids flowing in tubes and ducts are encountered in both hi-tech and low-tech situations. As pictured on page 78, tube/duct flows start out as boundary layer flows, but the boundary layers eventually span the entire cross section and thereby disappear. The diagram on page 78 depicts the merging thermal boundary layers, while the illustration below shows the way the velocity develops along a tube or duct.

Growing boundary layers Vanished boundary layer

Hydrodynamic development Fully developed region
As seen in the illustration, the velocity profile changes shape as the boundary layers grow. Because of the friction between the flowing fluid and the duct wall, the fluid very near the wall tends to slow down. Since the same mass flowrate passes through all cross sections, the fluid situated away from the wall speeds up to compensate for the near-wall slowdown. This process causes an initially flat velocity profile to become rounded. Beyond where the boundary layers meet, the velocity profile reaches a stable shape that is designated as fully developed. The length of tube/duct needed to achieve developed flow is different for laminar and turbulent conditions. Similarly, the shape of the fully developed velocity profile depends on whether the flow is laminar or turbulent.

In the real world, the velocity development along a tube or duct is affected by the shape of the tube/duct inlet. The following photo-
graphs show a sharp-edged inlet (at left) and a rounded inlet (at right). Since most fluids cannot turn a sharp corner, the fluid entering a sharp-edged inlet does not follow the contour of the duct wall in the region just downstream of the inlet. Rather, the flow tends to pinch inward to a minimum area and then expands to fill the entire cross section. When the flow does not follow the contour of the wall, it is called a separated flow.
Although the development of the velocity is of interest to many engineers in its own right, our concern here is how the velocity development affects the heat transfer. To clarify this point, it is useful to revisit briefly the local transfer coefficient \( h \) for tube or duct flows that was presented at the bottom of page 79. In that equation, the average fluid temperature at a cross section was denoted by \( T_{av} \). We now know that the proper \( T_{av} \) is the bulk temperature \( T_b \). With this,

\[
h = \frac{q}{T_w - T_b}
\]

where all quantities are specific to a given axial location.

From the diagram at the top of page 78 and from our knowledge of boundary layers, the thickening of the thermal boundary layer in the flow direction makes physical sense. As a thermal boundary grows thicker,
The thermal resistance to heat flow across it increases. In our study of heat conduction earlier in the course, we saw that the thermal resistance for convection is inversely proportional to the heat transfer coefficient $h$. Therefore, axially increasing thermal resistance corresponds to axially decreasing $h$, i.e.,

The question mark at the end of the $h$ versus $x$ curve is there to alert us to the possibility that something special may happen when the thermal boundary layers meet (top of page 78).

The immediate question is how the $h$ versus $x$ curve is affected by the axial development of the velocity distribution. This question may be answered...
by considering two different patterns of velocity development.

1. The velocity development shown at the bottom of page 81 occurs simultaneously with the development of the thermal boundary layer. That is,

   ![Diagram]

   Thermal and velocity development both begin here.

2. The velocity development is fully completed before the thermal development begins. That is,

   ![Diagram]
Illustrative $h$ versus $x$ distributions for the foregoing cases 1 and 2 are pictured in the graph.

As seen there, the $h$ values are higher for simultaneous development of the velocity and temperature distributions than for thermal development which occurs after the velocity profile has become fully developed. The reason for this result can be understood by looking at the diagram at the bottom of page 81. Pick a certain distance from the tube/duct wall and observe the magnitude of the velocity at that distance at any $x$ station in the hydrodynamic development region. Then, observe the velocity magnitude at the same distance from the wall in the hydro-
dynamic developed region. At near-wall locations, the developing velocity is always higher than the fully developed velocity. In general, higher near-wall velocities mean higher local heat transfer coefficients.

Up to now, we have left open the question of what happens to the \( h \) versus \( x \) graph downstream of where the thermal boundary layers have met. Strictly speaking, the answer depends on the thermal conditions at the walls of the pipe or duct. However, loosely speaking, the \( h \) versus \( x \) graph for a flow which is turbulent tends to level off as \( x \) increases, i.e.,

As indicated in the figure, the portion
of the tube or duct where \( h \) changes with \( x \) is called the thermal entrance region, while the portion where \( h \) is virtually independent of \( x \) is called the thermally developed region.

In engineering practice, it is common to assume that the \( h \) vs. \( x \) distribution at the bottom of the preceding page is suitable for turbulent flow. On the other hand, for laminar flow, this type of \( h \) vs. \( x \) distribution is achieved only for some special thermal boundary conditions which will be described shortly.

The preceding discussion featured the local heat transfer coefficient because of its close connection with issues of thermal development. The average heat transfer coefficient \( \bar{h} \) is also of great importance in practical design. From page 80u,

\[
Q = \bar{h}A(LMTD)
\]

The LMTD is defined on page 80u, and
A is the surface area of the wall which bounds the tube/duct flow.

At this point, the general discussion of tube/duct heat transfer will be made more specific to three fundamental thermal boundary conditions. The conditions happen to be the ones which lead to the existence of a fully developed heat transfer coefficient, \( h = \text{constant} \).

**Fundamental Boundary Conditions**

1. The temperature of the tube/duct wall is uniform.

The attainment of uniform wall temperature can occur when the outside of the tube/duct is surrounded by a fluid.
whose temperature is uniform and when the heat transfer coefficient at the outside surface of the tube/duct is very large. This is achieved when steam condenses at constant pressure on the outside of a tube/duct.

Suppose, for concreteness, that the wall temperature $T_w$ is greater than the entering bulk temperature $T_{bin}$. As the fluid flows through the tube, its temperature increases. If the tube is very long, the fluid temperature will finally become equal to the wall temperature, i.e.,

![Graph showing $T$ vs. $x$ with $T_w$ and $T_b$]

and

![Graph showing $h$ vs. $x$]

Note: $h$ becomes fully developed before $T_b \approx T_w$. 
2. The fluid in the tube is uniformly heated. This condition can be graphed as

\[
\begin{array}{ccc}
q \\
\hline \\
x
\end{array}
\]

Any two sections of equal length receive the same amount of heat when \( q \) = constant.

From page 80K,

\[
\frac{dT_b}{dx} = \frac{qC}{mC_p}
\]

where \( C \) denotes the circumference of the cross section. The quantities \( C, m, \) and \( C_p \) are all constant with \( x \). When \( q \) = constant , Then \( dT_b/dx = \)
constant, that is,

\[ \frac{dT_b}{dx} = \text{constant} = \frac{q C}{mc_p} \]

From this, the linear rise of the bulk temperature is given by

\[ T_b = T_{b, \text{in}} + \frac{q C}{mc_p} x \]

For the fully developed situation, \( h = \text{constant} \), and with \( q = h(T_w - T_b) \),

\[ T_w = T_b + \left( \frac{q}{h} \right) \text{constant} \]

Therefore,

developing

\[ T \]

fully developed

\[ T_w \]
3. There is external convection (at the outside surface of the tube/duct) with $T_\infty = \text{constant}$ and $h_\infty = \text{constant}$.

This situation is, in fact, closer to practice than the other fundamental thermal boundary conditions. It is exemplified by a pipe carrying hot water passing through a room. As the water moves through the pipe, its temperature drops. If the pipe is long enough, its temperature approaches $T_\infty$, i.e.,

The heat transfer passing from the
surroundings to the fluid in the tube/duct encounters three resistances in series. For a round pipe, the total resistance for a short length $\Delta x$ is

$$R = \frac{1}{h \pi D_1 \Delta x} + \frac{\ln \frac{D_2}{D_1}}{2 \pi \Delta x k_{pipe}} + \frac{1}{h_\infty \pi D_2 \Delta x}$$

The heat transfer rate $Q$ for the length $\Delta x$ is

$$Q = \frac{T_\infty - T_b}{R}$$

where $T_b$ is the local bulk temperature. The local heat transfer rate $q$
per unit inside tube wall area is

\[ q = \frac{Q}{\pi D_1 \Delta x} \]

When \( Q \) and the resistance \( R \) are substituted from the preceding page,

\[ q = \frac{T_{\infty} - T_b}{\frac{1}{h} + \frac{D_i \ln(D_2/D_i)}{2K_{pipe}} + \frac{D_i}{D_2} \frac{D_i}{h_{\infty}}} \]

Since \( T_b \rightarrow T_{\infty} \) at large \( x \), then \( q \rightarrow 0 \). Actually, theory shows that

\[ |T_{\infty} - T_b| \rightarrow C_1 e^{-\text{const.}x} \]

so that \( |q| \rightarrow C_2 e^{-\text{const.}x} \)

Then, the ratio

\[ h = \frac{|q|}{|T_{\infty} - T_b|} = \frac{C_2 e^{-\text{const.}x}}{C_1 e^{-\text{const.}x}} = \text{a constant} \]

Therefore, a fully developed \((h = \text{const})\) regime is achieved.
LAMINAR FLOW

The first order of business is to establish when a flow is laminar. A criterion commonly used is the magnitude of the Reynolds number. For a round pipe with inside diameter D in which the mean velocity is $\bar{U}$, the Reynolds number is

$$Re_D = \frac{\rho \bar{U}D}{\mu} = \frac{\bar{U}D}{\nu}$$

Another form that is more useful for practical calculations can be arrived at by noting that

$$m = \rho A \bar{U} = \rho \frac{\pi D^2}{4} \bar{U}, \text{ or } \rho \bar{U}D = \frac{4m}{\pi D}$$

so that

$$Re_D = \frac{4m}{\mu \pi D}$$

The cross section of a circular tube is completely described by a single quantity - the diameter D. However, for a non-circular duct, more than one
A dimension is needed to characterize the shape of the cross section. For example, for a rectangular cross section,

\[ \text{both } H \text{ and } W \text{ have to be specified.} \]

Since the Reynolds number has space for only a single dimension, how such a dimension should be chosen is open.

For non-circular ducts, the hydraulic diameter \( D_H \) (or the equivalent diameter \( D_e \)) is used as the characteristic dimension of the cross section. For the definition of \( D_H \), consider a general cross-sectional shape whose cross-section area is \( A \) and circumference is \( C \).
With these, \[ D_H = \frac{4A}{C} \]

Then, \[ Re_{D_H} = \frac{\rho \bar{U} D_H}{\mu} = \frac{\bar{U} D_H}{\mu} \]

This can be rewritten as \[ Re_{D_H} = \frac{\rho A \bar{U}}{A} \left( \frac{4A}{C} \right) \frac{1}{\mu} = \frac{4 m}{\mu C} \]

Note that for a round pipe, \( C = \pi D \), and \( Re_{D_H} = Re_D \)

Now, it is appropriate to ask for the range of Reynolds numbers for which laminar flow is to be expected. It is commonly assumed that for circular tubes, the flow is laminar for \[ Re < \sim 2300 \]

Actually, this criterion is valid for industrial applications where there are elbows, valves, tees, and other fittings which cause flow disturbances. In a laboratory setting, where great care may
be taken to avoid disturbances, laminar flow may actually occur at \( \text{Re}_D \) values as high as 50,000.

For rectangular ducts, \( \text{Re}_{Dh} < \sim 2300 \) is a suitable criterion for laminar flow under industrial conditions.

**Fully Developed Laminar Heat Transfer**

For a circular tube, the fully developed heat transfer results for the three fundamental thermal boundary conditions are:

1. Uniform tube wall temperature

\[
\text{Nu}_D = \frac{hD}{k} = 3.657, \quad h = 3.657 \frac{k}{D}
\]

2. Uniform heating

\[
\text{Nu}_D = \frac{hD}{k} = 4.364, \quad h = 4.364 \frac{k}{D}
\]

3. External convection

The \( \text{Nu}_D \) results depend on a parameter \( \text{Bi}^* \) defined as
\[ B_i^* = \frac{\frac{h_0 D_2}{k_{pipe}}}{2 + \frac{h_0 D_2}{k_{pipe}} \ln \left( \frac{D_2}{D_1} \right)} \]

<table>
<thead>
<tr>
<th>( B_i^* )</th>
<th>( Nu_0 )</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>4.364</td>
</tr>
<tr>
<td>0.1</td>
<td>4.330</td>
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<td>4.284</td>
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<td>2</td>
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<td>5</td>
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<tr>
<td>10</td>
<td>3.758</td>
</tr>
<tr>
<td>100</td>
<td>3.663</td>
</tr>
<tr>
<td>8</td>
<td>3.657</td>
</tr>
</tbody>
</table>

For various non-circular ducts, the values of the fully developed Nusselt number \( Nu_{D_H} \) are listed on the next page for the uniform temperature and uniform heating boundary conditions. The last column lists values of \( f Re_{D_H} \), where \( f \) is the friction factor (dimensionless pressure drop).
Nusselt numbers and the product of friction factor times Reynolds number for fully developed laminar flow in ducts of various cross-sections.

<table>
<thead>
<tr>
<th>Cross Section</th>
<th>Nu* (D_h)</th>
<th>Constant Axial Wall Heat Flux</th>
<th>Constant Axial Wall Temperature</th>
<th>(fRe_{D_h})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td>3.1</td>
<td>2.4</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>Circle</td>
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<td>3.657</td>
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<td></td>
</tr>
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<td>Square</td>
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<td>2.976</td>
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</tr>
<tr>
<td>Rectangle</td>
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<td>3.1</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>1 1/4</td>
<td>4.1</td>
<td>3.4</td>
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</tr>
<tr>
<td>2</td>
<td>4.8</td>
<td>4.0</td>
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<td></td>
</tr>
<tr>
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<td>4.4</td>
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<td>6.5</td>
<td>5.6</td>
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<td>8</td>
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<td>4.861</td>
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<tr>
<td>Insulated</td>
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</tr>
</tbody>
</table>
Thermal entrance region results are available in the research literature. To illustrate what is available, results are presented here for a circular tube with either uniform wall temperature or uniform wall heating.

\[
\frac{x/D}{Re_D Pr}
\]

Note that \( N_{uD} = hD/K \) and \( \overline{N_{uD}} = \bar{h}D/K \). The figure is for thermal development with an already developed velocity.
TURBULENT FLOW

A flow does not become fully turbulent when the laminar flow breaks down. There is a range of Reynolds numbers $>2,300$ where the flow is mixed laminar and turbulent. For $Re_0 > 10,000$, the flow is fully turbulent. Because of this, the $Nu_0$ equation for turbulent pipe flow is slightly altered for $Re_0 > 10,000$ and $Re_0 < 10,000$.

With regard to practice, tube/duct flows are much more likely to be turbulent than laminar. As a consequence, there is a considerable amount of experimental data in the literature for turbulent pipe flows.

Strictly speaking, fully developed heat transfer ($h = \text{constant}$) is achieved in turbulent flow for the three fundamental boundary conditions (uniform wall temperature, uniform wall heating, and external convection).
However, because of the good mixing inherent in turbulent flows, $h$ is not very sensitive to the specifics of the thermal boundary conditions. Because of this, it is acceptable to assume that $h = \text{constant}$ is achieved in almost all turbulent pipe/duct flows. The determination of $h = \text{constant}$ is a two-step process:

1. Friction factor

A graph illustrating the friction factor $f$ is shown, indicating that it depends on the flow regime (laminar or turbulent) and eventually approaches a constant value.

2. Nusselt number $\text{Nu}_D = \frac{hD}{k}$

The Nusselt number equation is written as:

$$\text{Nu}_D = \frac{(f/8) \text{Re}_D \text{Pr}}{1.07 + 12.7 \sqrt{f/8} (\text{Pr}^{1/3} - 1)(\mu_b^{1/3})}$$

- $0 \leq \frac{\mu_b}{\mu_w} \leq 40$
- $10^4 < \text{Re}_D < 5 \times 10^6$

- $0.5 < \text{Pr} < 200$
- $200 < \text{Pr} < 2000$

- $n = 0.11$ for 6% accuracy
- $n = 0.25$ for 10% accuracy
- $n = 0$ for $T_w > T_b$
- $n = 0$ for $T_w < T_b$

Notes: All fluid properties are evaluated at the bulk temperature $T_b$, except that $\mu_w$ is at $T_w$. For $\text{Re}$ between 2300 and 10,000, $\text{Re}_D \rightarrow (\text{Re}_D - 1000)$ and $1.07 \rightarrow 1.00$ in the $\text{Nu}_D$ equation.
The foregoing equations for fully developed turbulent heat transfer, which were developed for round pipes, can also be used for non-circular ducts by the substitutions

\[ \text{Re}_D \rightarrow \text{Re}_{Dh}, \quad \text{Nu}_D \rightarrow \text{Nu}_{Dh} \]

The length of the thermal entrance region is relatively short for turbulent flows, e.g.,

\[
\frac{\text{Pr}}{0.7} \quad \frac{X_{\text{ent}}}{D} = \begin{cases} 
15 & 15 \\
5 & \frac{3}{10} & 3
\end{cases}
\]

Because of this, it is quite common to use the fully developed \( h \) all the way forward to the beginning of the duct. The largest entrance region effects are for gases such as air (Pr \( \approx 0.7 \)), as shown at the right.
EXAMPLE

Air flowing in a rectangular duct I is to be heated by steam condensing in duct II. The wall separating the two ducts is at a uniform temperature of 100°F. The entering and exit bulk temperatures of the air are 75°F and 78.85°F, respectively.

Other information about the airflow:

\( \dot{m} = 0.117 \text{ lb}_m/\text{sec}, \quad W = 10 \text{ in.}, \quad H = 0.5 \text{ in.}, \quad L = 2 \text{ ft} \)

\( \rho = 0.0735 \text{ lb}_m/\text{ft}^3, \quad \nu = 1.69 \times 10^{-4} \text{ ft}^2/\text{sec} \)

\( k = 0.0152 \text{ Btu/hr-ft-}^\circ\text{F}, \quad c_p = 0.240 \text{ Btu/lb}_m-^\circ\text{F}, \quad Pr = 0.708 \)

(a) Calculate \( \overline{N} \) for the airflow using the appropriate Nusselt number correlation from the notes.

(b) Compute the value of \( Q \) in two different ways.

It may be assumed that the external surface of the entire heat exchanger is perfectly insulated.
a) First task in finding \( \dot{Q} \) is to find the appropriate Reynolds Number. For a non-circular duct we need to use the hydraulic diameter.

\[
Re_D = \frac{UD_D}{\nu} = \frac{\dot{m}}{\rho A} = \frac{117 \text{ lbm/s}}{0.0735 \text{ lb/s} \left( \frac{10}{12} \times \frac{12}{12} \right) \text{ ft}^2} = 45.84 \text{ ft/s}
\]

\[
D_D = \frac{4A}{C} = \frac{4(10 \text{ in})(0.5 \text{ in})}{2(10 \text{ in} + 0.5 \text{ in})} = 0.9524 \text{ in} = 0.0794 \text{ ft}
\]

\[
Re_D = \frac{45.84 \text{ ft/s}}{0.0794 \text{ ft}} = 581 \cdot 10^{-4} \text{ ft}^3/\text{lb} \cdot \text{s} \text{ >> 2300}
\]

Use two-step equation

You should get \( f = 0.02563 \) and \( Nu = 52.96 \).

Now \( \frac{\dot{Q}}{h} = \frac{Nu}{D_D} \cdot f \cdot \frac{1}{\frac{0.0152 \text{ Btu/hr}^{\circ F}}{0.0794 \text{ ft}}} = 10.14 \text{ Btu/hr}^{\circ F} \)

b) Only ways to find \( \dot{Q} \) are:

\[
\dot{Q} = mC_p(\Delta T) = 117 \text{ lbm/s} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \cdot (0.24 \text{ Btu/lbm})(78.85 - 75) \text{ °F} = 389.2 \text{ Btu/hr}
\]

\[
\dot{Q} = \dot{Q} = \dot{Q} (\text{LMTD})
\]

here, since the outer wall of duct is insulated, \( A \) = surface area of the heated wall between the two ducts.

\[
A = 2 \frac{10}{12} \text{ ft}^2 = 1.667 \text{ ft}^2
\]

\[
\text{LMTD} = \frac{(T_w - T_{in}) - (T_w - T_{out})}{\ln \left[ \frac{T_{out} - T_{in}}{T_w - T_{in}} \right]} = \frac{(100 - 75) - (100 - 78.85)}{\ln \left[ \frac{100 - 75}{100 - 78.85} \right]}
\]

\[
\text{LMTD} = 23.02
\]

\[
\dot{Q} = 10.14 \text{ Btu/hr}^{\circ F} \cdot 1.667 \text{ ft}^2 (23.02 \text{ °F}) = 389.1 \text{ Btu/hr}
\]
EXAMPLE

Air flows in a circular tube whose inner diameter $D = 2.45$ cm. The mass flow rate $\dot{m}$ of the airflow is $0.0617$ kg/sec. At the inlet of the tube, the air temperature and pressure are, respectively, $76.85^\circ$C and 1 atm. The wall temperature $T_w$ is maintained uniform at $103.17^\circ$C by steam condensing on the outer surface of the tube. What is the rate of heat transfer $Q$ from the tube wall to the air between $x = 0$ and $1.26$ m?

Solution

At the inlet, with $T_{in} = T_{bin} = 350$K,

$$Re = \frac{4\dot{m}}{\pi D} = \frac{4\dot{m}}{\pi D} = 154 \times 10^5 \gg 2300 \rightarrow \text{Flow is turbulent}.$$  

We need $\bar{T}_w = \frac{1}{2}(T_{bin} + T_{bwall})$. However, we don't know $T_{bwall}$. This is an iterative process problem.

Procedure:

1. Guess at $T_{bwall}$, find $\bar{T}_w = \frac{1}{2}(T_{bin} + T_{bwall})$, and find relevant properties at $\bar{T}_w$.
2. Find $Re = 4\dot{m}/\pi D$
3. Find $f$ and $Nu_D$, and $h = Nu_D K/D = -\frac{hA}{mC_p}$
4. Find $T_{bwall}$ from $T_{bwall} = Tw + (T_{bin} - Tw)e$
5. Find $Q$ from $Q = \dot{m}C_p (T_{bwall} - T_{bin})$
6. Using the newly obtained $T_{bwall}$, repeat steps 1 through 5 until the properties evaluated in step 1 no longer change.

Following the procedure outlined above.

$$h = 287.66 \text{ W/m}^2\text{K}, \quad T_{bwall} = 86.36^\circ\text{C}$$

$$Q = 592 \text{ W}$$
TUBE BANKS

A tube bank is an array of tubes in crossflow which are arranged in a regular pattern — either staggered or in-line with respect to the flow direction. Heat transfer occurs between the fluid that flows outside the tubes and the fluid that flows inside the tubes.

In-Line Array

Staggered Array
Attention will now be focused on the heat transfer and flow outside of the tubes.

The crossflow enters the tube bank at a temperature $T_{b,in}$ and leaves the tube bank at a temperature $T_{b,out}$ (both bulk temperatures). The outside surface temperature of all the tubes is $T_w$. $A$ is the total outside surface area of all the tubes, equal to

$$A = \pi D L \times \text{(number of tubes)}$$

where $D$ = outside diameter of the tubes and $L$ = tube length. $h$ is the average heat transfer coefficient for the entire tube bank. $Q$ is the overall rate of heat transfer from the tube bank to the crossflow.

The rate of heat transfer is given by

$$Q = h A \text{ (LMTD)}$$

where

$$\text{LMTD} = \frac{(T_w - T_{b,in}) - (T_w - T_{b,out})}{\ln \left( \frac{T_w - T_{b,in}}{T_w - T_{b,out}} \right)}$$

The average heat transfer coefficient $h$ can be found from the following equations.
\[ \overline{Nu_D} = Pr^{0.34}(Pr/Pr_w)^n fn(Re_D) \]
\[ n = 0 \text{ for gases} \]
\[ n = \frac{1}{3} \text{ for liquids} \]

The function \( fn(Re_D) \) takes the following form for the various circumstances of flow and tube configuration:

\begin{align*}
10 \leq Re_D \leq 100: & \quad fn(Re_D) = 0.8 Re_D^{0.4}, \text{ aligned rows} \\
fn(Re_D) = 0.9 Re_D^{0.4}, \text{ staggered rows} \\
100 < Re_D < 10^3: & \quad \text{treat tubes as though they were isolated} \\
10^3 \leq Re_D \leq 2 \times 10^3: & \quad fn(Re_D) = 0.27 Re_D^{0.63}, \text{ aligned rows} \\
S_T/S_L < 0.7 \\
\text{For } S_T/S_L \geq 0.7, \text{ heat exchange is much less effective. Therefore, tube bundles are not designed in this range and no correlation is given} \\
fn(Re_D) = 0.35(S_T/S_L)^{0.2} Re_D^{0.6}, \text{ staggered rows} \\
S_T/S_L < 2 \\
fn(Re_D) = 0.40 Re_D^{0.6}, \text{ staggered rows} \\
S_T/S_L \geq 2 \\
Re_D > 2 \times 10^3: & \quad fn(Re_D) = 0.021 Re_D^{0.84}, \text{ aligned rows} \\
fn(Re_D) = 0.022 Re_D^{0.84}, \text{ staggered rows} \\
Pr > 1 \\
\overline{Nu_D} = 0.019 Re_D^{0.84}, \text{ staggered rows} \\
Pr = 0.7
\end{align*}

Properties at:
\[ \bar{T} = \frac{1}{2}(T_{in} + T_{out}) \]
\[ Pr_w \text{ is at } Tw. \]

For tube banks with few rows, correct the \( \overline{h} \) from these equations with a factor \( < 1 \) from the following graph.

The Reynolds number \( Re_D \) appearing in these equations has a special definition:
$Re_D = \frac{\rho U_{\text{max}} D}{\mu}$

$U_{\text{max}}$ is the velocity in the minimum free flow area. The minimum free flow area is defined for in-line and staggered arrays in the diagrams below.

**In line:**

Minimum free flow area per lane

**A lane**

**Staggered:**

**A lane**

The minimum free flow area per lane is the smaller of $ab$ and $(cd + ef)$. 
There are two alternative ways of calculating $Q$. One is $Q = \dot{m}A\text{ (LMTD)}$ as already discussed. The other, which comes from the first law of thermodynamics, is

$$Q = \dot{m} c_p (T_{b,\text{out}} - T_{b,\text{in}})$$

where $\dot{m}$ is the rate of mass flow through the tube bank. Since the two ways of computing $Q$ give the same value

$$\dot{m}A\text{ (LMTD)} = \dot{m} c_p (T_{b,\text{out}} - T_{b,\text{in}})$$

If the LMTD is substituted into this equation, then it is obtained

$$T_{b,\text{out}} = T_w + (T_{b,\text{in}} - T_w) e^{-\left(\frac{\dot{m}A}{\dot{m} c_p}\right)}$$

This equation provides a way of determining $T_{b,\text{out}}$ if it is unknown.

**TUBE BANK PRESSURE DROP**

A knowledge of the pressure drop through the tube bank is of equal importance to a knowledge of the heat transfer rate.
The pressure drop per row in a tube bank is given by

$$\Delta P_{row} = \left(\rho \frac{U_{\text{max}}}{2}\right) \cdot X \cdot f$$

where $\rho$ is at $T = \frac{1}{2}(T_{\text{in}} + T_{\text{out}})$ as before.

The quantities $X$ and $f$ are found from the figures on the next page—upper figure for in-line and lower figure for staggered. The parameters for these figures are

$$P_t = \frac{S_t}{D}, \quad P_l = \frac{S_l}{D}, \quad \text{Re}_{D,\text{max}} \rightarrow \text{Re}_D \text{ from page 98}$$

$S_t$ and $S_l$ are the transverse and longitudinal center-to-center distances as illustrated in the lower figures on page 1.

The quantity $X$ is obtained from the small inserted diagram at the upper right of each figure on the next page, while $f$ is obtained from the main part of the figure.

To find $X$, $P_t$, and $P_l$ are first computed and then used to enter the abscissa of the inserted diagram. $f$ for the in-line case is read from the main figure using $P_l$ as the
Friction factor $f$ and correction factor $\chi$
Curve parameter, while \( f \) for the staggered case is read from the other main figure using \( P_f \) as the curve parameter.

When there are 10 or more rows, the overall pressure drop is calculated from

\[
\Delta p = (\Delta p)_{row} \times (\text{number of rows})
\]

For arrays with fewer rows, corrections for inlet and exit effects should be made, but are often neglected.
EXAMPLE

In a tube bank in which the tubes are positioned in a staggered pattern, the geometry of the array is defined by $D = 1$ in., $S_t = 1.5$ in., $S_l = 0.9$ in. The velocity upstream of the array is $U_o = 31.5$ ft/sec. Compute the Reynolds number $Re_D$ for the array. Use $\nu = 1.69 \times 10^{-4}$ ft$^2$/sec.

\[ Re_{D,max} = \frac{U_{max}D}{\nu} \]

You need to find the minimum free-flow area to determine $U_{max}$. Compare $\overline{ab}$ to $(\overline{cd} + \overline{ef})$.

\[ \overline{ab} = S_t - D = 1.5 - 1 = 0.5 \text{ in} \]
\[ \overline{cd} = \overline{ef} = \sqrt{\left(\frac{S_t}{2}\right)^2 + S_l^2} - D = \sqrt{\left(\frac{1.5}{2}\right)^2 + (0.9)^2} - 1 = 0.17154 \text{ in} \]

Therefore, $\overline{cd} + \overline{ef} = 0.3431 \text{ in}$. This is SMALLER than $\overline{ab}$.

Find $U_{max}$ using continuity (mass conservation).

\[ \frac{A_{in}U_o}{A_{min}} = \frac{A_{in}U_{max}}{A_{min}} \]

\[ U_{max} = U_o \frac{A_{in}}{A_{min}} = 31.5 \text{ ft/sec} \frac{1.5 \text{ in}}{0.3431 \text{ in}} = 137.73 \text{ ft/sec} \]

\[ Re_{D,max} = \frac{U_{max}D}{\nu} = \frac{137.73 \text{ ft/sec}}{1.69 \times 10^{-4} \text{ ft}^2/\text{sec}} = 67912 \]

\[ T_{c, out} = T_{c, un} = \left( \frac{m_{cP}}{m_{cP}} \right) n \left( T_{in} - T_{c, in} \right) \]

1. $U_f \left( \frac{m_{cP}}{m_{cP}} \right) h > \left( \frac{m_{cP}}{m_{cP}} \right) c$

2. $C_{P} \left( \frac{m_{cP}}{m_{cP}} \right) h > \left( \frac{m_{cP}}{m_{cP}} \right) c$
Heat exchangers are devices which facilitate the transfer of heat from one thermal medium to another. In current practice, heat exchangers are generally regarded as involving two fluids, one at a higher temperature and the other at a lower temperature. There are three generic categories of heat exchangers. They are: (a) recuperator, (b) regenerator, and (c) direct contact.

In a recuperator, the two participating fluids are kept separate from each other by stationary walls. The essence of a recuperator is displayed in Fig HX-1. The figure shows a segment of wall and the respective fluids separated by the wall.
Fig. HX-1

As pictured there, the fluids are flowing parallel to each other. However, this is one of the many possible relative orientations of the fluids, as will be discussed shortly.

A regenerator uses the heat capacity of a matrix to cyclically store energy imparted by a hot fluid passing through it and then distribute the energy to a cold fluid passed through the matrix.

Heating period

\[ T_{\text{hot, in}} \rightarrow \text{matrix} \rightarrow T_{\text{hot, out}} \]

Fig. HX-2

Cooling period

\[ T_{\text{cold, out}} \rightarrow \text{matrix} \rightarrow T_{\text{cold, in}} \]
Figure HX-2 provides an illustration of a regenerator and its operation. As shown there, there is a period during which hot fluid passes through the matrix. The matrix absorbs heat from the fluid, and its temperature rises. When the matrix temperature reaches a desired value, the hot stream is turned off and the cold stream initiated. The cold stream draws energy out of the matrix, thereby causing the matrix temperature to drop to a preselected value. At that point, the cold stream is turned off, and the hot stream is initiated, and the cycle is repeated. During the period in which the matrix absorbs energy from the hot stream, $T_{\text{out}} < T_{\text{out, in}}$. During the other part of the cycle, when the cold stream draws energy out of the matrix, $T_{\text{cold, out}} > T_{\text{cold, in}}$. Thus, by means of the just-described two-step process, heat is transferred from the hot fluid to the cold fluid.

In a **direct-contact** heat exchanger, the participating fluids are brought together without there being walls to
Separate them. Figure HX-3 is a schematic diagram of a representative direct-contact heat exchanger. The two participating fluids in this case are delivered to a drum. Water, being the heavier fluid, enters near the top of the drum, and the oil, being lighter, enters near the bottom. An agitator is rotated to enhance water/oil contact. The oil tends to rise to the top of the drum, while the water tends to drop to the bottom.

Among the three generic categories of heat exchangers, the recuperator is, by far, the most common type employed in practice. In fact, it is not uncommon for practitioners to equate heat exchangers and recuperators. In light of this, only recuperators will be considered here. Information on regenerators can be found in Section 3.15 of the
most complete collection of heat exchanger information:


Some information on direct-contact devices is available in Section 2.6.8 of The Handbook.

Representative Recuperators

A commonly encountered recuperator is the shell and tube heat exchanger, which is shown schematically in Fig. HX-4.

Fig. HX-4
As seen there, the main components of the exchanger are a cylindrical shell or drum, an array of tubes, a pair of tube sheets, and baffles. Each tube sheet is a circular disk in which a regular array of holes has been drilled. The tubes are positioned by being inserted in these holes. The presence of the tube sheets creates plenum chambers at each end of the exchanger through which the fluid which flows in the tubes enters and leaves the exchanger. The shell-side fluid is guided by the baffles. The baffles force the shell-side fluid to pass in crossflow across the tubes in order to increase the shell-side heat transfer coefficient.

In Fig. HX-4, the tube fluid passes directly from one end of the heat exchanger to the other. Similarly, the shell fluid passes from one end to the other. This situation is described as having one tube pass and one shell pass. To better understand what this means, it is useful to look at multipass situations. In Fig. HX-5, the tube fluid passes
from one end to the other and then turns around and returns to the end where it entered. On the other hand, the shell fluid goes directly from one end to the other. This arrangement is described as two tube passes and one shell pass.

Fig. HX-5

An even larger number of passes is illustrated in Fig. HX-6. Here, there are four passes between the tube fluid's inlet and outlet and two passes between the shell fluid's inlet and outlet.

Shell and tube heat exchangers are
often large devices, several feet in diameter and ten or more feet in length, although smaller versions are sometimes encountered. Other, less massive heat exchangers are typified by the plate-fin and tube configuration shown at the right. Such an arrangement is used in air-conditioning machines where the refrigerant passes through the tubes and air passes through the channels formed by the plate fins. The tubes in a plate-fin and tube heat exchanger are not always round as indicated in Fig. HX-8, where they are shown flattened. The most common

Fig. HX-7

Fig. HX-8
application of flattened tubes is in automobile radiators, where, also, the fins are crinkled rather than flat.

Another common type of exchanger is the stack which is shown at the right. It consists of rectangular ducts placed one atop the other, with the flow direction alternating from duct to duct, giving Fig. HX-9 a crossflow pattern.

In Fig. HX-9, fins have been inserted in each of the ducts, but in other applications, there are no fins.

Tube banks, described in an earlier chapter, are also heat exchangers. For reference purposes, a tube bank is pictured in Fig. HX-10. The dotted-line outline in the figure is meant to define the space through the crossflow passes. It is interesting to compare the heat
exchangers pictured in Figs. HX-7 and HX-10. Both are crossflow heat exchangers involving flow over a tube bank. However, the flow pattern for the fluid which passes outside the tubes is quite different in the two cases. Fig. HX-10

In Fig. HX-7, the fluid is compartmentalized into separate channels by the fins. The flows in the separate channels are more or less independent of each other until the downstream end of the fins is reached. In this case, the external fluid is said to be unmixed. On the other hand, in Fig. HX-10, the external fluid can mix freely within itself, if it so chooses. This situation is described as having the external fluid mixed.

In both cases, the in-tube fluid is compartmentalized in the individual tubes.
Consequently, the tube fluid is said to be unmixed.

Perhaps the physically simplest of all heat exchangers is the double-pipe heat exchanger. As shown in Figs. HX-11 and HX-12, a double-pipe heat exchanger is, in essence, a pipe within a pipe.

Fig. HX-11. Parallel Flow

Fig. HX-12. Counterflow

The participating fluids (A and B respectively) may flow parallel to each other or in opposite directions.
There are various methodologies for analyzing and designing heat exchangers. Two of the commonly used methods are:

1. LMTD method
2. $\varepsilon$-NTU method

LMTD is an abbreviation for log mean temperature difference. The LMTD method is very convenient when the inlet and exit temperatures of both of the participating fluids are known. $\varepsilon$ stands for the effectiveness, and NTU is an abbreviation for number of transfer units. The $\varepsilon$-NTU method is advantageous when the fluid outlet temperatures are unknown.

**LMTD Method**

The most convenient approach to presenting the LMTD method is to illustrate its use for analyzing the double-pipe heat exchanger.

Figure HX-13 is a longitudinal schematic of a double-pipe heat exchanger oper-
ating in the counterflow mode, and under the diagram is a graph displaying representative longitudinal variations of the bulk temperatures of the participating fluids. As seen, the two
participating fluids enter at opposite ends of the heat exchanger. At any position \( x \) in the exchanger, the initially hotter fluid (subscript \( h \)) will be at a higher temperature than the initially cooler fluid (subscript \( c \)). Therefore, no matter what the position, heat will flow from the \( h \) fluid to the \( c \) fluid. The temperature of the \( h \) fluid drops continuously in the direction of \( h \) fluid flow, while the temperature of the \( c \) fluid rises continuously in the direction of \( c \) fluid flow.

The \( h/c \) temperature differences at each end of the exchanger are indicated in Fig. HX-14 as \( \Delta T_I \) and \( \Delta T_H \). These \( \Delta T \)'s can be used to define a log mean temperature difference (LMTD) as follows:

\[
LMTD = \frac{\Delta T_I - \Delta T_H}{\ln \frac{\Delta T_I}{\Delta T_H}} \quad (HX-1)
\]

A longitudinal diagram and associated temperature graph for a parallel-flow heat exchanger are displayed in Fig. HX-14.
Fig. HX-14

At any longitudinal station $x$, heat flows from the $h$ stream to the $c$ stream. Since both fluids flow in the same
direction, their respective temperatures tend to approach each other. However, temperature equality of the two streams will not be achieved unless the heat exchanger is very long.

The hot-to-cold fluid temperature differences at the respective ends of the heat exchanger are defined in Fig. HX-14 as $\Delta T_1$ and $\Delta T_2$. This is the same notation previously used in Fig. HX-13 for the counterflow case, but the meanings of $\Delta T_1$ and $\Delta T_2$ are different for the respective cases. Specifically, for counterflow:

$$\Delta T_1 = T_{h, in} - T_{c, out}$$

$$\Delta T_2 = T_{h, out} - T_{c, in}$$

and for parallel flow

$$\Delta T_1 = T_{h, in} - T_{c, in}$$

$$\Delta T_2 = T_{h, out} - T_{c, out}$$

The LMTD has the same definition for
counterflow and parallel flow, namely, equation (HX-1).

In analyzing double-pipe heat exchangers, it is widely assumed that the outer surface of the outermost pipe is very well insulated. The presence of the insulation is represented by the pattern of random dots surrounding the heat exchangers in Figs. HX-13 and HX-14. The insulation is assumed to totally block any possible heat losses or gains between the outermost pipe and the ambient. Therefore, any and all heat transfer out of the hotter fluid is totally absorbed by the cooler fluid.

The rate of heat transfer from the hotter to the cooler fluid is denoted by $Q$. If $T_{h, in}$ and $T_{h, out}$ are the inlet and outlet bulk temperatures of the hot fluid, then

$$Q = (m_{c_p})_h (T_{h, in} - T_{h, out}) \quad (HX-4)$$

where $(m_{c_p})_h$ is the mass flow - specific heat product of the hotter fluid. It is
usual to call $mc_p$ the capacity rate $C$, so that

$$C = mc_p, \quad C_h = (mc_p)_h, \quad C_c = (mc_p)_c$$

Therefore, equation (HX-5) becomes

$$Q = C_h (T_{h,\text{in}} - T_{h,\text{out}})$$

(HX-6)

Since this same $Q$ is absorbed by the cooler fluid

$$Q = (mc_p)_c (T_{c,\text{out}} - T_{c,\text{in}}) = C_c (T_{c,\text{out}} - T_{c,\text{in}})$$

(HX-7)

Upon equating the $Q$'s in equations (HX-4) and (HX-7), there follows

$$\frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{h,\text{out}}} = \frac{(mc_p)_b}{(mc_p)_c}$$

(HX-8)

This equation indicates that the temperature rise of the cooler fluid and the temperature drop of the hotter fluid are, in general, not equal. Only when $(mc_p)_c = (mc_p)_h$ are the magnitudes of the two
temperature changes equal.

One of the main uses of equation (HX-8) is for the following problem scenario: The inlet temperatures $T_{i,h}$ and $T_{i,c}$ are given, as are the capacity rates $(m_{cp})_h$ and $(m_{cp})_c$. The outlet temperature of one of the fluids, say $T_{o, out}$, is specified as a design goal. The outlet temperature of the other fluid, say $T_{h, out}$, can be determined by using equation (HX-8), Also, $Q$ can be found from equations (HX-4) and (HX-7). The main unknown is the size of the heat exchanger needed to achieve the specified outlet temperature. None of the equations (HX-1) to (HX-8) contain any parameters related to the size of the exchanger. Therefore, other equations have to be identified.

Experience with heat transfer between a fluid flowing in a duct and the duct wall provides guidance as to how to proceed. If the respective wall and bulk temperatures at stations 1 and 2 are $T_{w1}$, $T_{w2}$ and $T_{b1}$ and $T_{b2}$, then the LMTD can
be evaluated as

\[ LMTD = \frac{(T_w - T_b)_1 - (T_w - T_b)_2}{\ln \frac{(T_w - T_b)_1}{(T_w - T_b)_2}} \] (HX-9)

and

\[ Q = \bar{h}A (LMTD) \] (HX-10)

where \( \bar{h} \) is the average heat transfer coefficient and \( A \) is the area of the duct surface which interfaces with the fluid.

In the case of the double-pipe heat exchanger, the heat transfer is between two fluids (separated by a wall) rather than between a fluid and a wall. Therefore, the duct-related LMTD of equation (HX-9) has to be replaced by the LMTD for a double-pipe heat exchanger, equation (HX-1). Furthermore, the thermal resistance, \( 1/\bar{h}A \), that appears in equation (HX-10) has to be replaced by a thermal resistance that accounts for all the series resistances encountered by heat passing from one fluid to the other.
Figure HX-15 illustrates the cross section of a double-pipe heat exchanger. As heat passes from the hotter to the cooler fluid—a radial heat flow—it encounters three resistances in series. This situation has been dealt with earlier, and the thermal resistance was written as

\[ R = \frac{1}{h_{\text{in}} 2\pi R_{\text{in}} L} + \frac{\ln \left( \frac{R_{\text{out}}}{R_{\text{in}}} \right)}{2\pi k_{\text{pipe}} L} + \frac{1}{h_{\text{out}} 2\pi R_{\text{out}} L} \]  

(HX-11)

where \( L \) is the length of the heat exchanger. It is commonly assumed that \( R \) is constant throughout the heat exchanger. Then, for the double-pipe heat exchanger,
Equation (HX-12) is perfectly adequate for connecting the heat transfer rate with the fluid temperatures, the size of the heat exchanger, and the individual heat transfer coefficients. However, in the public literature, a cosmetically different form of equation (HX-12) is used. To convert the equation, let

\[ UA = \frac{1}{R} \quad (HX-13) \]

In this equation, \( U \) is called the **overall heat transfer coefficient**, and \( A \) is the surface area normal to radial heat transfer. The difficulty here is that two different surface areas can be identified,

\[ A_{\text{in}} = 2\pi R_{\text{in}} L, \quad A_{\text{out}} = 2\pi R_{\text{out}} L \quad (HX-14) \]

Therefore,

\[ UA = U_{\text{in}} A_{\text{in}} = U_{\text{out}} A_{\text{out}} = \frac{1}{R} \quad (HX-15) \]
Since the UA product has the same value regardless of whether $A_{in}$ or $A_{out}$ is used, i.e., $= \frac{1}{R}$, then the choice of $A$ is at the choice of the user.

**EXAMPLE**

A double-pipe heat exchanger having a cross section identical to that of Fig. HX-15 is to be analyzed. The inner fluid is the hotter fluid, and the outer fluid is the cooler fluid, i.e.,

$\text{in} \sim \text{hotter} \sim h_1; \quad \text{out} \sim \text{cooler} \sim c$

The two fluids flow in counterflow. Heat losses at the outermost pipe are to be neglected. The dimensions of the exchanger are:

$D_{in} = 2.72 \text{ in}, \quad D_{out} = 2.80 \text{ in}, \quad L = 5.83 \text{ ft}$

The inner pipe is made of 0.5% carbon steel. The inner fluid is air, with fluid properties:

$K = 0.01602 \frac{\text{Btu}}{\text{hr}-\text{ft}^{-2}-\text{oF}}, \quad C_p = 0.24 \frac{\text{Btu}}{\text{lb}_m^{-1} \cdot \text{oF}}$
\( \mu = 1.312 \times 10^{-5} \frac{\text{lbm}}{\text{ft-sec}} \), \( \nu = 1.917 \times 10^{-4} \frac{\text{ft}^2}{\text{sec}} \)

\( Pr = 0.709 \)

The outer fluid is air. The heat transfer coefficient for the outer fluid was found by prior calculations to be \( h_{\text{out}} = 5.12 \) Btu/hr-ft\(^2\)-°F. The inlet temperatures of the participating fluids are

\[ T_{h,\text{in}} = 133^\circ\text{F}, \quad T_{c,\text{in}} = 60^\circ\text{F} \]

Find the velocity \( U \) of the air in the inner pipe which yields

\[ T_{h,\text{out}} = 107^\circ\text{F}, \quad T_{c,\text{out}} = 88.5^\circ\text{F} \]

**Solution**

With the use of equations (HX-2),

\[ \Delta T_1 = 133 - 88.5 = 44.5, \quad \Delta T_2 = 107 - 60 = 47^\circ\text{F} \]

and, with these, the LMTD of equation (HX-1) becomes

\[ \text{LMTD} = \frac{44.5 - 47}{\ln \frac{44.5}{47}} = 45.74^\circ\text{F} \]
and from equation (HX-12)

\[ Q = \frac{45.74}{R} \]

and the equation for \( R \), equation (HX-11), can be partially evaluated as

\[ R = \frac{1}{h_m 2\pi \frac{2.72}{2.12} \frac{5.83}{5.83}} + \frac{\ln \frac{2.80}{2.72}}{2\pi (30.5)(5.83)} \approx 0.0000259 \]

\[ + \frac{1}{(5.12) 2\pi \frac{2.80}{2.12} \frac{5.83}{5.83}} \approx 0.0457 \]

Clearly, the resistance of the pipe wall can be neglected. Then

\[ R = \frac{1}{4.152 h_m} + 0.0457 \]

The next step is to determine the coefficient \( h_m \) for the tube flow. Since the velocity of the tube flow is unknown, it is not possible to know if the flow is laminar or turbulent.
Because of this, it is necessary, at first, to guess the flow regime (laminar or turbulent) and then, when the problem is fully solved, to check the value of the Reynolds number.

If laminar flow is guessed, then by inspecting the duct flow section of the presentation of the convection results, it is seen that for a round pipe, the fully developed Nusselt numbers are between 3.657 (uniform wall temperature) and 4.364 (uniform wall heat flux). The average of these will be used, i.e.,

\[ \text{Nu}_D = \frac{3.657 + 4.364}{2} = 4.0105 = \frac{h_{in} D_{in}}{k} \]

so that

\[ h_{in} = \frac{(4.0105)(0.01602)}{(2.72/12)} \]

\[ h_{in} = 0.2834 \text{ Btu/hr-ft}^2-\text{OF} \] (small value!)

and

\[ R = \frac{1}{(4.152)(0.2834)} + 0.0457 \]

\[ R = 0.8954 = \frac{\text{OF}}{\text{Btu/hr}} \]

and

\[ Q = \frac{45.74}{0.0894} = 51.08 \text{ Btu/hr} \]
To find $\bar{U}$ for the flow in the inner pipe, it is useful to restate equation (Hx-4) as:

$$m_h = \frac{Q}{c_{ph}(T_{h,\text{in}} - T_{h,\text{out}})}$$

and then to recall:

$$m_h = (\rho AV)h = \rho \left( \pi D_{in}^2 / 4 \right) \bar{U}$$

Then, solving for $\bar{U}$ and introducing numbers, there follows:

$$\bar{U} = \frac{4 \left( \frac{51.08}{3600} \right)}{(0.24)(133-107) \left( \frac{1.312 \cdot 10^{-5}}{1.917 \cdot 10^{-4}} \right) \pi \left( \frac{2.72}{12} \right)^2 \rho = \mu / \nu}$$

$$\Rightarrow \bar{U} = 0.823 \text{ ft/sec} \quad \text{(very low velocity)}$$

Now, compute $Re$ to check on laminar flow assumption:

$$Re_D = \frac{\bar{U} D_{in}}{\nu} = \frac{0.823 \left( \frac{2.72}{12} \right)}{1.917 \cdot 10^{-4}} = 973.1$$

Since $973.1 < 2300$, the laminar assumption is verified.
Other types of heat exchangers

The analysis of other types of heat exchangers using the LMTD is based on modifying the approach used for counterflow, double-pipe heat exchangers. The main modification is that the counterflow LMTD is multiplied by a correction factor $F$ to take account of geometrical effects. The formula for analyzing other exchangers is

$$Q = UA F \left( \frac{T_{h,\text{in}} - T_{c,\text{out}}}{\ln \frac{T_{h,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{c,\text{in}}}} \right)$$

\begin{equation}
(HX-16)
\end{equation}

$F$ values for crossflow heat exchangers (e.g., Figs. HX-7, HX-8, HX-9, and HX-10) are provided by Figs. HX-16 and HX-17. The first of these Fig. HX-16 is for cases in which both participating fluids are unmixed (e.g., Figs. HX-7, HX-8, and HX-9). Figure HX-17 is for cases in which one of the fluids is mixed and the other fluid is unmixed.
Both fluids unmixed

**Fig. HX-16**

\[ R = \frac{T_i - T_o}{t_o - t_i} \]

\[ P = \frac{t_o - t_i}{T_i - t_i} \]

One mixed, one unmixed

**Fig. HX-17**

\[ R = \frac{T_i - T_o}{t_o - t_i} \]

\[ P = \frac{t_o - t_i}{T_i - t_i} \]
To use these figures, it is first necessary to evaluate the dimensionless temperatures $P$ and $R$

$$P = \frac{t_0 - t_i}{T_u - t_i}, \quad R = \frac{T_x - T_0}{t_0 - t_i} \quad (HX-17)$$

where $T_u$, $t_0$, $T_x$, and $T_0$ are defined in the diagram associated with each figure. Note that $F < 1$.

$F$ values for shell and tube exchangers are presented in Figs. $HX-18$ and $HX-19$.

$h = 2g\Delta t$

**Fig.**

**$HX-18$**

Figure $HX-18$ is for one shell pass and
Two, four, six, etc. tube passes, as indicated in the diagram above the graph. Figure HX-19 is for two shell passes and for four, eight, twelve, etc. tube passes. The parameters \( P \) and \( R \) in the figures have already been defined in equation (HX-17).
Other resistances: fins, fouling

When finned surfaces are used, care has to be taken in evaluating the thermal resistance for convective heat transfer at that surface. As pictured in Fig. HX-20, the area exposed to fluid flow consists of two parts: the area of the unfinned base surface $A_{ubs}$ and the surface area of the fins $A_{fin}$.

As discussed and analyzed in the exposition of fins, there is a temperature drop (or rise) from the fin base to the
fin tip. Therefore, a real-world fin will transfer less heat than an ideal fin which is at a uniform temperature throughout. It may be recalled that the fin efficiency \( \eta \) compares real fins with ideal fins, i.e.,

\[
\eta = \frac{Q_{\text{fin}}}{Q_{\text{ideal}}} \quad (\text{HX}-18)
\]

where \( \eta \leq 1 \). Another way of looking at this situation is to say that a real fin of surface area \( A_{\text{fin}} \) will transfer the same amount of heat as an ideal fin whose surface area is \( A_{\text{ideal}} \), where

\[
A_{\text{ideal}} = \eta A_{\text{fin}} \quad (\text{HX}-19)
\]

In other words, a real fin of surface area \( A_{\text{fin}} \) can, for purposes of analysis, be replaced by a uniform temperature fin of surface area \( A_{\text{ideal}} \).

The temperature of an ideal fin is equal to the temperature of the base surface to which the fin is attached. Therefore, the actual fin and base surface can be modeled by an isothermal surface.
composed of the unfinned base and the ideal fin. The area of such an isothermal surface is (eff ~ effective)

\[ A_{\text{eff}} = A_{\text{ubs}} + \eta A_{\text{fin}} \quad (\text{HX-20}) \]

and the corresponding thermal resistance is

\[ R = \frac{1}{hA_{\text{eff}}} = \frac{1}{h(A_{\text{ubs}} + \eta A_{\text{fin}})} \quad (\text{HX-21}) \]

This is the equation which is commonly used for the evaluation of the thermal resistance of a finned surface.

In the analysis of heat exchangers presented so far, it has been assumed that all the participating surfaces are free of deposits of foreign matter. Examples of such deposits include rust, soot, mineral precipitates, biological growths, etc. The presence of such deposits gives rise to additional thermal resistances which have to be included in equation (HX-11). When a wall is coated by such
Fig. HX-21

deposits it is said to be fouled.

It is very difficult to get a highly accurate value for the fouling resistance because such decisive factors as the thickness of the deposit, the density and composition of the deposited layer, and the degree of adhesion of the deposit to the surface are not well known. In fact, many of the factors are a function of time.

For preliminary design, it is common to make use of empirically based, tabulated estimates of fouling resistances. Tables HX-1, HX-2, and HX-3 convey fouling resistances reproduced from three different widely respected textbooks.
In these tables, the listed numbers correspond to the quantity

\[ R_{\text{fouling}} \times A_{\text{contact}} = R_f A_c \quad (HX-22) \]

where \( A_{\text{contact}} \) is the area of the interface between the deposit and the wall. The units of \( R_{\text{fouling}} \) are

\[ \frac{m^2 \cdot ^\circ C}{W} \text{ or } \frac{ft^2 \cdot ^\circ F}{Btu/hr} \quad (HX-23) \]

with the conversion factor

\[ 1 \frac{ft^2 \cdot ^\circ F}{Btu/hr} = 0.17612 \frac{m^2 \cdot ^\circ C}{W} \quad (HX-24) \]

Table HX-1

<table>
<thead>
<tr>
<th>Type fluid</th>
<th>(h ft^2 °F/Btu)</th>
<th>(m^2 °C/W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seawater:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below 50°C</td>
<td>( 5 \times 10^{-4} )</td>
<td>( 9 \times 10^{-5} )</td>
</tr>
<tr>
<td>Above 50°C</td>
<td>( 1 \times 10^{-3} )</td>
<td>( 2 \times 10^{-4} )</td>
</tr>
<tr>
<td>Treated boiler feedwater above 50°C</td>
<td>( 1 \times 10^{-3} )</td>
<td>( 2 \times 10^{-4} )</td>
</tr>
<tr>
<td>Fuel oil</td>
<td>( 5 \times 10^{-3} )</td>
<td>( 9 \times 10^{-4} )</td>
</tr>
<tr>
<td>Quenching oil</td>
<td>( 4 \times 10^{-3} )</td>
<td>( 7 \times 10^{-4} )</td>
</tr>
<tr>
<td>Alcohol vapors</td>
<td>( 5 \times 10^{-4} )</td>
<td>( 9 \times 10^{-5} )</td>
</tr>
<tr>
<td>Steam, non-oil-bearing</td>
<td>( 5 \times 10^{-4} )</td>
<td>( 9 \times 10^{-5} )</td>
</tr>
<tr>
<td>Industrial air</td>
<td>( 2 \times 10^{-3} )</td>
<td>( 4 \times 10^{-4} )</td>
</tr>
<tr>
<td>Refrigerating liquid</td>
<td>( 1 \times 10^{-3} )</td>
<td>( 2 \times 10^{-4} )</td>
</tr>
</tbody>
</table>
### Table HX-2

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( R_f A_c [\text{m}^2 \cdot \text{°C}/\text{W}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seawater and treated boiler</td>
<td>0.0001</td>
</tr>
<tr>
<td>feedwater (below 50°C)</td>
<td></td>
</tr>
<tr>
<td>Seawater and treated boiler</td>
<td>0.0002</td>
</tr>
<tr>
<td>feedwater (above 50°C)</td>
<td></td>
</tr>
<tr>
<td>River water (below 50°C)</td>
<td>0.0002–0.001</td>
</tr>
<tr>
<td>Fuel oil</td>
<td>0.0009</td>
</tr>
<tr>
<td>Refrigerating liquids</td>
<td>0.0002</td>
</tr>
<tr>
<td>Steam (nonoil bearing)</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

### Table HX-3

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( R_f A_c [\text{m}^2 \cdot \text{°C}/\text{W}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel oil</td>
<td>0.005</td>
</tr>
<tr>
<td>Transformer oil</td>
<td>0.001</td>
</tr>
<tr>
<td>Vegetable oils</td>
<td>0.003</td>
</tr>
<tr>
<td>Light gas oil</td>
<td>0.002</td>
</tr>
<tr>
<td>Heavy gas oil</td>
<td>0.003</td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.005</td>
</tr>
<tr>
<td>Gasoline</td>
<td>0.001</td>
</tr>
<tr>
<td>Kerosene</td>
<td>0.001</td>
</tr>
<tr>
<td>Caustic solutions</td>
<td>0.002</td>
</tr>
<tr>
<td>Refrigerant liquids</td>
<td>0.001</td>
</tr>
<tr>
<td>Hydraulic fluid</td>
<td>0.001</td>
</tr>
<tr>
<td>Molten salts</td>
<td>0.0005</td>
</tr>
<tr>
<td>Engine exhaust gas</td>
<td>0.01</td>
</tr>
<tr>
<td>Steam (non-oil-bearing)</td>
<td>0.0005</td>
</tr>
<tr>
<td>Steam (oil-bearing)</td>
<td>0.001</td>
</tr>
<tr>
<td>Refrigerant vapors (oil-bearing)</td>
<td>0.002</td>
</tr>
<tr>
<td>Compressed air</td>
<td>0.002</td>
</tr>
<tr>
<td>Acid gas</td>
<td>0.001</td>
</tr>
<tr>
<td>Solvent vapors</td>
<td>0.001</td>
</tr>
<tr>
<td>Seawater</td>
<td>0.0005–0.001</td>
</tr>
<tr>
<td>Brackish water</td>
<td>0.001–0.003</td>
</tr>
<tr>
<td>Cooling tower water (treated)</td>
<td>0.001–0.002</td>
</tr>
<tr>
<td>Cooling tower water (untreated)</td>
<td>0.002–0.005</td>
</tr>
<tr>
<td>River water</td>
<td>0.001–0.004</td>
</tr>
<tr>
<td>Distilled or closed-cycle condensate water</td>
<td>0.0005</td>
</tr>
<tr>
<td>Treated boiler feedwater</td>
<td>0.0005–0.001</td>
</tr>
</tbody>
</table>

Tables HX-1 and HX-2 are in good agreement with each other, but Table HX-3 deviates somewhat from the others.
E - NTU Method

The effectiveness \( \epsilon \) is often used to characterize the performance of a heat exchanger. The effectiveness is defined as

\[
\epsilon = \frac{Q}{Q_{\text{max}}}
\]  

(HX-25)

In this ratio, \( Q \) denotes the actual rate of heat transfer in a specific type of heat exchanger (e.g., parallel flow, counterflow; crossflow - both fluids unmixed; crossflow - one fluid unmixed and the other mixed; etc.) corresponding to given values of

\[
T_{h,\text{in}}, T_{c,\text{in}}, \quad C_h = (m c_p)_h, \quad C_c = (m c_p)_c
\]  

(HX-26)

\( Q_{\text{max}} \) is the maximum possible rate of heat transfer that can be achieved in the given heat exchanger type for the same operating conditions as are specified in equation (HX-26).
At first glance, the conditions for the attainment of $Q_{\text{max}}$ appear obvious, that is,

$$T_{h,\text{out}} \rightarrow T_{c,\text{in}} \quad \text{and} \quad T_{c,\text{out}} \rightarrow T_{h,\text{in}}$$

(HX-27)

If this is so, then

$$Q_h = (m c_p)_h (T_{h,\text{in}} - T_{h,\text{out}}) \max C_h (T_{h,\text{in}} - T_{c,\text{in}})$$

and

$$Q_c = (m c_p)_c (T_{c,\text{out}} - T_{c,\text{in}}) \max C_c (T_{h,\text{in}} - T_{c,\text{in}})$$

(HX-28)  

(HX-29)

However, it is necessary that $Q_{h,\text{max}} = Q_{c,\text{max}} = Q_{\text{max}}$. From equations (HX-28) and (HX-29), it is seen that this requirement is not fulfilled when $C_c \neq C_h$. Therefore, the assumed conditions, equations (HX-27), for the attainment of $Q_{\text{max}}$ are not correct when $C_c \neq C_h$.

From the left and middle terms of each of equations (HX-28) and (HX-29), with $Q_h = Q_c = Q$, 


\[
\frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{c,\text{out}} - T_{c,\text{in}}} = \frac{(mc_p)_c}{(mc_p)_h} = \frac{C_c}{C_h} \quad \text{(HX-30)}
\]

This equation states that the temperature changes of the hot and cold streams are not equal when the two capacity rates \( mc_p \) are not equal.

Suppose that \( C_c > C_h \), then equation (HX-30) states that

\[
(T_{h,\text{in}} - T_{h,\text{out}}) > (T_{c,\text{out}} - T_{c,\text{in}}) \quad \text{(HX-31)}
\]

Alternatively, if \( C_h > C_c \), then

\[
(T_{c,\text{out}} - T_{c,\text{in}}) > (T_{h,\text{in}} - T_{h,\text{out}}) \quad \text{(HX-32)}
\]

From these equations, it is seen that that fluid stream whose capacity rate is the smaller of \( C_c \) and \( C_h \) will experience the larger temperature change.

With this insight, attention can be returned to the identification of \( Q_{\text{max}} \).
The largest temperature change that can be experienced by either stream in a heat exchanger is \((T_{h,\text{in}} - T_{c,\text{in}})\). When \(C_c \neq C_h\), only the fluid stream with the smaller capacity rate can actually attain this temperature change. Let \(C_{\text{min}}\) be the smaller of \(C_h\) and \(C_c\). Then,

\[
Q_{\text{max}} = C_{\text{min}} (T_{h,\text{in}} - T_{c,\text{in}}) \quad \text{(HX-33)}
\]

\[
= (m c_p)_{\text{min}} (T_{h,\text{in}} - T_{c,\text{in}})
\]

Consequently, from equation (HX-25),

\[
\varepsilon = \frac{Q}{C_{\text{min}} (T_{h,\text{in}} - T_{c,\text{in}})} \quad \text{(HX-34)}
\]

Furthermore, since \(Q = Q_h = Q_c\) and using equations (HX-28) and (HX-29), equation (HX-34) becomes

\[
\varepsilon = \frac{C_h (T_{h,\text{in}} - T_{h,\text{out}})}{C_{\text{min}} (T_{h,\text{in}} - T_{c,\text{in}})} = \frac{C_c (T_{c,\text{out}} - T_{c,\text{in}})}{C_{\text{min}} (T_{h,\text{in}} - T_{c,\text{in}})} \quad \text{(HX-35)}
\]

For the special case in which the capacity rates of the two streams are
equal, then

$$
\epsilon = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}}
$$

(AH-36)

A heat exchanger which operates in the range $\epsilon \approx 0.7$ is thought to be performing well. Higher values of $\epsilon$, even approaching 1.0, can be achieved if desired, but at a price. One way to increase $\epsilon$ is to make the area of the heat exchange surfaces larger; another way is to lower the thermal resistance $R$ by increasing the heat transfer coefficients. This would involve a larger heat exchanger, a larger pump or blower, and higher pumping power expenditures. Economic considerations often discourage the pursuit of $\epsilon$ values greater than $\sim 0.7$.

Besides $\epsilon$, the other parameter of relevance in heat exchanger design is the NTU defined as

$$
\text{NTU} = \frac{UA}{C_{\text{min}}} = \frac{1}{RC_{\text{min}}}
$$

(AH-37)
The NTU is dimensionless. The letters N, T, and U come from the designation "number of transfer units." Although an occasional author has attempted to provide an explanation of the meaning of "number of transfer units" as a descriptor of $\frac{UA}{C_{\text{min}}}$, they have not been very convincing. It is probably best to regard NTU as the name for $\frac{UA}{C_{\text{min}}}$ and nothing more.

The relationship between $E$ and NTU has been developed for a number of heat exchanger types. There is no need to go into depth with respect to these derivations here. However, it is important to outline a typical derivation to demonstrate that the $E$-NTU relationships are not pulled out of a magician's hat.

Consider the case of a parallel-flow heat exchanger. The rate of heat transfer $Q$ from one fluid to the other is given by equation (HX-12), into which equation (HX-1) for the LMTD and equation (HX-13) for $R$ have been substituted.
\[ Q = UA \ln \left[ \frac{\Delta T_I - \Delta T_{II}}{\ln \frac{\Delta T_I}{\Delta T_{II}}} \right] \]  

(HX-38)

where, from Fig. HX-14, \( \Delta T_I \) and \( \Delta T_{II} \) are

\[ \Delta T_I = T_{h,\text{in}} - T_{c,\text{in}}, \quad \Delta T_{II} = T_{h,\text{out}} - T_{c,\text{out}} \]  

(HX-39)

It readily follows that

\[ \Delta T_I - \Delta T_{II} = (T_{h,\text{in}} - T_{h,\text{out}}) + (T_{c,\text{out}} - T_{c,\text{in}}) \]

\[ = \frac{Q}{C_h} + \frac{Q}{C_c} \]  

(HX-40)

\[ \frac{\Delta T_{II}}{\Delta T_I} = \frac{(T_{h,\text{out}} - T_{h,\text{in}}) + (T_{h,\text{in}} - T_{c,\text{in}}) - (T_{c,\text{out}} - T_{c,\text{in}})}{T_{h,\text{in}} - T_{c,\text{in}}} \]

(HX-41)

The last term in the numerator of equation (HX-41) can be replaced by

\[ T_{c,\text{out}} - T_{c,\text{in}} = \frac{C_h}{C_c} (T_{h,\text{in}} - T_{h,\text{out}}) \]  

(HX-42)

The final result will be the same regardless of whether \( C_h = C_{\text{min}} \) or
\[ C_e = C_{\text{min}}. \text{ For concreteness let } C_p = C_{\text{min}}. \text{ Then, from the first two terms of equation (HX-35),} \]
\[ \varepsilon = \frac{T_{h, \text{in}} - T_{h, \text{out}}}{T_{h, \text{in}} - T_{c, \text{in}}} \quad \text{(HX-43)} \]

The substitution of equations (HX-40), (HX-41), (HX-42), and (HX-43), together with the definition of the NTU equation (HX-37), into equation (HX-40), leads to
\[ \varepsilon = \frac{1}{2} \left[ 1 - e^{-\zeta \times \text{NTU}} \right], \quad \zeta = 1 + \frac{C_{\text{min}}}{C_{\text{max}}} \quad \text{(HX-44)} \]
which completes the parallel flow derivation.

An extensive listing of \( \varepsilon \)-NTU relations for a variety of heat exchanger types is presented in Table HX-4. For compactness, the notation \( C_r \) has been introduced
\[ \frac{C_{\text{min}}}{C_{\text{max}}} = C_r \quad \text{(HX-45)} \]

It is seen that the first entry in the table is identical to equation (HX-44).
after a simple change of notation has been made.

Table HX-4

<table>
<thead>
<tr>
<th>FLOW ARRANGEMENT</th>
<th>RELATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentric tube</td>
<td></td>
</tr>
<tr>
<td>Parallel flow</td>
<td>$\varepsilon = \frac{1 - \exp \left[-\text{NTU}(1 + C_r)\right]}{1 + C_r}$</td>
</tr>
<tr>
<td>Counterflow</td>
<td>$\varepsilon = \frac{1 - \exp \left[-\text{NTU}(1 - C_r)\right]}{1 - C_r \exp \left[-\text{NTU}(1 - C_r)\right]}$</td>
</tr>
<tr>
<td>Shell and tube</td>
<td></td>
</tr>
<tr>
<td>One shell pass (2, 4, ... tube passes)</td>
<td>$\varepsilon_l = 2 \left(1 + C_r + (1 + C_r^2)^{1/2}\right)^{-1}$</td>
</tr>
<tr>
<td>$2n$ Shell passes (2n, 4n, ... tube passes)</td>
<td>$\varepsilon = \left[\left(\frac{1 - \varepsilon_l C_r}{1 - \varepsilon_l}\right)^n - 1\right] \left[\left(\frac{1 - \varepsilon_l C_r}{1 - \varepsilon_l}\right)^n - C_r\right]^{-1}$</td>
</tr>
<tr>
<td>Cross flow (single pass)</td>
<td></td>
</tr>
<tr>
<td>Both fluids unmixed</td>
<td>$\varepsilon = 1 - \exp \left[\left(\frac{1}{C_r}\right)(\text{NTU})^{0.22} \left{\exp \left[-C_r(\text{NTU})^{0.78}\right] - 1\right}\right]$</td>
</tr>
<tr>
<td>$C_{\text{max}}$ (mixed), $C_{\text{min}}$ (unmixed)</td>
<td>$\varepsilon = \left(\frac{1}{C_r}\right)(1 - \exp \left{-C_r[1 - \exp (-\text{NTU})]\right})$</td>
</tr>
<tr>
<td>$C_{\text{min}}$ (mixed), $C_{\text{max}}$ (unmixed)</td>
<td>$\varepsilon = 1 - \exp (-C_r^{-1}[1 - \exp (-C_r(\text{NTU})))$</td>
</tr>
<tr>
<td>All exchangers ($C_r = 0$)</td>
<td>$\varepsilon = 1 - \exp (-\text{NTU})$</td>
</tr>
</tbody>
</table>

Note the careful specification of the flow patterns for the crossflow-type heat ex-
In certain types of design calculations it is more convenient to have NTU as a function of $\varepsilon$ rather than $\varepsilon$ as a function of NTU. Table HX-5 conveys some information in this regard.

**Table HX-5**

<table>
<thead>
<tr>
<th>FLOW ARRANGEMENT</th>
<th>RELATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentric tube</td>
<td></td>
</tr>
<tr>
<td>Parallel flow</td>
<td>$\text{NTU} = -\frac{\ln[1 - \varepsilon(1 + C_T)]}{1 + C_T}$</td>
</tr>
<tr>
<td>Counterflow</td>
<td>$\text{NTU} = -\frac{1}{C_T - 1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C_T - 1}\right)$</td>
</tr>
<tr>
<td>Cross flow (single pass)</td>
<td></td>
</tr>
<tr>
<td>$C_{\text{max}}$ (mixed), $C_{\text{min}}$ (unmixed)</td>
<td>$\text{NTU} = -\ln\left[1 + \left(\frac{1}{C_T}\right)\ln(1 - \varepsilon C_T)\right]$</td>
</tr>
<tr>
<td>$C_{\text{min}}$ (mixed), $C_{\text{max}}$ (unmixed)</td>
<td>$\text{NTU} = -\left(\frac{1}{C_T}\right)\ln[C_T\ln(1 - \varepsilon) + 1]$</td>
</tr>
<tr>
<td>All exchangers ($C_T = 0$)</td>
<td>$\text{NTU} = -\ln(1 - \varepsilon)$</td>
</tr>
</tbody>
</table>

Both to facilitate rapid calculations and to provide visual evidence of trends, the $\varepsilon$-NTU relations of Table HX-4 have been evaluated and are plotted in Figs.
HX-22 through HX-27.

Effectiveness of a parallel-flow heat exchanger

**Fig. HX-22**

Effectiveness of a counterflow heat exchanger

**Fig. HX-23**

Effectiveness of a single-pass, cross-flow heat exchanger with both fluids unmixed

**Fig. HX-24**

Effectiveness of a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed

**Fig. HX-25**
From an inspection of these figures it is seen that the effectiveness increases as the NTU increases. The increase in \( \varepsilon \) is most rapid at small NTU. The increase is physically plausible when the quantities which constitute the NTU are examined, i.e., \( \frac{UA}{C_{\text{min}}} \). Suppose \( C_{\text{min}} \) (and \( C_{\text{max}} \)) is (are) regarded as fixed. Then, NTU increases when \( UA \) increases and, since \( UA = \frac{1}{R} \), \( \varepsilon \) increases as the thermal resistance decreases. It is natural that devices with lower thermal resistance will be more efficient.
The $e$ vs. NTU curves are stacked one above the other with decreasing values of $C_{min}/C_{max}$. To explain this trend, suppose that $U$, $A$, and $C_{min}$ are fixed, i.e., fixed NTU. Then, decreases in $C_{min}/C_{max}$ imply increasing values of $C_{max}$. It is natural that the increase in $C_{max}$ will make the heat exchanger more efficient.

Figure HX-25 is different from the others in that it contains results for two cases. In one case, the mixed stream is associated with $C_{max}$, whereas in the other, the mixed stream is associated with $C_{min}$.

The $e$-NTU method is a convenient way for determining the rate of heat transfer when the inlet temperatures are known. The technique for using the $e$-NTU method will now be illustrated by an example.
EXAMPLE

A counterflow heat exchanger is to cool oil by using water as the coolant. The oil \((\text{subscript } h)\) enters at \(T_{h,\text{in}} = 100^\circ\text{C}\), whereas the water \((\text{subscript } c)\) enters at \(T_{c,\text{in}} = 20^\circ\text{C}\). The mass flowrates of the two fluids are

\[
m_{\text{oil}} = m_h = 2 \text{ kg/sec}, \quad m_{\text{water}} = m_c = 0.48 \text{ kg/sec}
\]

The heat transfer surface area is 12.5 m² (tube wall sufficiently thin so there is no need to consider changes in \(A\) across wall thickness). The average value of the overall heat transfer coefficient is \(U = 400 \text{ W/m}^2\cdot\text{K}\).

Determine the heat transfer rate \(Q\) and the exit temperature of the water \(T_{c,\text{out}}\).

Solution

The first step is to calculate \(C_h\) and \(C_c\) and to compare their magnitudes. The needed values of the specific heat \(c_p\) are read from the tables as

\[
c_{p,\text{oil}} = c_{p,h} = 2000 \frac{\text{J}}{\text{Kg}\cdot^\circ\text{C}}, \quad c_{p,\text{water}} = c_{p,c} = 4170 \frac{\text{J}}{\text{Kg}\cdot^\circ\text{C}}
\]
Then,
\[ C_{oil} = C_h = 2 \times 2000 = 4000 \text{ W/}^\circ\text{C} \]
\[ C_{water} = C_c = 0.48 \times 4170 = 2002 \text{ W/}^\circ\text{C} \]

and
\[ \frac{C_{water}}{C_{oil}} = \frac{C_c}{C_h} = \frac{2002}{4000} = \frac{C_{min}}{C_{max}} \]

Since \( C_{min} = C_c \), equation (HX-35) becomes
\[ \epsilon = \frac{T_{c, out} - T_{c, in}}{T_{h, in} - T_{c, in}} = \frac{T_{c, out} - 20}{100 - 20} \]

where the given data have been introduced. If \( \epsilon \) can be calculated, then \( T_{c, out} \) can be found. From the given information,

\[ NTU = \frac{UA}{C_{min}} = \frac{400 \times 12.5}{2002} = 2.50 \]

With \( NTU = 2.50 \) and \( C_r = C_{min} / C_{max} = 0.5005 \), the second entry in Table HX-4 is used to calculate

\[ \epsilon = 0.833 \]

which is indicative of very good performance.
With this,

\[ 0.833 = \frac{T_{c,\text{out}} - 20}{80} \]

or

\[ \Rightarrow T_{c,\text{out}} = 86.64 \, ^\circ \text{C} \]

Then,

\[ Q = C_c (T_{c,\text{out}} - T_{c,\text{in}}) \]

\[ = 2002 (86.64 - 20) \]

\[ \Rightarrow Q = 133,410 \, \text{W} = 133.4 \, \text{KW} \]

This is an appreciable amount of thermal power.
Natural convection denotes flows induced by temperature-created density variations throughout the fluid. For example, suppose that a horizontal pipe carrying hot water is situated in a room. Since the pipe wall is also hot, the room air that is adjacent to the pipe wall will be heated to a higher temperature than the room air that is far from the pipe. For virtually all fluids, the density $p$ decreases as the temperature increases. Therefore, the air next to the pipe has a lower density than the air farther away. It is well known that hot, light fluids tend to rise, and this is what happens to the air next to the pipe. Of course, the space left vacant by the rising air is immediately occupied by other air. The replacement is in turn heated, and it, too, subsequently rises. This is the way natural convection is created.
A photograph of the temperature field about a hot horizontal pipe was already presented on page 2 of these notes.

Another commonly encountered natural convection situation is the vertical plate. For example, consider a window subject to winter cold at its outer surface. The surface of the window that faces the room has a temperature $T_w$ which is lower than the room temperature $T_{room}$. As a consequence, the room air that is adjacent to the window is lower in temperature and higher in density than the more remote room air. Because of this, the air that is next to the window moves downward as suggested in the diagram. Of course, in the summer, especially if the room is air conditioned, the air next to the window will move upward.

Since natural convection flows are self-
inducing, i.e., they are free of charge. This may be regarded as a bonus when the enhanced heat transfer due to natural convection is desired. On the other hand, natural convection is a nuisance in situations when heat transfer is to be minimized.

It is seen that the difference between the density of the fluid in contact with the wall and the density of the fluid far from the wall is a key factor in the creation of a natural convection flow. Let

\[ \rho_w = \text{density of fluid in contact with wall} \]

\[ \rho_0 = \text{density of fluid far from wall} \]

Suppose an experiment were to be performed in a laboratory on earth and a second experiment were to be performed in a laboratory on the surface of the moon. Both are identical natural convection experiments involving the same fluids, the same geometry, and the same operating conditions, i.e., same temperatures
and pressures. Therefore, the $p_w$ and $p_0$ values for the two experiments would be the same, as would $|p_w - p_0|$. The logical question to be asked is: "Is the vigor of the natural convection the same in the two experiments?" The answer is a definite no, and the reason is that the gravity force acting on objects on the surface of the moon is much smaller than that acting on objects on the surface of the earth. In light of this, the driving force for natural convection is more properly written as

$$g |p_w - p_0|$$

than $|p_w - p_0|$. It is reasonable to expect that in any analysis of natural convection heat transfer, the quantity $g |p_w - p_0|$ will be involved in an important way. In fact, it would be no surprise if this quantity were to be involved in a key dimensionless group which governed the Nusselt number for natural convection.
If a dimensionless group is to be user-friendly, all the quantities which comprise the group should be readily measurable or be able to be determined by look-ups in tables. There is no meter available for measuring the density \( \rho \). Therefore, \( |\rho_w - \rho_0| \) is not able to be determined directly by experiment. Because of this, it is usual to replace \( |\rho_w - \rho_0| \) by \( |T_w - T_0| \), as follows.

Every fluid has a property called the coefficient of thermal expansion \( \beta \), which is defined as

\[
\beta = -\frac{\text{fractional change of density}}{\text{change of temperature}} \quad \text{const}_{\rho}
\]

or

\[
\beta = -\frac{\Delta \rho / \rho}{\Delta T} = -\frac{1}{\rho} \frac{\Delta \rho}{\Delta T}
\]

The minus sign in the definition takes account of the fact that, almost always \( \Delta \rho / \Delta T \) is negative. Note that \( \beta \) has dimensions of \((\text{temperature})^{-1}\). From the foregoing equation,

\[
|\rho_w - \rho_0| = \beta \rho |T_w - T_0|
\]
Therefore, instead of $g \beta \rho (T_w - T_0)$, it is more convenient to use
\[ \frac{g \beta \rho}{T_w - T_0} \]
as the cornerstone of the key dimensionless group that governs natural convection.

Other parameters which should affect natural convection flow and heat transfer include:

- $\mu$ - viscosity of the fluid. It is reasonable to expect that the viscosity will oppose the natural convection motion.
- $\ell$ - a characteristic dimension of the surface over which the flow passes.
- $k$ - thermal conductivity of the fluid.
- $c_p$ - specific heat at constant pressure of the fluid.

When these four parameters are brought together with $g \beta \rho (T_w - T_0)$, a dimen-
sionless group called the Rayleigh number, abbreviated $Ra$, emerges

$$Ra_\ell = \frac{g \beta \ell (T_w - T_0) \ell^3}{\nu^2} \frac{c_p \mu}{k} \frac{\ell}{Pr}$$

where the subscript $\ell$ indicates the characteristic dimension. For example, for natural convection about a horizontal cylinder of diameter $D$, the characteristic dimension $\ell = D$.

There has been a great deal of both experimental and analytical work on natural convection heat transfer. Almost always, the end result of this work has been expressible in the following functional form

$$\overline{Nu}_\ell = \frac{h \ell}{k} = \text{function of } Ra_\ell \text{ and } Pr$$

Nusselt number results will be presented in this form for several important natural convection systems.
Vertical Plane Surface (Flat Plate)

The vertical flat plate in natural convection is a generic configuration which has applications for windows, walls, equipment cabinets, fins, etc. If \( T_w > T_o \), the natural convection flow along the plate is upward as indicated at the lower right. If \( T_w = T_o \), the flow is downward. The flow is of the boundary-layer type. The layer grows in thickness in the flow direction. The upflow (or downflow) \( T_w > T_o \) is fed by fluid that is drawn in from the ambient.
horizontally.

The rate of heat transfer between the plate and the adjacent fluid due to natural convection is

\[ Q = \overline{h}A \Delta T \]

where
\[ A = LW \]
\[ \Delta T = T_w - T_\infty, \quad Q \text{ from plate to fluid} \]
\[ \Delta T = T_\infty - T_w, \quad Q \text{ from fluid to plate} \]

The most complete correlation of \( \overline{h} \) results for the vertical plate is

\[ \overline{Nu}_L = 0.68 + \frac{0.67 Ra_{L}^{1/4}}{[1 + (0.492 \Pr)^{9/16}]^{4/9}} \]
In this equation,

\[ \overline{Nu}_L = \frac{hL}{k}, \quad \overline{Ra}_L = \frac{9 \beta (T_w - T_\infty) k^3}{v^3} \Pr \]

The fluid properties \( k, \gamma, \) and \( \Pr \) are to be looked up at \( T = \frac{1}{2} (T_w + T_\infty) \). Also, recall that since \( \gamma = \mu/\rho \), then \( \gamma \) is pressure dependent. The thermal expansion coefficient \( \beta \) is evaluated from the rules:

(a) Gases. \( \beta = \frac{1}{T_\infty}, \quad T_\infty \) in absolute units

(b) Liquid. Look up \( \beta \) at \( \overline{T} = \frac{1}{2} (T_w + T_\infty) \)

The equation for \( \overline{Nu}_L \) is plotted on the figure on the preceding page along with experimental data. The accuracy of the equation is confirmed by its very good agreement with the data.

It is interesting to examine how the \( h \) for a plate with height \( L_1 \), compares with a plate of height \( L_2 = 2L_1 \). To make the comparison easy, ignore the free-standing constant 0.68 in the \( \overline{Nu}_L \) equation. Then, after substituting the
Definitions of $\bar{N}u_L$ and $Ra_L$,

$$\frac{\bar{h}_2(2L)}{k} \frac{L_1}{\bar{h}_1} = \left[ \frac{9\beta(T_w - T_0)(2L)^3}{\nu^2} \right]^{1/4}$$

so that

$$\frac{\bar{h}_2}{\bar{h}_1} \approx \frac{1}{2} \left( 2^{3/4} \right) = \frac{1}{2^{1/4}} = 0.707$$

which shows that $\bar{h}$ is smaller for the taller plate than for the shorter plate.

**Vertical Cylinder**

The vertical cylinder in natural convection is frequently encountered in practice when calculating the heat loss from the outside surface of vertical pipes, either bare or covered with insulation. The natural convection heat transfer between the cylinder and the fluid is strongly affected by whether the thermal boundary layer is relatively thin or relatively thick compared with the diameter of the
cylinder. These situations are illustrated in the figure. When $\delta_r \ll D$, the fluid in

the boundary layer thinks that it is flowing upward along a flat plate. On the other hand, when $\delta_r$ is not $\ll D$, the fluid is aware of the curvature of the cylinder surface.

To compute the rate of heat transfer at the surface of the cylinder,

$$Q = \bar{h}A\Delta T, \quad A = \pi DL$$

The determination of $\bar{h}$ for the vertical cylinder is a two-step process. First, the $\bar{h}$ for vertical plate of height $L$ equal to the height of the cylinder
is calculated from the $\overline{Nu_L}$ equation on page 165. Then, a correction is applied. The correction depends on the value of $\delta_f/D$. Theory gives that

$$\frac{\delta_f}{D} \sim \frac{5.66 \, (L/D)}{(Ra_L/Pr)^{1/4}}$$

and the correction factor is read from

$$\frac{\overline{h}_{cylinder}}{\overline{h}_{plate}}$$
Note that $\overline{h_{cylinder}}/\overline{h_{plate}} = 1$.

**Horizontal Cylinder**

The generic horizontal cylinder encompasses the entire range from very fine wires to large diameter pipes. For a horizontal cylinder of outside diameter $D$,

$$Ra_D = \frac{g \beta (Tw - To) D^3}{\nu^2 Pr}, \quad Nu_D = \frac{h_D}{k}$$

Because $Ra_D$ varies as $D^3$, it is not surprising that the available data for natural convection about a horizontal cylinder span an enormous range of $Ra_D$.

A hot horizontal cylinder is sheathed with a layer of relatively hot, light fluid which buoys up above the cylinder and rises as a plume (like the smoke plume which rises from a hot cigarette). If the cylinder is at a temperature lower than ambient, the plume moves downward.
The rate of heat transfer $Q$ at the surface of a horizontal cylinder is given by

$$Q = \bar{h}A\Delta T,$$

where $A = \pi DL$

The $\bar{h}$ is obtained via the Nusselt number

$$\bar{h} = \frac{k}{D} Nu_D$$

outside diameter $D$

where

$$Nu_D = 0.36 + \frac{0.518 Ra_D^{1/4}}{\left[1 + \left(0.559 Pr^{9/16}\right)^{4/9}\right]}$$

The properties $k$, $\nu$, and $Pr$ are looked up at $\bar{T} = \frac{1}{2} (T_w + T_\infty)$. For liquids $\beta$ is also read out at $\bar{T}$, while for gases $\beta = \frac{1}{2}$. The comparison of the equation to data shows satisfactory agreement.

![Graph](image-url)
Horizontal Plate

There are numerous applications involving convective heat transfer at a horizontal plane surface. In the so-called radiant heat concept for the space heating of basementless homes, pipes carrying hot water are embedded in the concrete slab. The upper surface of the slab is, thereby, maintained at a temperature higher than the air in the house. Heat is transferred from the slab surface to the air by natural convection and to objects in the room by radiation.

Natural convection motions induced by horizontal surfaces depend on whether it is natural for a layer of relatively heavy fluid to be resting atop a layer of lighter fluid or whether the opposite arrangement is more consistent with nature. This is a matter of stability and instability, which can be analyzed theoretically. On the other hand, common sense suggests that light above heavy is the stable
configuration. It is also reasonable to believe that if, somehow heavy were to be on top of light, nature would take steps, via fluid motions, to set things to rights.

With this as background, attention will be turned to several specific cases involving horizontal flat surfaces.

(a) Hot surface facing upward $T_w > T_\infty$

When $T_w > T_\infty$, then $P_w < P_\infty$. This is an unstable situation which nature resolves by the illustrated pattern of upflow and downflow.

(b) Cold surface facing downward $T_w < T_\infty$
Case (b) is also unstable because $\rho_w > \rho_\infty$ means that heavy is atop light. The flow pattern for case (b) is a mirror image of that for case (a).

(c) Cold surface facing upward $T_w < T_\infty$

Since $\rho_w > \rho_\infty$, instability is not an issue. However, due to leakage of the heavy fluid around the sides, there is downward flow induced above the surface to replace that leaked away.

(d) Hot surface facing downward $T_w > T_\infty$

This is an exact upside-down image of (c).

Although only side views have been displayed in the foregoing, the actual surfaces are two-dimensional when viewed either from above or below, as appropriate.
The pictured surface has surface area $A$ and circumference $C$. In bringing together data for different surface shapes, it was found advantageous to use a characteristic dimension $\ell$ defined as

$$\ell = \frac{A}{C}$$

The correlating equations for $\overline{Nu}_f$ for horizontal surfaces are

**Cases (a) and (b):**

$$\overline{Nu}_f = 0.54 \, Ra_f^{1/4}, \quad 10^4 < Ra_f < 10^7$$

$$\overline{Nu}_f = 0.15 \, Ra_f^{1/3}, \quad 10^7 < Ra_f < 10^{11}$$

**Cases (c) and (d):**

$$\overline{Nu}_f = 0.27 \, Ra_f^{1/4}, \quad 10^5 < Ra_f < 10^{11}$$
The fluid properties which appear in these equations are to be looked up at 
\[ T = \frac{1}{2} (T_w + T_0) \].

**Enclosed Spaces**

Applications involving natural convection in enclosed spaces include the space between multi-pane windows, the interior of a canister which houses heat-generating electronic components, the interior of a room, and many others. For example, consider the space between two parallel surfaces which bound a narrow gap. The temperatures of the long vertical walls are \( T_1 \) and \( T_2 \), respectively. When the gap width \( W \) is very small, the heat transfer across the gap is by pure conduction. If \( k \) is the thermal conductivity of the fluid in the gap, and \( A \) is the face area of each of the parallel walls, then
\[ Q = -kA \frac{T_2 - T_i}{W} = \frac{kA(T_i - T_2)}{W} \]

For thicker gaps, natural convection will be induced, so that
\[ Q = hA(T_i - T_2) \]

Or, by defining an effective thermal conductivity which includes convection,
\[ Q = k_{\text{eff}} A \frac{T_i - T_2}{W} \]

From a comparison of the last two equations,
\[ \frac{k_{\text{eff}}}{W} = h \quad \text{or} \quad \frac{k_{\text{eff}}}{k} = \frac{hW}{k} \]

But, \( hW/k = \overline{Nu}_w \), so that
\[ \frac{k_{\text{eff}}}{k} = \overline{Nu}_w \]

Theoretical studies have shown that
\[
\frac{K_{eff}}{K} = 1 + \frac{W \cdot R_{aw}}{L} \div 720
\]

If calculations which are accurate to 5% are satisfactory, then

\[K_{eff} \approx K\]

when

\[
\frac{W}{L} \div \frac{R_{aw}}{720} \leq 0.05
\]

or when

\[
R_{aw} \leq 36 \div \frac{L}{W}
\]

For \(R_{aw}\) greater than this value, the actual value of \(K_{eff}\) should be used from the \(K_{eff}/K\) equation at the top of the page. The properties in the \(K_{eff}/K\) equation are to be read at \(T = \frac{1}{2}(T_1 + T_2)\).

**EXAMPLE**

A vertical plate made of copper separates two large air spaces. The plate is 1-foot high and 5-feet wide. The
air space at the left is at 120°F and 746 mm Hg, while the air space at the right is at 40°F and 735 mm Hg. What is the rate of heat transfer $Q$ from the hotter air space to the cooler air space? Neglect radiation.

The heat transferred from the left-side air space to the plate is equal to that transferred from the plate to the right-side air space. It is reasonable to neglect the temperature drop across the thickness of the copper plate, i.e., the plate has a single temperature $T_w$, which remains to be found. If I denotes the left-hand space and II denotes the right-hand space,

$$Q_I = h_I A (T_{0I} - T_w) = h_{II} A (T_w - T_{0II})$$

so that

$$T_w = \frac{h_I T_{0I} + h_{II} T_{0II}}{h_I + h_{II}}$$

Once $T_w$ has been found, then $Q$ follows directly.
Since $\overline{h}_i$ depends on $T_{oi} - T_w$ and on $\overline{T}_i = \frac{1}{2} (T_{oi} + T_w)$ for property evaluation, and $\overline{h}_\infty$ depends on $T_w - T_{oi}$ and on $\overline{T}_\infty = \frac{1}{2} (T_{oi} + T_w)$, then the solution scheme must be iterative. That is, a value of $T_w$ is guessed and updated by means of the equation at the bottom of the preceding page.

Start with a guess of $T_w = 80^\circ\text{F}$ (average of 40$^\circ\text{F}$ and 120$^\circ\text{F}$). First work on $\overline{h}_i$. With $\overline{T}_i = \frac{1}{2} (120 + 80) = 100^\circ\text{F}$,

\[ K = 0.01557 \text{ Btu/hr-ft-}^\circ\text{F} \]
\[ \nu = 0.6489 \text{ ft}^2/\text{hr at 1 atmosphere} \]
\[ = 0.6489 \left( \frac{760}{746} \right) = 0.6611 \text{ ft}^2/\text{hr at 746 mm Hg} \]
\[ Pr = 0.710 \]

Then, \( Ra_c = \frac{9 \beta (T_{oi} - T_w) L^3}{\nu^2 Pr} \)
\[ Ra_c = \frac{(32.2)(1/579.7)(120 - 80)(1)^3}{(0.6611/3600)^2} \]
\[ Ra_c = 46.8 \times 10^6 \]

The appropriate $\overline{Nu}_c$ equation is
\[ \overline{Nu_L} = 0.68 + \frac{0.67 \, Ra_L^{4/9}}{\left[ 1 + \left( \frac{0.492}{Pr} \right)^{9/16} \right]^{4/9}} \]

With \( Pr = 0.710 \) and \( Ra_L = 46.8 \times 10^6 \),
\[ \overline{Nu_L} = 43.13 \quad \text{or} \quad \overline{h_i} = 43.13 \cdot 0.01557 \]
\[ \overline{h_i} = 0.672 \ \text{Btu/hr-ft}^2\text{-}\text{°F} \]

Proceeding along similar lines
\[ \overline{h_ii} = 0.695 \ \text{Btu/hr-ft}^2\text{-}\text{°F} \]

Then, returning to the \( T_w \) equation at the bottom of page 179,
\[ T_w = \frac{(0.672)(120) + (0.695)(40)}{0.672 + 0.695} \]
\[ T_w = 79.3^\circ F \]

In this example problem, this is close enough to the guess of \( T_w = 80^\circ F \).

Finally,
\[ Q = \overline{h_i}A(T_{ooi} - T_w) = \]
\[ Q = (0.672)(1)(5)(120 - 79.3) = 136.8 \ \text{Btu} \]
Boiling occurs in power-producing processes (steam power plants), petroleum refining, distillation, vapor-compression refrigeration processes and many others.

Nukiyama's famous pool boiling experiment. He suspended a horizontal wire in a pool of liquid water at its saturation temperature. The wire was heated by passing an electric current through it. The wire surface temperature is called $T_s$, and the water saturation temperature is $T_{sat}$. In the experiment, the heating power was increased and the wire temperature was measured. The first experiment was performed with a nichrome wire, and the second experiment was performed with a platinum wire.

Test setup:

(Note: $q'' \rightarrow q$)

Nukiyama's power-controlled heating apparatus for demonstrating the boiling curve.
Results:

Nukiyama's boiling curve for saturated water.

Both wires followed the curve a→b→c as the heating power was increased. At c, the nichrome wire burned out. For the platinum wire, the heat flux was decreased from its value at c, and the wire followed the path c→d→e→a. The path b→d or d→b was not encountered in these experiments.

The next significant experiment was that of Drew and Mueller. They controlled the wire temperature (actually, they used a tube rather than a wire) and were able to map out the entire boiling curve.
Here is the currently accepted boiling curve for water:

Nucleate boiling is the most important boiling regime. The most useful equation for evaluating \( q \) in the nucleate regime is:

\[
q_s'' = \frac{1}{\mu_s} h_{sg} \left[ \frac{g(pL-p_v)}{\sigma} \right]^{1/3} \left( \frac{C_{pL} \Delta T_e}{C_{sg} h_{sg} P_{vL}} \right)^3
\]

\( \sigma \) is the surface tension, and \( h_{sg} \) is the latent heat of evaporation. The coefficients \( C_{sg} \) are listed in the table.
for various fluid-surface combinations.

<table>
<thead>
<tr>
<th>FLUID-SURFACE COMBINATION</th>
<th>$C_a$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water-copper</td>
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<tr>
<td>Mechanically polished</td>
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<td>1.0</td>
</tr>
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<td>Benzene-chromium</td>
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<tr>
<td>Ethyl alcohol-chromium</td>
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</tbody>
</table>

The peak heat flux on the boiling curve, $q''_{max}$, can be estimated from:

$$q''_{max} = 0.149 h fg \frac{L_f}{\rho_f} \left[ \frac{\sigma g (P_2 - P_1)}{\rho_f} \right]^{1/4}$$
CONDENSATION

Condensation is a change of phase in which a vapor becomes a liquid, giving up the latent heat of condensation. This is the process. Condensation is an important heat transfer mode in power plants, chemical processors, water desalination, etc.

- As an initial problem, consider a cooled vertical plate situated adjacent to a pure, saturated vapor. The vapor is at a temperature $T_v$, and the plate temperature $T_w$ is lower than $T_v$. These temperature conditions will cause the vapor to condense on the plate. The liquid condensate will form either discrete drops on the plate or will become a continuous sheet. Although drop-type condensation gives rise to high heat transfer coefficients, it is difficult to achieve in practice. The more common mode of condensation is film condensation. It, too, yields high values of the heat transfer coefficient, but not quite as high as those for the drop mode.

- Schematically, film condensation can be depicted as
As shown in the diagram, the condensate runs downward along the plate under the action of gravity. The film thickness increases in the downward direction as additional condensate is deposited in the film along its length. At a station $x$, 

[Diagram of condensation process with labeled parts: condensate, film thickness, and upward trend]
The mass flowrate of the condensate is \( m \), and it increases with \( x \). Each unit mass of vapor that condenses gives up \( h_{fg} \) units of energy. The condensed liquid at the edge of the condensate layer is at the temperature \( T_{sat} \). As the liquid runs down the plate, it is cooled to temperatures below \( T_{sat} \) (i.e., between \( T_w \) and \( T_{sat} \)). This subcooling of the liquid also liberates energy (often called sensible heat).

The rate of heat transfer \( Q \) at the plate surface between \( x = 0 \) and \( x = x \) is the sum of the rates of energy liberated as latent heat and sensible heat in the length between 0 and \( x \).

\[
Q = \text{rate of latent heat liberation} + \text{rate of sensible heat liberation}
\]

The rate of latent heat liberation is \( m h_{fg} \), while the rate of sensible heat liberation is \( m c_p (T_{sat} - T_b) \), where \( T_b \) is the bulk temperature at \( x \). From analysis, it has been found that

\[
(T_{sat} - T_b) = 0.68 (T_{sat} - T_w)
\]
so that
\[ Q = \dot{m}h_{fg} + 0.68 \dot{m}c_p(T_{sat} - T_w) \]

or
\[ Q = \dot{m}h'_{fg} \quad \text{Specific heat of liquid condensate} \]

\[ h'_{fg} = h_{fg} + 0.68 c_p(T_{sat} - T_w) \]

An average heat transfer coefficient for the length of plate between \( x = 0 \) and \( x = x \) may be defined as
\[ \bar{h} = \frac{Q}{A(T_{sat} - T_w)} = \frac{\dot{m}h'_{fg}}{xW(T_{sat} - T_w)} \]

It now remains to provide information about the heat transfer coefficient \( \bar{h} \). Experiments have demonstrated that there are different formulas for \( \bar{h} \) depending on the flow regime. To characterize the flow regime, a Reynolds number is introduced
\[ Re = \frac{4 \dot{m}/W}{\mu} \]

The flow regimes are
I. \( Re \leq 30 \) - laminar flow without waves
II. \( 30 < Re < 1800 \) - laminar flow with waves
III. \( Re > 1800 \) - turbulent flow
Even though the flow regimes seem to be quantitatively defined via the value of \( Re \), there is a difficulty in evaluating \( Re \) because it is not known beforehand. Therefore, it is necessary to make an educated guess about the flow regime and then to use the corresponding \( h \) equation to check the guess.

In all three regimes, the \( h \) equation has the form
\[
\frac{h \left( \frac{y}{s} \right)^{\frac{1}{3}}}{k} = f_i(Re)
\]

where
\[
f_i = \frac{1.47}{Re^{\frac{1}{8}}}
\]

\[
f_{II} = \frac{Re}{1.08 Re^{1.22} - 5.2}
\]

\[
f_{III} = \frac{Re}{8750 + 58(Re^{3/4} - 253)/Pr^{1/2}}
\]

The \( f_i \) is a theoretical result (first obtained by Nusselt himself), while \( f_{II} \) and \( f_{III} \) are from experiment. To proceed, note that...
\[ h = \frac{m h'_{fg}}{x W (T_{sat} - T_w)} = Re \frac{\mu h'_{fg}}{4 x (T_{sat} - T_w)} \]

So that

\[ Re \frac{\mu h'_{fg} (\mu^2 / g)^{1/3}}{4 k x (T_{sat} - T_w)} = f_\alpha (Re) \]

where \( f_\alpha \) is one of the three formulas from the preceding page, depending on the flow regime. Once the flow regime is selected, \( Re \) can be solved for (the solution does not require trial and error). Then, this \( Re \) can be used to check that the guessed flow regime is actually correct.

Once \( Re \) has been found, then \( m \) follows directly, and \( Q = m h'_{fg} \).

Note that all of the properties in the preceding equations are those of the liquid condensate and are looked up at

\[ T = \frac{1}{2} (T_w + T_{sat}) \]

However, \( h'_{fg} \) is the latent heat corresponding to \( T_{sat} \).
### Properties of Dry Air at Atmospheric Pressure

**English Units**

<table>
<thead>
<tr>
<th>$T$ °F</th>
<th>$c_p$ Btu/lb-m-T</th>
<th>$p \times 10^2$ lb/in²</th>
<th>$u \times 10^2$ lb/in²-ft</th>
<th>$w$ ft/h</th>
<th>$k \times 10^2$ Btu/m²-ft°F</th>
<th>$Pr$</th>
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</table>

Under pressure $P$, $c_p$, $\mu$ are independent of temperature. $\nu = 1/P$. If pressure exceeds atmospheric, $\nu < 1$. The pressure $P$ is written in atm.
<table>
<thead>
<tr>
<th>T°F</th>
<th>c_v \text{ Btu/lb}_\text{lb}°\text{F}</th>
<th>\rho \times 10^2 \text{ lb}_\text{lb} \text{/ft}^3</th>
<th>\mu \times 10^3 \text{ lb}_\text{lb} \text{/ft-h}</th>
<th>\gamma</th>
<th>k \times 10^2 \text{ Btu}_\text{lb} \text{/ft} \text{-h}°\text{F}</th>
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### Property Values of Gases at Atmospheric Pressure

#### Helium

| $T$, F | $ho$, lb/ft$^3$ | $c_p$, Btu/lb F | $u_{in}$, lb/sec ft | $u_{ex}$, ft$^2$/sec | $h_{in}$, Btu/hr ft F | $a$, ft/sec | $Pr$ |
|--------|-------------------|----------------|---------------------|------------------|------------------|----------|-----|
| -456   | 1.242             | 5.66 x 10$^{-7}$ | 3.68 x 10$^{-4}$   | 0.0061           | 0.0024          | 0.74     |     |
| -400   | 1.212             | 33.7            | 39.85               | 0.0553           | 0.0560          | 0.70     |     |
| -350   | 1.212             | 105.2           | 96.30               | 0.0660           | 3.596           | 0.694    |     |
| -300   | 1.119             | 122.1           | 102.8               | 0.0784           | 5.299           | 0.70     |     |
| 0      | 1.242             | 154.9           | 186.9               | 0.0977           | 9.490           | 0.71     |     |
| 400    | 1.242             | 184.8           | 289.9               | 0.114            | 14.40           | 0.72     |     |
| 600    | 1.242             | 260.9           | 404.5               | 0.130            | 20.21           | 0.72     |     |
| 800    | 1.242             | 233.5           | 501.9               | 0.145            | 23.81           | 0.72     |     |
| 1000   | 1.242             | 256.5           | 602.5               | 0.159            | 34.00           | 0.72     |     |
| 1200   | 1.242             | 279.9           | 841.0               | 0.172            | 41.98           | 0.72     |     |

#### Hydrogen

| $T$, F | $ho$, lb/ft$^3$ | $c_p$, Btu/lb F | $u_{in}$, lb/sec ft | $u_{ex}$, ft$^2$/sec | $h_{in}$, Btu/hr ft F | $a$, ft/sec | $Pr$ |
|--------|-------------------|----------------|---------------------|------------------|------------------|----------|-----|
| -400   | 2.589             | 1.079 x 10$^{-4}$ | 2.040 x 10$^{-1}$  | 0.0132           | 0.0966          | 0.759    |     |
| -370   | 2.586             | 1.601           | 5.253               | 0.0209           | 0.282           | 0.721    |     |
| -320   | 2.662             | 3.430           | 18.45               | 0.0384           | 0.933           | 0.712    |     |
| -200   | 3.101             | 3.760           | 36.79               | 0.0567           | 1.84            | 0.718    |     |
| -100   | 3.234             | 4.578           | 59.77               | 0.0741           | 2.99            | 0.719    |     |
| 0      | 3.419             | 6.023           | 117.9               | 0.105            | 6.82            | 0.706    |     |
| 70     | 3.446             | 6.689           | 152.7               | 0.119            | 7.87            | 0.697    |     |
| 250    | 3.461             | 7.300           | 190.6               | 0.132            | 9.85            | 0.600    |     |
| 350    | 3.463             | 7.915           | 232.1               | 0.145            | 12.26           | 0.682    |     |
| 440    | 3.465             | 8.491           | 276.6               | 0.157            | 14.79           | 0.675    |     |
| 530    | 3.471             | 9.055           | 324.6               | 0.169            | 17.50           | 0.663    |     |
| 620    | 3.477             | 9.599           | 376.4               | 0.182            | 20.56           | 0.664    |     |
| 700    | 3.481             | 10.68           | 489.9               | 0.203            | 26.75           | 0.659    |     |
| 980    | 3.505             | 11.69           | 612.2               | 0.222            | 33.18           | 0.664    |     |
| 1160   | 3.540             | 12.62           | 743.5               | 0.238            | 39.59           | 0.676    |     |
| 1340   | 3.575             | 13.55           | 885.0               | 0.254            | 46.49           | 0.686    |     |
| 1520   | 3.622             | 14.42           | 1039.0              | 0.265            | 53.19           | 0.703    |     |
| 1700   | 3.670             | 15.29           | 1192.0              | 0.282            | 60.00           | 0.715    |     |
| 1880   | 3.720             | 16.18           | 1370.0              | 0.296            | 67.60           | 0.733    |     |
| 1940   | 3.735             | 16.42           | 1429.0              | 0.300            | 69.80           | 0.736    |     |

#### Oxygen

<p>| $T$, F | $ho$, lb/ft$^3$ | $c_p$, Btu/lb F | $u_{in}$, lb/sec ft | $u_{ex}$, ft$^2$/sec | $h_{in}$, Btu/hr ft F | $a$, ft/sec | $Pr$ |
|--------|-------------------|----------------|---------------------|------------------|------------------|----------|-----|
| -280   | 0.2692            | 5.220 x 10$^{-8}$ | 2.095 x 10$^{-1}$  | 0.00522          | 0.00252         | 0.815    |     |
| -190   | 0.2192            | 7.721           | 4.722               | 0.00790          | 0.2204         | 0.773    |     |
| -100   | 0.2192            | 9.979           | 8.173               | 0.01064          | 0.3958         | 0.745    |     |
| -10    | 0.2192            | 12.91           | 12.32               | 0.01305          | 0.6120         | 0.725    |     |
| 90     | 0.2189            | 13.86           | 17.07               | 0.01546          | 0.8652         | 0.709    |     |
| 170    | 0.2199            | 15.56           | 22.39               | 0.01774          | 1.150          | 0.702    |     |
| 260    | 0.2250            | 17.16           | 28.18               | 0.02000          | 1.450          | 0.695    |     |
| 350    | 0.2265            | 18.86           | 34.43               | 0.02212          | 1.786          | 0.694    |     |
| 440    | 0.2222            | 20.10           | 41.27               | 0.02411          | 2.132          | 0.697    |     |
| 530    | 0.2260            | 21.48           | 48.49               | 0.02610          | 2.496          | 0.700    |     |
| 620    | 0.2399            | 22.79           | 56.13               | 0.02792          | 2.867          | 0.704    |     |</p>
<table><thead><tr><th>T, °F</th><th>ρf, lb/ft³</th><th>c_p, Btu/lb F</th><th>μf, lb/sec ft</th><th>v_f, ft/sec</th><th>h, Btu/hr ft F</th><th>ε_f, ft²/hr</th><th>Pr</th></tr>
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## Steam Properties (English Units)

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### 300°F
- $c_p = 0.4748$
- $\rho = 0.02376$
- $\mu \times 10^6 = 3.421$
- $\lambda \times 10^6 = 1.659$
- $\text{Pr} = 0.98$

### 400°F
- $c_p = 0.4729$
- $\rho = 0.02884$
- $\mu \times 10^6 = 3.957$
- $\lambda \times 10^6 = 1.952$
- $\text{Pr} = 0.96$

### 500°F
- $c_p = 0.4716$
- $\rho = 0.02579$
- $\mu \times 10^6 = 4.508$
- $\lambda \times 10^6 = 2.771$
- $\text{Pr} = 0.93$

### 600°F
- $c_p = 0.4820$
- $\rho = 0.02333$
- $\mu \times 10^6 = 5.64$
- $\lambda \times 10^6 = 2.610$
- $\text{Pr} = 0.94$

### 700°F
- $c_p = 0.4945$
- $\rho = 0.02131$
- $\mu \times 10^6 = 6.24$
- $\lambda \times 10^6 = 2.344$
- $\text{Pr} = 0.93$

### 800°F
- $c_p = 0.4977$
- $\rho = 0.01961$
- $\mu \times 10^6 = 6.736$
- $\lambda \times 10^6 = 2.376$
- $\text{Pr} = 0.91$

### 900°F
- $c_p = 0.5065$
- $\rho = 0.01846$
- $\mu \times 10^6 = 7.336$
- $\lambda \times 10^6 = 2.416$
- $\text{Pr} = 0.91$

### 1000°F
- $c_p = 0.5155$
- $\rho = 0.01691$
- $\mu \times 10^6 = 7.828$
- $\lambda \times 10^6 = 4.164$
- $\text{Pr} = 0.91$

### 1100°F
- $c_p = 0.5245$
- $\rho = 0.01583$
- $\mu \times 10^6 = 8.325$
- $\lambda \times 10^6 = 4.565$
- $\text{Pr} = 0.90$

### 1200°F
- $c_p = 0.5335$
- $\rho = 0.01487$
- $\mu \times 10^6 = 8.829$
- $\lambda \times 10^6 = 4.856$
- $\text{Pr} = 0.89$

---

**Note:**
- $c_p$ in Btu/lb\(\text{mass}\)°F
- $\rho$ in lb/ft\(^3\)
- $\mu$ in lb-ft/s
- $\lambda$ in ft-sec/°F
# Saturated Liquids

## Prop Eng

<table>
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<tr>
<th>$T$ (°F)</th>
<th>$\rho$ (lb/ft$^3$)</th>
<th>$c_p$ (Btu/lb °F)</th>
<th>$\frac{v}{\sqrt{g}}$ (ft/sec)</th>
<th>$\frac{k}{\rho}$ (Btu/hr ft °F)</th>
<th>$\frac{a}{v}$ (ft$^2$/hr)</th>
<th>Pr</th>
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### Carbon Dioxide (CO$_2$)

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<th>$\frac{a}{v}$ (ft$^2$/hr)</th>
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### Dichlorodifluoromethane (Freon) (CCl$_2$F$_2$)

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### Saturated Liquids Properties

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<th>Latent Heat (Btu/ft²)</th>
<th>Velocity (ft/sec)</th>
<th>Fr</th>
<th>Pr</th>
<th>Viscosity (1/ft)</th>
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**Dichlorodifluoromethane (Freon) (CCl₂F₂) (Continued)**

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<th>Pr</th>
<th>Viscosity (1/ft)</th>
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**Eutectic calcium chloride solution (29.9% CaCl₂)**

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<th>Viscosity (1/ft)</th>
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**Glycerin [C₃H₈(OH)₃]**

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<th>Velocity (ft/sec)</th>
<th>Fr</th>
<th>Pr</th>
<th>Viscosity (1/ft)</th>
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**Ethylene glycol [C₃H₈(OH)₂]**

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<th>Heat of Fusion (Btu/lb)</th>
<th>Latent Heat (Btu/ft²)</th>
<th>Velocity (ft/sec)</th>
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<th>Pr</th>
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**Engine oil (unused)**
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<th>$h$ Btu/lb$_{water}$</th>
<th>$u \times 10^3$ ft$^2$/h</th>
<th>$l$ Btu/lb$_{water}$°F</th>
<th>$a \times 10^3$ ft$^2$/h</th>
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## Properties of Liquid Metals

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<th>κ₀ ft²/sec</th>
<th>λ₀ Btu/hr ft F</th>
<th>α₀ ft²/sec</th>
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<th>κ₀ ft²/sec</th>
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<th>$\nu  \cdot 10^6$ (m$^2$/s)</th>
<th>$k  \cdot 10^3$ (W/m · K)</th>
<th>$\alpha  \cdot 10^6$ (m$^2$/s)</th>
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### Table A.5  Thermophysical properties of saturated fluids

**Saturated liquids**

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<th>(\nu \cdot 10^6) (m²/s)</th>
<th>(k \cdot 10^3) (W/m · K)</th>
<th>(a \cdot 10^7) (m²/s)</th>
<th>(Pr)</th>
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<th>Specific Heat (kJ/kg · °C)</th>
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*Approximation for 647.3 K (270°C)."
### Table A.7  Thermophysical properties of liquid metals

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<tr>
<td>PbBi, (44.5%/55.5%)</td>
<td>398</td>
<td>422</td>
<td>10,524</td>
<td>0.147</td>
<td>—</td>
<td>9.05</td>
<td>0.584</td>
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<td></td>
<td></td>
<td>644</td>
<td>10,236</td>
<td>0.147</td>
<td>1.496</td>
<td>11.86</td>
<td>0.790</td>
<td>0.189</td>
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<td></td>
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<td>922</td>
<td>9,813</td>
<td>—</td>
<td>1.171</td>
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<tr>
<td>Dimension</td>
<td>English Units</td>
<td>SI Units</td>
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<tr>
<td>Acceleration</td>
<td>1 ( \text{ft/s}^2 ) ( = 3.048 \times 10^{-1} \text{m/s}^2 ) ( = 2.3519 \times 10^{-2} \text{m/s}^2 )</td>
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<tr>
<td></td>
<td>1 ( \text{ft/s} ) ( = 9.2903 \times 10^{-2} \text{m/s} ) ( = 6.4516 \times 10^{-4} \text{m/s} )</td>
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<tr>
<td>Area</td>
<td>1 ( \text{ft}^2 ) ( = 9.2903 \times 10^{-2} \text{m}^2 ) ( = 6.4516 \times 10^{-4} \text{m}^2 )</td>
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<td></td>
<td>1 ( \text{in}^2 ) ( = 6.4516 \times 10^{-6} \text{m}^2 ) ( = 1.0 \times 10^{-10} \text{m}^2 )</td>
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<tr>
<td>Conductance, thermal</td>
<td>1 ( \text{Btu/h-ft}^2 \cdot ^\circ \text{F} ) ( = 5.6784 \text{W/m}^2 \cdot ^\circ \text{C} )</td>
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<tr>
<td>Conductivity, thermal</td>
<td>1 ( \text{Btu/h-ft} \cdot ^\circ \text{F} ) ( = 1.7308 \text{W/m} \cdot ^\circ \text{C} )</td>
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<tr>
<td>Density</td>
<td>1 ( \text{lbm/ft}^3 ) ( = 1.6018 \times 10^{-3} \text{kg/m}^3 ) ( = 0.001 \times 10^{-3} \text{kg/m}^3 )</td>
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<tr>
<td>Diffusivity, thermal</td>
<td>1 ( \text{ft}^2/\text{s} ) ( = 9.2903 \times 10^{-2} \text{m}^2/\text{s} ) ( = 6.4516 \times 10^{-4} \text{m}^2/\text{s} )</td>
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<td></td>
<td>1 ( \text{ft}^2/\text{h} ) ( = 2.5936 \times 10^{-2} \text{m}^2/\text{s} ) ( = 1.6018 \times 10^{-4} \text{m}^2/\text{s} )</td>
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<tr>
<td>Energy</td>
<td>1 ( \text{Btu} ) ( = 1.0551 \text{kJ} ) ( = 10.551 \times 10^3 \text{kJ} )</td>
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<tr>
<td></td>
<td>1 ( \text{kW-h} ) ( = 3.6000 \times 10^3 \text{kJ} ) ( = 1.3558 \times 10^{-3} \text{kJ} )</td>
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<tr>
<td></td>
<td>1 ( \text{ft-lb} ) ( = 1.3823 \times 10^{-4} \text{kJ} ) ( = 3.6404 \times 10^{-4} \text{kJ} )</td>
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<tr>
<td></td>
<td>1 ( \text{hp-h} ) ( = 2.6845 \times 10^3 \text{kJ} ) ( = 1.0551 \times 10^{-4} \text{kJ} )</td>
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<tr>
<td>Force</td>
<td>1 ( \text{lb} ) ( = 4.4482 \text{N} ) ( = 1.0551 \text{kJ} )</td>
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<tr>
<td>Heat</td>
<td>1 ( \text{Btu} ) ( = 1.0551 \times 10^3 \text{W} ) ( = 10.551 \times 10^3 \text{W} )</td>
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<tr>
<td>Heat flow rate</td>
<td>1 ( \text{Btu/s} ) ( = 1.0551 \times 10^3 \text{W} ) ( = 10.551 \times 10^3 \text{W} )</td>
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<tr>
<td>Heat flux</td>
<td>1 ( \text{Btu/ft}^2 ) ( = 3.1546 \times 10^{-1} \text{W/m} ) ( = 9.6152 \times 10^{-1} \text{W/m} )</td>
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<tr>
<td>(unit area)</td>
<td>1 ( \text{Btu/ft} ) ( = 3.1546 \times 10^{-1} \text{W/m} ) ( = 9.6152 \times 10^{-1} \text{W/m} )</td>
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<tr>
<td>Heat generation rate</td>
<td>1 ( \text{Btu/ft} \cdot \text{lb}_{\text{m}} ) ( = 6.4612 \times 10^{-1} \text{W/kg} ) ( = 0.0036 \times 10^{-1} \text{W/kg} )</td>
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<tr>
<td>(unit mass)</td>
<td>1 ( \text{Btu/ft}^3 ) ( = 1.055 \times 10^{-3} \text{W/m}^3 ) ( = 3.1546 \times 10^{-3} \text{W/m}^3 )</td>
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<tr>
<td>Heat transfer coefficient</td>
<td>1 ( \text{Btu/ft}^2 \cdot ^\circ \text{F} ) ( = 5.6784 \text{W/m}^2 \cdot ^\circ \text{C} )</td>
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<tr>
<td>Latent heat</td>
<td>1 ( \text{Btu/lbm} ) ( = 2.3260 \text{kJ/kg} ) ( = 4.5539 \times 10^{-1} \text{kg} )</td>
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<tr>
<td>Length</td>
<td>1 ( \text{ft} ) ( = 3.048 \times 10^{-1} \text{m} ) ( = 1.0 \times 10^{-6} \text{m} )</td>
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<tr>
<td></td>
<td>1 ( \text{\mu m} ) ( = 3.048 \times 10^{-3} \text{m} ) ( = 1.0 \times 10^{-9} \text{m} )</td>
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<tr>
<td></td>
<td>1 ( \text{in} ) ( = 2.5400 \times 10^{-2} \text{m} ) ( = 1.0 \times 10^{-3} \text{m} )</td>
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<td></td>
<td>1 ( \text{mile} ) ( = 1.6093 \times 10^3 \text{m} ) ( = 1.0 \times 10^9 \text{m} )</td>
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<tr>
<td>Mass</td>
<td>1 ( \text{lbm} ) ( = 4.5359 \times 10^{-1} \text{kg} ) ( = 4.5359 \times 10^{-1} \text{kg} )</td>
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<tr>
<td>Mass flow rate</td>
<td>1 ( \text{lbm/s} ) ( = 4.5359 \times 10^{-1} \text{kg/s} ) ( = 1.2600 \times 10^{-4} \text{kg/s} )</td>
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<tr>
<td>Mass flux</td>
<td>1 ( \text{lbm/s} \cdot \text{ft}^2 ) ( = 4.8824 \text{kg/s-m}^2 ) ( = 1.3558 \times 10^{-3} \text{kg/s-m}^2 )</td>
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<td></td>
<td>1 ( \text{lbm/s} \cdot \text{ft} ) ( = 1.3562 \times 10^{-3} \text{kg/s-m}^2 ) ( = 7.0352 \times 10^{-4} \text{kg/s-m}^2 )</td>
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<td></td>
<td>1 ( \text{lbm/s} \cdot \text{in} \cdot \text{in} ) ( = 1.9545 \times 10^{-3} \text{kg/s-m}^2 ) ( = 1.9545 \times 10^{-3} \text{kg/s-m}^2 )</td>
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<tr>
<td>Momentum, linear</td>
<td>1 ( \text{lbm-ft/s} ) ( = 1.3823 \times 10^{-1} \text{kg-m/s} ) ( = 3.6404 \times 10^{-3} \text{kg-m/s} )</td>
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<td></td>
<td>1 ( \text{lbm-ft/ft} ) ( = 1.3823 \times 10^{-1} \text{kg-m/s} ) ( = 3.6404 \times 10^{-3} \text{kg-m/s} )</td>
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<tr>
<td>Power</td>
<td>1 ( \text{Btu/s} ) ( = 1.0551 \times 10^3 \text{W} ) ( = 1.3558 \text{W} )</td>
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<td></td>
<td>1 ( \text{ft-lb/s} ) ( = 1.0551 \times 10^3 \text{W} ) ( = 1.3558 \text{W} )</td>
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<td></td>
<td>1 ( \text{Btu/ft} ) ( = 2.9308 \times 10^{-1} \text{W} ) ( = 7.4570 \times 10^2 \text{W} )</td>
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<tr>
<td>Dimension</td>
<td>English Units</td>
<td>SI Units</td>
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<tr>
<td>Pressure</td>
<td>1 lb./ft²</td>
<td>(4.7880 \times 10^{-2}) kN/m²</td>
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<tr>
<td></td>
<td>1 lb./in.²</td>
<td>(6.8948) kN/m²</td>
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<tr>
<td></td>
<td>1 standard atmosphere</td>
<td>(1.0133 \times 10^{2}) kN/m²</td>
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<tr>
<td></td>
<td>1 ft. water</td>
<td>(2.4609 \times 10^{-1}) kN/m²</td>
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<tr>
<td></td>
<td>1 ft. mercury</td>
<td>(2.9891) kN/m²</td>
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<td></td>
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<td>(3.866) kN/m²</td>
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<tr>
<td>Resistance, thermal</td>
<td>1 lb.-°F/Btu</td>
<td>(1.8956) °C/W</td>
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<tr>
<td>(total) (unit)</td>
<td>1 lb.-ft²-°F/Btu</td>
<td>(1.7611 \times 10^{-1}) m²·°C/W</td>
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<tr>
<td>Specific energy</td>
<td>1 Btu/lbₘ</td>
<td>(2.3260) kJ/kg</td>
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<tr>
<td></td>
<td>1 ft-lb/lbₘ</td>
<td>(2.9891 \times 10^{-3}) kJ/kg</td>
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<tr>
<td>Specific heat</td>
<td>1 Btu/lbₘ-°F</td>
<td>(4.1868) kJ/kg·°C</td>
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<tr>
<td>Specific volume</td>
<td>1 ft³/lbₘ</td>
<td>(6.2428 \times 10^{-2}) m³/kg</td>
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<tr>
<td>Surface tension</td>
<td>1 lb./in.</td>
<td>(1.7513 \times 10^{2}) N/m</td>
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<tr>
<td>Temperature</td>
<td>°R</td>
<td>°K = (\frac{5}{9}) × °R</td>
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<tr>
<td></td>
<td>°F</td>
<td>°C = (\frac{5}{9}(°F - 32))</td>
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<tr>
<td>Temperature difference</td>
<td>1°F(°R)</td>
<td>°C(°K)</td>
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<tr>
<td>Time</td>
<td>1 h</td>
<td>(3.6000 \times 10^{3}) s</td>
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<td></td>
<td>1 min</td>
<td>(6.0000 \times 10^{3}) s</td>
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<tr>
<td>Velocity</td>
<td>1 ft/s</td>
<td>(3.0480 \times 10^{-1}) m/s</td>
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<td></td>
<td>1 ft/h</td>
<td>(8.4667 \times 10^{-2}) m/s</td>
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<td></td>
<td>1 mph</td>
<td>(4.4704 \times 10^{-1}) m/s</td>
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<tr>
<td>Viscosity, dynamic</td>
<td>1 poise (g/cm·s)</td>
<td>(1.0000 \times 10^{-1}) kg/m·s (N·s/m²)</td>
<td></td>
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<tr>
<td></td>
<td>1 lbₘ·ft·s</td>
<td>(1.4882) kg·m·s</td>
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<td></td>
<td>1 lbₘ·ft·h</td>
<td>(4.1338 \times 10^{-4}) kg·m·s</td>
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<td></td>
<td>1 lbₘ·s·in.²</td>
<td>(6.8947) kg·s·m</td>
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<tr>
<td></td>
<td>1 lbₘ·ft²</td>
<td>(1.7237) kg·m·s</td>
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<tr>
<td>Viscosity, kinematic</td>
<td>1 stoke (cm²·s)</td>
<td>(1.0000 \times 10^{-4}) m²/s</td>
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<tr>
<td></td>
<td>1 ft²/s</td>
<td>(9.2903 \times 10^{-2}) m²/s</td>
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<tr>
<td></td>
<td>1 ft²/h</td>
<td>(2.5806 \times 10^{-3}) m²/s</td>
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<tr>
<td>Volume</td>
<td>1 ft³</td>
<td>(2.8317 \times 10^{-2}) m³</td>
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<tr>
<td></td>
<td>1 in.³</td>
<td>(1.6387 \times 10^{-5}) m³</td>
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<tr>
<td>Volume flow rate</td>
<td>1 ft³/s</td>
<td>(2.8317 \times 10^{-2}) m³/s</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>1 ft³/min</td>
<td>(4.7195 \times 10^{-4}) m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 ft³/h</td>
<td>(7.8658 \times 10^{-6}) m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problems

1. The velocity boundary layer is identified and described on page 7, and the layer development on a flat plate is pictured on page 11. The thickness \( \delta \) of the boundary layer increases with \( x \). The equation for \( \delta \) is (BL-2).
   
   (a) Suppose \( U_\infty = 25 \text{ feet/sec} \). Make a table of \( \delta \) and \( \delta/x \) versus \( x \) for \( 0 \leq x \leq 2 \text{ feet} \) (use about 10 values of \( x \)). Take note of the magnitudes of \( \delta \) and \( \delta/x \) and comment.

   (b) Repeat part (a) for \( U_\infty = 50 \text{ feet/sec} \) and note how the boundary layer thickness responds to the magnitude of the velocity.

   (c) The local Reynolds number (dimensionless) is \( \text{Re}_x = U_\infty x/\nu \). The flow usually is laminar for \( x \) locations where \( \text{Re}_x < 5 \times 10^5 \). Calculate \( \delta/x \) corresponding to \( \text{Re}_x = 5 \times 10^5 \). Sometimes, another kind of local Reynolds is used: \( \text{Re}_\delta = U_\infty \delta/\nu \). Compute \( \text{Re}_\delta \) corresponding to \( \text{Re}_x = 5 \times 10^5 \).
2. A flat plate is aligned parallel to a freestream airflow whose velocity \( U_\infty = 30 \text{ m/s} \), temperature \( T_\infty = 30^\circ C \), and pressure is one atmosphere. The plate surface temperature \( T_w = 50^\circ C \). The length of the plate in the flow direction is 90 cm, and its width (transverse to the flow) is 60 cm.

(a) Find the rate of heat transfer \( Q \) from the plate to the air between \( x = 2 \text{ cm} \) and \( x = 4 \text{ cm} \).

(b) Find \( Q \) between \( x = 12 \text{ cm} \) and \( x = 14 \text{ cm} \).

Hints: Refer to equation (BL-56) for \( Q \). Equation (BL-59) suggests a way to get \( Q \) for part of a plate. Evaluation of properties is discussed at the bottom of page 53 and on page 54.

3. Air flows parallel to a flat plate. Two cases \( I \) and \( II \) are to be considered. These cases are identical in every way, except that \( (U_\infty)_I = 2(U_\infty)_I \). The heat transfer rates \( Q_I \) and \( Q_{II} \) are measured, and it is found that \( Q_{II} = 1.827Q_I \). It is also known that for case \( I \), the flow is laminar over the entire length of the plate.

(a) For case \( II \), is the flow entirely laminar or has transition to turbulence taken place somewhere along the length of the plate?

(b) Upon noting that \( (Re_L)_I = 2(Re_L)_I \), find the numerical value of \( (Re_L)_I \).
4. An airflow passes parallel to a thin flat plate of dimensions \( L = 3 \) feet and \( W = 4 \) feet. The freestream conditions are \( U_\infty = 23.6 \) feet/sec, \( T_\infty = 60^\circ F \), and \( p = 727.4 \) mm Hg. The plate surface temperature is uniform; \( T_w = 180^\circ F \). A trip wire positioned at \( x = 1 \) foot causes the boundary layer which was laminar for \( 0 \leq x \leq 1 \) ft to be turbulent for \( x > 1 \) ft.
(a) What is the value of \( Q_{\text{lam}} \)?
(b) What is the value of \( Q_{\text{turb}} \)?
(c) If the trip wire were removed, at which \( x \) value would laminar to turbulent transition occur?

5. A flat plate aligned parallel to an airflow is equipped with a trip rod at its leading edge \( x = 0 \) to ensure turbulent boundary layer flow for all \( x \geq 0 \). The freestream velocity \( U_\infty = 126 \) ft/sec, the pressure is 725 mm Hg, and the freestream temperature \( T_\infty = 73.2^\circ F \). The plate is heated by an embedded heating element. The surface temperature \( T_w \) is measured at several locations along the length of the plate. A curve fit of the data is

\[
T_w = 73.2 + 12.12x^{0.2}
\]

where \( x \) is in feet and \( T_w \) is in \(^\circ F\). The plate length \( L = 68 \) in. and the width \( W = 41 \) in.

(a) What is \( Q \) between \( x = 6 \) and 10 in?
(b) What is \( Q \) between \( x = 48 \) and 52 in?
(c) What is \( Q \) for the entire surface?
Note: All \( Q \)'s are for one side of the plate.
6. The cylinder in crossflow is shown on p. 64.

(a) A simplified representation for $\overline{Nu}_D$ for a circular cylinder situated in crossflow in air is

$$\overline{Nu}_D = 0.148 Re_D^{0.633} \text{ for } Re_D \text{ between } 5 \times 10^3 \text{ and } 5 \times 10^4$$

$$\overline{Nu}_D = 0.0208 Re_D^{0.814} \text{ for } Re_D \text{ between } 5 \times 10^4 \text{ and } 2 \times 10^5$$

In operating mode I, $Re_D = 2.1 \times 10^4$. In mode II, everything except the cylinder diameter is kept the same as in mode I. The diameter $D_{II} = 4.3D_I$.

(a) $\overline{Nu}_{DII}/\overline{Nu}_{DI} = ?$

(b) $\overline{h}_{II}/\overline{h}_I = ?$

(c) $Q_{II}/Q_I = ?$

Next, return attention to mode I and consider a mode III which is the same as mode I except for a change in velocity. The velocity $(U_\infty)_{III} = 2(U_\infty)_I$.

(d) $\overline{Nu}_{DIII}/\overline{Nu}_{DI} = ?$

(e) $\overline{h}_{III}/\overline{h}_I = ?$

(f) $Q_{III}/Q_I = ?$

(b) For $Re_D = 10^4$ and $10^5$, compute $\overline{Nu}_D$ from the simplified representation in part (a). Then, compute $\overline{Nu}_D$ for these $Re_D$ values from the appropriate formula from those at the bottom of page 69. Please make comparisons.
7.

Please refer to the table on page 72 of the convection notes.

(a) Table entries 1 and 3, and entries 6 and 7 show two ways of deploying a bar of square cross section. Which deployment should be used to achieve the highest heat transfer coefficient?

(b) Repeat part (a), but now the comparison is to be made between entries 2 and 10.

(c) Again repeat part (a). Compare the deployments of entry 4 and entries 5, 9.
8.

Air with a flowrate \( \dot{m} = 0.01094 \text{ lb}_{m}/\text{sec} \) enters a circular tube whose internal diameter is 1.5 inches. The local rate of heat transfer \( q \) per unit area varies along the length of the tube according to the formula

\[
q = 130 + 56x - 18x^2
\]

where \( q \) is in Btu/hr-ft\(^2\) and \( x \) is in feet. The bulk temperature of the air at the tube inlet is 63.5°F. Note that \( q \) is positive when heat flows from the wall to the fluid.

Calculate the bulk temperature of the air at \( x = 1 \) foot and at \( x = 3 \) feet.

**Note that**

\[
Q = \int_0^x q \, dA = \int_0^x q \pi D \, dx
\]
9.

Air flows in a circular tube in which there is uniform heating $q_f$ along the length of the tube. The left-hand diagram on page 80L shows the corresponding $T_b$ vs $x$ diagram. The wall temperature at four locations is measured as

<table>
<thead>
<tr>
<th>$x$ (ft)</th>
<th>$T_w$ ($^\circ$F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>51.12</td>
</tr>
<tr>
<td>4</td>
<td>54.45</td>
</tr>
<tr>
<td>5</td>
<td>57.78</td>
</tr>
<tr>
<td>6</td>
<td>61.11</td>
</tr>
</tbody>
</table>

Plot these numbers and compute the slope.

The tube i.d. (inner diameter) is 0.5 in. These measurements were made at locations where the local $h$ is a constant. For developed laminar flow with uniform $q_f$, $h = 4.36 \frac{Btu}{k/D}$ where $k$ is the air conductivity $0.016 \text{ hr-ft-}^\circ\text{F}$. The local heat transfer coefficient $h$ is defined as $h = \frac{q_f}{T_w - T_b}$ where $q_f$, $T_w$, and $T_b$ are all local values at a selected $x$. In the problem being studied, both $h$ and $q_f$ are constant, so that $T_w - T_b$ is the same at the selected $x$'s. (a) If $m = 2.82 \text{ lbm/hr}$, $C_p = 0.24 \frac{Btu}{\text{lbm-}^\circ\text{F}}$, find the value of $q_f$. (b) What are the values of $T_b$ at $x = 3$, 4.5, and 6 ft?
10. Air flows in a circular tube whose inner diameter $D = 2.45$ cm. The mass flow rate $\dot{m}$ of the airflow is $0.0617$ kg/sec. At the inlet of the tube, the air temperature and pressure are, respectively, $76.85^\circ$C and 1 atm. The wall temperature $T_w$ is maintained uniform at $103.17^\circ$C by steam condensing on the outer surface of the tube. What is the rate of heat transfer $Q$ from the tube wall to the air between $x = 0$ and $1.26$ m?

11. Air at a velocity of $35.2$ ft/sec and a pressure of $793.1$ mm Hg flows in a long circular pipe whose inside diameter is $0.78$ in. The air temperature at inlet is $70.3^\circ$F. Heat is transferred from the pipe wall to the air along an axial length between the inlet and a station $10.3$ feet downstream of the inlet. Thereafter, the wall of the pipe is perfectly insulated. The bulk temperature is measured at the exit of the insulated section as $129.7^\circ$F (an easy measurement because the air temperature in a sufficiently-long insulated section becomes uniform).

(a) What is the rate of heat transfer $Q$ to the air between the inlet and the $10.3$ ft station?
(b) What is the value of the log-mean $\Delta T$ for the heated section?
12. A circular tube is heated electrically at its outside wall to provide the \( q = \) constant boundary condition. Air at a pressure of 25.63 psi flows through the tube. The bulk temperature is given by

\[
T_b = 70.1 + 8.13x
\]

with \( x \) in feet and \( T_b \) in °F. The inside diameter of the tube is 0.800 inches. The mass flowrate \( \dot{m} = 0.0216 \text{ lb}_m/\text{sec} \).

What is the value of the wall temperature \( T_w \) at \( x = 4, 8, 12, 16, \) and 20 inches?

20. Hot air passes through a horizontal pipe which is situated in a room where the air temperature \( T_{\infty} = 70^\circ \text{F} \). The air flowing inside the pipe has a mean velocity of 46 ft/sec. At a given axial station, the pipe wall temperature \( T_w = 130^\circ \text{F} \). The inner and outer diameters of the pipe are 1 inch and 1.29 inches, respectively. To simplify the problem: (a) neglect the thermal resistance of the pipe wall and (b) assume that the heat loss from the outside of the pipe is by natural convection only. For the room air, the properties are:

\[
k = 0.0157 \text{ Btu/hr-ft-}^\circ \text{F}, \quad \nu = 1.79 \times 10^{-4} \text{ ft}^2/\text{sec}
\]

\[
\beta = 0.00178^\circ\text{F}^{-1}, \quad \rho = 0.0709 \text{ lb}_m/\text{ft}^3, \quad Pr = 0.706
\]

For the air inside the pipe,

\[
k = 0.0167 \text{ Btu/hr-ft-}^\circ \text{F}, \quad \nu = 1.59 \times 10^{-4} \text{ ft}^2/\text{sec}
\]

\[
\rho = 0.0846 \text{ lb}_m/\text{ft}^3, \quad Pr = 0.699
\]

(a) At the given axial station, find the rate of heat loss \( q \) per unit outside surface area.

(b) Find the bulk temperature \( T_b \) of the airflow in the pipe at the given axial station.
13. Refer to the problem statement on page 126. That problem statement is to be used here. However, replace the \( T_{\text{h, out}} \) and \( T_{\text{c, out}} \) values in the middle of page 127 with

\[
T_{\text{h, out}} = 118.3^\circ \text{F} \\
T_{\text{c, out}} = 65.5^\circ \text{F}
\]

Find the velocity \( U \) of the air in the inner pipe which corresponds to these conditions.

Note: The background for this problem is given on pages 114-130.

14. In a certain double-pipe heat exchanger, the hot and cold fluids (\( h \) and \( c \), respectively) flow in the counterflow mode. Given data:

\[
\begin{align*}
(\dot{m}c_p)_h &= 65.97 \text{ Btu/hr} \cdot ^\circ \text{F}, & (\dot{m}c_p)_c &= 39.58 \text{ Btu/hr} \cdot ^\circ \text{F} \\
UA &= 10.47 \text{ Btu/hr} \cdot ^\circ \text{F} \\
T_{\text{h, in}} &= 173.1^\circ \text{F}, & T_{\text{c, in}} &= 40.1^\circ \text{F}
\end{align*}
\]

Find: (a) \( T_{\text{h, out}} \) and \( T_{\text{c, out}} \) (b) \( Q \)}
15. The counterflow heat exchanger analysis described on pages 126-130 does not take account of fouling. Fouling gives rise to additional thermal resistances at both the inner surface of the inner pipe and the outer surface of the inner pipe, as described on pages 137-140. Please redo the example and take account of the fouling resistances. Use the fouling resistance for industrial air listed in Table HX-1 on page 139.
16. Water boils at 1 atm pressure on a polished copper surface. The surface temperature is 390°C.

Find the rate of heat transfer per unit surface area due to boiling.

Property data at 373.15 K: \( \sigma = 58.9 \times 10^3 \text{ N/m}, \)
\( h_{fg} = 2.257 \times 10^6 \text{ J/kg}, \)
\( k_e = 0.681 \text{ W/m} \cdot \text{K}, \)
\( c_{pe} = 4.212 \text{ J/kg} \cdot \text{K}, \)
\( \Pr_e = 1.76, \)
\( \rho_e = 958 \text{ kg/m}^3, \)
\( \rho_v = 0.598 \text{ kg/m}^3. \)

Find the peak heat flux on the boiling curve.
A vertical cylinder is 0.33 inches in diameter and 11.43 inches high. The surface of the cylinder is isothermal at 163°F. The air surrounding the cylinder is at $T_0 = 77°F$ and at a pressure of 724 mm Hg (1 atm = 760 mm Hg). Find the rate of heat transfer from the cylinder surface to the air by natural convection. Compare this heat transfer rate with that for an isothermal vertical plate whose surface temperature is 163°F. The height of the plate is 11.43 inches and its width $W$ is 1.037 inches ($\pi \times 0.33 = 1.037$).


18. A copper vertical plate 1-foot high and 5-feet wide separates two large air spaces. The air space at the left is at $120°F$ and 746 mm Hg, while the air space at the right is at $40°F$ and 735 mm Hg. What is the rate of heat transfer $Q$ from the hotter air space to the cooler air space? Neglect radiation.
19. A long, horizontal cylinder has an outside diameter of 2.97 cm. The operating conditions are $T_w = 76.85^\circ C$, $T_0 = 26.85^\circ C$, $P_0 = 0.872$ atm. Radiation is to be neglected. 
(a) What is $\bar{h}$ for natural convection? 
(b) Suppose now that the cylinder is situated in a forced-convection crossflow. (no natural convection!). At what freestream velocity would the forced convection $\bar{h}$ be equal to that of part (a)? The participating fluid is air.
20. [Diagram showing a fluid flow with dimensions labeled: 20 in., 60 in., x.]

**Given data:**
- $T_w = 120^\circ F$, $T_\infty = 80^\circ F$
- Pressure = 743 mm Hg
- Fluid is air

<table>
<thead>
<tr>
<th>$x$ (in.)</th>
<th>$q$ (Btu/hr ft$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>482.5</td>
</tr>
<tr>
<td>2.5</td>
<td>215.8</td>
</tr>
<tr>
<td>5</td>
<td>120.6</td>
</tr>
<tr>
<td>10</td>
<td>94.6</td>
</tr>
<tr>
<td>20</td>
<td>76.3</td>
</tr>
<tr>
<td>25</td>
<td>328.2</td>
</tr>
<tr>
<td>30</td>
<td>316.4</td>
</tr>
<tr>
<td>40</td>
<td>298.7</td>
</tr>
</tbody>
</table>

**Find**

(a) $Q$ between $x = 0$ and $x = 10$ in.
(b) $Q$ between $x = 20$ and $x = 30$ in.
(c) $Q$ between $x = 30$ and $x = 40$ in.
(d) $Q$ between $x = 40$ and $x = 60$ in.
(e) $Q$ between $x = 0$ and $x = 60$ in.
(f) Suppose that it is desired to double the value of $Q$ found in (e). Describe how that might be done. Show calculations.
21.

A heating element is installed in a flat plate in such a way that the air is heated uniformly. The uniform heating condition can be pictured as

\[ q^+ \]

where \( q \) is the local rate of heat transfer per unit area. From physical reasoning, it can be expected that the wall temperature will increase in the \( x \)-direction. Laminar boundary layer theory, when applied to the uniformly heated plate, gives

\[ Nu_x = \frac{h x}{k} = 0.459 \frac{Re_x^{1/2}}{Pr^{1/3}} \]

Suppose that \( u^* = 65.1 \text{ ft/sec} \), \( v = 1.7 \times 10^{-4} \text{ ft}^2/\text{sec} \), \( Pr = 0.91 \), \( k = 0.0157 \text{ Btu/hr-ft-°F} \), \( T_w = 67.3 \text{°F} \), and \( q = 980 \text{ Btu/hr-ft}^2 \).

(a) Derive an equation which relates \( T_w \) and \( x \).

(b) What is the largest value of \( x \) for which this equation can be used?
22. The rate of heat transfer from one face of an isothermal flat plate to a forced-convection air flow parallel to the plate is 1996 Btu/hr. The temperature difference \((T_w - T_∞)\) driving the heat transfer is 38.3°F. The plate width \(W\), transverse to the flow, is 6.3 ft. To simplify the work, do not look up the air properties. Instead, use \(k = 0.0158\) Btu/hr-ft-°F, \(v = 0.6916\) ft²/hr, and \(Pr = 0.71\).

(a) Use calculations to determine whether the air flow over the plate is either all laminar, part laminar and part turbulent, or all turbulent.

(b) Next, suppose that the freestream velocity \(u_0\) were doubled compared with that for which \(Q = 1996\) Btu/hr. What would be the new value of \(Q\)?

(c) Then, suppose that \(u_0\) were halved compared with that for which \(Q = 1996\) Btu/hr. What would be the new value of \(Q\)?
23. Airflow at the rate of 302.7 lbm/hr flows through a 2-in.-diameter pipe. The air is heated uniformly (pages 81K and 81L). The air temperature at inlet \((x=0)\) is 55.3 \(^\circ\)F. For \(x \geq 80\) in., fully developed conditions prevail; that is, \(h = \text{constant} = \frac{h_{fd}}{\sqrt{f}}\) for \(x \geq 80\) in. Also, it is given that \(T_b = 100^\circ\)F at \(x = 80\) in.

Local heat transfer coefficients \(h(x)\) in the thermal entrance region are listed as follows:

<table>
<thead>
<tr>
<th>(x) (in.)</th>
<th>(\frac{h(x)}{h_{fd}})</th>
<th>(\frac{[T_w(x) - T_b(x)]}{T_b(x)})</th>
<th>(T_b(x))</th>
<th>(T_w(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.15</td>
<td></td>
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<tr>
<td>20</td>
<td>1.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1.025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Find the value of \(g\) which corresponds to the uniform heating.
(b) Find the value of \(h_{fd}\).
(c) Fill in the last three columns of the table.
A piping system experiences a sudden contraction as shown:

Both $D_1$ and $D_2$ are internal diameters. $D_1 = 2.50$ in, $D_2 = 0.50$ in. The fluid is air, with $m = 0.00416$ lbm/sec. Let the mean bulk temperature in the 2.50 in. pipe be 100°F, and that in 0.50 in. pipe be 160°F. The pressure in both pipes is 737.6 mm Hg (1 atm = 760 mm Hg).

(a) Compute the fully developed $h$ values in the two sections of the pipe.

(b) The air temperature at the inlet of the 2.50 in. section is 87.2°F. The wall temperature of the 2.50 in. section is uniform and equal 120°F. What is the bulk temperature at a location 6 ft downstream of the inlet? What is the rate of heat transfer $Q$ for the 6 foot length?
25. Air at 65°F and 760 mm Hg enters a \( \frac{1}{2} \)-in.-diameter tube where it is heated uniformly at the rate \( q = 58.2 \text{ Btu/hr-ft}^2 \). The mass flowrate of the air is \( 15 \text{ m}^3/\text{hr} \). (a) What is the fluid bulk temperature at \( x = 3 \) feet? (b) What is the tube wall temperature at \( x = 3 \) feet? At \( x = 4 \) feet, the \( \frac{1}{2} \)-in.-diameter pipe undergoes an abrupt enlargement and becomes a 1.50-in.-diameter pipe. In the 1.50-in.-diameter pipe, wall heating continues but nonuniformly for the condition of uniform wall temperature. At a certain \( x \) location in the 1.50 in. pipe: \( T_b = 140^\circ \text{F} \) and \( T_w = 157.2^\circ \text{F} \). (c) What is value of \( q \) at this cross section?

26. A \( \frac{1}{2} \)-in.-diameter vertical cylinder is 1-foot high. The surface temperature of the cylinder is 15120°F. The cylinder is situated in an air environment where the temperature \( T_\infty = 80^\circ \text{F} \), and the pressure is 742.7 mm Hg. Consider natural convection only. (a) First, find \( h \) for a 1-foot-high vertical plate. (b) Find \( Q_{\text{cyl}} \).
27. A 14 Btu/hr heater is buried in a 2-inch diameter copper sphere whose outer surface is painted white. The sphere is situated in a room where the air temperature is 70°F. The walls of the room are also at 70°F. The air pressure is 736 mm Hg. In the steady state, what is the surface temperature of the sphere when:
(a) The heat loss at the surface of the sphere is only by natural convection.*
(b) The heat loss at the surface of the sphere is by only by radiation.
(c) The heat loss at the surface of the sphere is by simultaneous natural convection and radiation.

* \[ \text{Nu}_D = 2 + \frac{0.589 \cdot Ra^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{1/4}} \]

28. This problem is about a double-pipe heat exchanger in which the two fluids flow in parallel (Fig. HX-11, page 114, convection). The heat exchanger is very well insulated at its outside surface. The hotter fluid h flows through the annulus, while the cooler fluid c flows through the center pipe. The heat exchanger is
18-feet long. The inner and outer diameters of the center pipe are 1.00 and 1.25 inches respectively. The operating conditions are

\[
\begin{align*}
T_{h, in} &= 300^\circ F, \quad (mc_p)_h = 0.10 \frac{\text{Btu}}{\text{min} \cdot ^\circ F} \\
T_{c, in} &= 40^\circ F, \quad (mc_p)_c = 0.025 \frac{\text{Btu}}{\text{min} \cdot ^\circ F} \\
\varepsilon &= 0.78
\end{align*}
\]

(a) Find \( T_{h, out} \) and \( T_{c, out} \).

(b) Find the rate of heat transfer from the \( h \) fluid to the \( c \) fluid.

(c) Find the thermal resistance \( R \).

(d) If \( \dot{h} = 6.3 \frac{\text{Btu}}{\text{hr} \cdot ^\circ F} \), find \( T_c \).

Neglect the thermal resistance of the pipe wall.

29.

Air flows in a round pipe at the rate \( \dot{m} = 0.0216 \text{ lbm/sec} \). The inner diameter of the pipe is 0.800 inches. The axial variation of the bulk temperature \( T_b \) is

\[
T_b = 50 + 24x
\]

where \( T_b \) is in \( ^\circ F \) and \( x \) is in feet.

(a) Let \( Q(x) \) denote the rate of heat
transfer (Btu/hr) between $x = 0$ and $x = x$. Write a formula for $Q(x)$ as a function of $x$. Use all of the given information to evaluate the numerical constants in this equation.

(b) What is the value of $q$ in Btu/hr-ft² at any $x$?

(c) What is the value of the Reynolds number? Use the $T_b$ at $x = 25$ inches for the property evaluation?

(d) What is the value of $h$ at $x = 25$ inches?

(e) What is the value of the wall temperature at $x = 25$ inches?
30. The diagram shows a counter-flow heat exchanger. Heat flows from the "h" fluid to the "c" fluid through the common wall that separates them. That separating wall has a surface area $A_{\text{surf}} = W \times L$. All the other walls are insulated as shown in the diagram.

Given information:

- Both fluids are air
- $T_{h,\text{in}} = 99.5 \, ^\circ F$, $T_{c,\text{in}} = 47.5 \, ^\circ F$
- $W = 5 \, \text{in.}$, $H = 0.5 \, \text{in.}$
- $m_h = m_c = 84.2 \, \text{lb}_m/\text{hr}$
- $(C_p)_h = (C_p)_c = 0.2403 \, \text{Btu}/\text{lb}_m \cdot ^\circ F$

Find the length $L$ that is needed to achieve $E = 0.75$ for the heat exchanger.

To guide the solution for $L$, find

(a) The value of NTU
(b) The value of the resistance $R$
(c) The value of $L$

Neglect the conductive resistance of the wall which separates the fluids.