Given data:

\[ P_1 = 7.3 \text{ psia}; \quad P_2 = 51.28 \text{ psia}; \quad P_3 = 54.61 \text{ psia}; \]
\[ P_4 = 43.7 \text{ psia}; \quad A_{in} = 3.052 \text{ in}^2; \quad A_{out} = 3.58 \text{ in}^2; \]
\[ \frac{4L}{D} = 2.015; \quad \frac{4L}{D_{II}} = 1.057; \]
\[ T_0 = 612 \text{ R}; \]
\[ P_1 > P_2 \quad \text{Subsonic flow}. \]

For Pipe (1)\(-\)\(2)\):

Guess \( M_1 = 0.5 \) (From B4)

\[ \frac{p_1}{p^{**}} = 2.13809 \quad \Rightarrow \quad p^{**} = 34.14 \text{ psia} \]

\[ \frac{(4L \text{min})}{D} = 1.06906 \]

\[ \frac{P_2}{p^{**}} = \frac{51.28}{34.14} = 1.5019 \quad \Rightarrow \quad \frac{(4L \text{min})}{D_{II}} = 0.21006 \]

\[ \frac{4L}{D} = \frac{(4L \text{min})}{D} - \frac{(4L \text{min})}{D_{II}} \]

\[ = 0.859 \neq 2.015 \]

(Too low)
By guess of $M_1 = 0.3$

gave $4f_{Ⅰ}/D = 3.35 \neq 2.015$

(Too high)

$M_1$ lies between 0.3 and 0.5

The converged solution:

Guess: $M_1 = 0.37$

Form By

$$\left(\frac{4f_{Ⅰ} \text{mm}}{D}\right)_Ⅰ = 2.93198$$

$$\frac{p_Ⅰ}{p^{**}} = 2.92094 \Rightarrow p^{**} = 24.99195 \text{ Pa}$$

$$\Rightarrow \frac{p_Ⅰ}{p^{**}} = \frac{51.28}{24.99195} = 2.05186$$

From $4f_{Ⅱ}/D = M_2 = 0.52$ and $\left(\frac{4f_{Ⅱ} \text{mm}}{D}\right)_Ⅱ = 0.91742$

$$\Rightarrow \left(\frac{4f_{Ⅱ}}{D}\right) = \left(\frac{4f_{Ⅱ} \text{mm}}{D}\right)_Ⅰ - \left(\frac{4f_{Ⅱ} \text{mm}}{D}\right)_Ⅱ$$

$$= 2.93198 - 2.05186$$

$$\Rightarrow \frac{4f_{Ⅱ}}{D} = 2.01456 \approx 2.015$$

Thus $M_1 = 0.37$ and $M_2 = 0.52$
For nozzle (2) → (3):

Isentropic Table B.2

\[ \frac{P_3}{P_0} = 0.83166 \Rightarrow P_0 = 61.6598 \, \text{Pa} \]

\[ \text{at} \, (n = 0.52) \]

\[ \frac{P_3}{P_0} = 0.88567 \]

\[ \Rightarrow M_3 = 0.42 \]

\[ \frac{A}{A^*} = 1.3034 \Rightarrow A^* = \frac{3.652}{1.3034} \]

\[ \Rightarrow A^* = 2.8155 \, \text{cm}^2 \]

\[ \dot{m} = \frac{0.532 \, P_0 \, A^*}{\sqrt{T_0}} = \frac{0.532 \times 61.6598 \times 2.34155}{\sqrt{612}} \]

\[ \Rightarrow \dot{m} = 3.165 \, \text{lbm/sec} \]

For pipe (3) → (4):

\[ \frac{A_3}{A_4} = \frac{M_4}{M_3} \quad M_3 = 0.42 \]

\[ \text{from} \quad 84 \left( \frac{46 \, \text{lbm}}{\text{in}} \right)^{0.5} = 1.92937 \]
\[ \left( \frac{4 \ell \text{in}}{D} \right)_{\text{II}} = \left( \frac{4 \ell \text{in}}{D} \right)_{\text{III}} = \frac{4 \ell}{D} \]

\[ = 1.92437 \approx 1.057 \]

\[ \Rightarrow \left( \frac{4 \ell \text{in}}{D} \right)_{\text{IV}} = 0.91737 \approx 0.9174 \]

From B.9

\[ M_4 = 0.52 \]

\[ \frac{P_4}{P_{4+}} = 2.05187 \]

\[ \Rightarrow P_{4+} = \frac{43.7}{2.05187} = 21.384 \text{ PSI} \]

\[ = \Rightarrow P_{4+} = 21.384 \text{ PSI} \]

To cross-check:

\[ P_{4+} = 0.9935 \times \frac{m^2 \times \sqrt{T_0}}{A_1} \]

\[ = 0.9935 \times 8.105 \times \sqrt{612} \]

\[ = \frac{3.052}{3.052} \]

\[ = 25 \text{ PSI} \approx 24.99 \text{ PSI} \]
(a) \(M_1 = 0.37\)  \(M_2 = 0.52\)  \(M_3 = 0.52\)  \(M_4 = 0.52\)

(b) \(p_{1,2}^{*} = 24.992 \text{ psi}\)  \(p_{3,4}^{*} = 21.304 \text{ psi}\)

(c) \(A_{2,3}^{*} = 2.34155 \text{ in}^2\)

(d) \(P_{0,2,3} = 61.65981 \text{ psi}\)

(e) \(m = 3.105 \text{ lbm/sec}\)

(2)

- \(A_{\text{min}}\)
- \(A_x\)
- \(1.385\)
\[ \dot{m} = \frac{0.532 \, P_0 \, A^*}{\sqrt{T_0}} \]

\[ \Rightarrow A^* = \frac{2.46 \times \sqrt{573.2}}{0.532 \times 104} = 1.0694 \text{ in}^2 \]

\[ \Rightarrow A^* = 1.0644 \text{ in}^2 \]

\[ \text{case(i)} \]

Assume no shock:
This implies \( P_0 \) and \( A^* \) do not change for isentropic flow:

\[ \frac{P_2}{P_0} = \frac{77.21}{104} = 0.7429 \]

From \( B_2 \):

\[ M_2 = 0.667 \]

But:

\[ \frac{A_2}{A^*} = \frac{1.385}{1.0649} = 1.301088 \]

\[ \Rightarrow M_2 = 0.52 \text{ or } M_2 = 1.66 \]

\[ \Rightarrow \text{There is a shock to accommodate the pressure rise.} \]
For a shock:

\[ A_{min} = A^+ = 1.064494 \text{ in}^2 \]

\[ P_0 \times A_x^+ = 104 \times 1.064494 = P_0 \times A_y^+ \]

\[ \Rightarrow \quad \frac{P_0 \times A_x^+}{P_0 \times A_y^+} = \frac{77.21 \times 1.385}{110.707} = 0.965932 \]

From \( B_2 \) at \( \left( \frac{p_2 A_2}{P_0 A_x^+} = 0.965932 \right) \),

we get \( M_2 = 0.58 \)

\[ \Rightarrow \quad \frac{A_2}{A_x^+} = 1.213 \Rightarrow \frac{A_y^+}{A_x^+} = \frac{1.385}{1.213} \]

\[ \Rightarrow \quad A_y^+ = 1.141797 \text{ in}^2 \]

\[ \frac{A_x^+}{A_y^+} = \frac{1.064494}{1.141797} = 0.932297 \]

From \( B_3 \) (shock table)

at \( \frac{A_x^+}{A_y^+} = 0.932297 \) we get:

\[ M_x \approx 1.49 \]

\[ M_y \approx 0.7046 \]
To find $A_x = \frac{r}{A_x}$

From $x_2$:

$M_x = 1.49$

$\frac{r}{A_x} = 1.1695$

$\Rightarrow r = 1.1695 \times 1.064495$

$\Rightarrow A_x = 1.2449 \text{ in}^2$

(QR) from $M_y = 0.705$

$\frac{r}{A_y} = 1.09 \Rightarrow A_{shock} = 1.24455 \text{ in}^2$

Answers:

(a) $M$ at $A_{min} = 1$  \[ M = 1 \text{ at } A_{min} \]

(b) At $A = 1.385$  \[ M_a = 0.58 \text{ at } A = 1.385 \text{ in}^2 \]

(c) Yes There is a shock in the diverging section

(d) $M_x = 1.49$

$A_x = 1.2449 \text{ in}^2$