HOMEWORK PROBLEM 1

This problem is related to a Type (a) shock such as that is described on pages 100 - 102 of the notes. On those pages, the velocity of the V of the air is given, and the unknown is the shock velocity Vs. In problem 8, the shock velocity is given (Vs = (100ft)/(0.1036sec) = 965.25 ft/sec), and the desired result is the air velocity V. Pre-capped pressure and temperature are 12.31psia and 67.3°F, respectively.

(a) \( Vs = \frac{100}{0.1036} = 965.25 \text{ ft/sec} \) and the desired result is the air velocity V.

(b) \( Vy = 965.25 \text{ ft/sec} \)

(c) \( Mx = \frac{Vx}{49.02*(67.3 + 459.7)^{0.5}} = \frac{Vx}{1125.3} \)

Solution strategy:

Guess \( Mx = 1.400 \Rightarrow \frac{Vx}{Vy} = 1.6896 \)
\( Vx = (\frac{Vx}{Vy})*Vy = 1.6896*965.25 = 1631 \text{ ft/s} \)
\( Mx = \frac{Vx}{1125.3} = 1.449 \)

Guess \( Mx = 1.500 \Rightarrow \frac{Vx}{Vy} = 1.8621 \)
\( Vx = (\frac{Vx}{Vy})*Vy = 1.8621*965.25 = 1797 \text{ ft/s} \)
\( Mx = \frac{Vx}{1125.3} = 1.597 \rightarrow \) our answer is further from the previous guess, go in the other direction

Guess \( Mx = 1.300 \Rightarrow \frac{Vx}{Vy} = 1.5157 \)
\( Vx = (\frac{Vx}{Vy})*Vy = 1.5157*965.25 = 1463 \text{ ft/s} \)
\( Mx = \frac{Vx}{1125.3} = 1.300 \rightarrow \) our guess checks out

\( V_f = Vx - Vy = 1463 - 965.3 = 497.8 \text{ ft/sec} \)
HOMEWORK PROBLEM 2

For a Type (a) moving shock, the temperature and pressure in the pre-capped flow are 8.3°F and 15.1 psi, respectively. In the air that is brought to rest due to the capping of the end of the pipe, the temperature is 200.5°F and the pressure is 29.6 psi.

We know that there is an error because the temperature and pressure ratios are fixed. I sent out an email stating the temperature as 83°F, which is actually incorrect anyways as the temperature is 80.3°F. I will solve the problem using each of the three temperatures.

**Using 8.3°F:**
(a) What is the velocity of the pre-capped flow?

\[ V_x = V_s + V_f, \quad V_y = V_s \]

\[ V_x = M_x \times 49.02(T_f+459.7)^{0.5} \]

\[ P_x = 15.1 \text{ psi}, \quad P_y = 29.6 \text{ psi} \rightarrow \frac{P_y}{P_x} = 1.9603 \rightarrow \frac{T_y}{T_x} = 1.2226 \]

\[ \frac{P_y}{P_x} \rightarrow M_x = 1.350, \quad My = 0.7616 \]

However, \( \frac{T_y}{T_x} = \frac{(200.5+459.7)}{(8.3+459.7)} = 1.4107 \rightarrow \) which is a conflict with above

\[ V_x = 1.350 \times 49.02(468)^{0.5} = 1431.6 \text{ ft/s} \]

\[ V_y = 0.7615 \times 49.02(660.2)^{0.5} = 959.1 \text{ ft/s} \]

\[ V_y = V_x - V_f; \quad V_f = 472.5 \text{ ft/s} \]

(b) What is the velocity of the shock that is caused by the capping?

\[ V_s = V_y = 959.1 \text{ ft/s} \]

**Using 83°F:**

\[ P_x = 15.1 \text{ psi}, \quad P_y = 29.6 \text{ psi} \rightarrow \frac{P_y}{P_x} = 1.9603 \rightarrow \frac{T_y}{T_x} = 1.2226 \]

\[ \frac{P_y}{P_x} \rightarrow M_x = 1.350, \quad My = 0.7616 \]

However, \( \frac{T_y}{T_x} = \frac{(200.5+459.7)}{(83+459.7)} = 1.2165 \rightarrow \) which is a conflict with above

\[ V_x = 1.350 \times 49.02(542.7)^{0.5} = 1541.7 \text{ ft/s} \]

\[ V_y = 0.7615 \times 49.02(660.2)^{0.5} = 959.1 \text{ ft/s} \]

\[ V_y = V_x - V_f; \quad V_f = 582.6 \text{ ft/s} \]

\[ V_s = V_y = 959.1 \text{ ft/s} \]

**Using 80.3°F:**

\[ P_x = 15.1 \text{ psi}, \quad P_y = 29.6 \text{ psi} \rightarrow \frac{P_y}{P_x} = 1.9603 \rightarrow \frac{T_y}{T_x} = 1.2226 \]

\[ \frac{P_y}{P_x} \rightarrow M_x = 1.350, \quad My = 0.7616 \]

\[ \frac{T_y}{T_x} = \frac{(200.5+459.7)}{(80.3+459.7)} = 1.2226 \rightarrow \] which is a match!

\[ V_x = 1.350 \times 49.02(540)^{0.5} = 1537.8 \text{ ft/s} \]

\[ V_y = 0.7615 \times 49.02(660.2)^{0.5} = 959.1 \text{ ft/s} \]

\[ V_y = V_x - V_f; \quad V_f = 578.7 \text{ ft/s} \]

\[ V_s = V_y = 959.1 \text{ ft/s} \]
HOMEWORK PROBLEM 3

(a) What is the speed $V_r$ of the reflected shock?

Part 1

$M_x = \frac{V_x}{(20.05 \times T_x^{0.5})} = \frac{750}{(20.05 \times 283^{0.5})} = 2.22$

$P_y/P_x = 5.5831 \rightarrow P_y = 565.6 \text{ kPa}$

$T_y/T_x = 1.8746 \rightarrow T_y = 530.5 \text{ K}$

$V_x/V_y = 2.9783 \rightarrow V_y = 251.8 \text{ m/s}$

$V_{up} = V_x - V_y = 498.2 \text{ m/s}$

Part 2

$P_x = 565.5 \text{ kPa}, T_x = 530.5 \text{ K}$

Guess $M_x = 1.5 \rightarrow V_x/V_y = 1.8621 \rightarrow T_y/T_x = 1.3202$

$V_x = M_x \times 20.05 \times (530.5)^{0.5} = 692.7 \text{ m/s}$

$V_y = V_x - 498.2 \text{ m/s} = 194.5 \text{ m/s} \rightarrow V_x/V_y = 1.9746 \rightarrow \text{answer doesn't check, guess again}$

Guess $M_x = 1.85, V_x/V_y = 2.4381$ (B3)

$V_x = M_x \times 20.05 \times (530.5)^{0.5} = 854.3 \text{ m/s}$

$V_r = V_x - 498.2 \text{ m/s} = V_y = 356.1 \text{ m/s} \rightarrow V_x/V_y = 2.3990 \rightarrow \text{answer doesn't check, guess again}$

Guess $M_x = 1.8385, V_x/V_y = 2.4200$ (B3)

$V_x = M_x \times 20.05 \times (530.5)^{0.5} = 849.0 \text{ m/s}$

$V_r = V_x - 498.2 \text{ m/s} = V_y = 350.8 \text{ m/s} \rightarrow V_x/V_y = 2.4200 \rightarrow \text{answer matches!}$

$b) What is the pressure $p$ in the rest gas behind the reflected shock$

$M_x = 1.8385, P_y/P_x = 3.7768$

$P_y = (P_y/P_x) \times P_x = 2,136 \text{ kPa (21.1 atm)}$