Essay 9

Hydraulic Diameter for Non-Circular Ducts

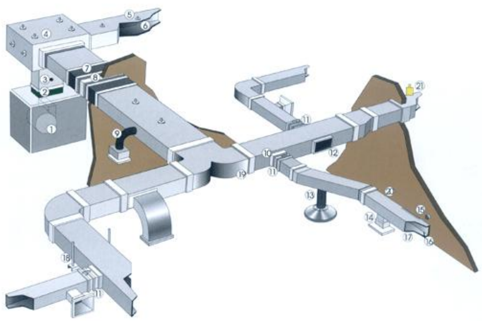

Fig. 9. 1 Complex ducting system involving non-circular ducts

Although the round pipe is a very common conveyance for flowing fluids, ducts of other cross sections are commonly encountered. Figure 9.1 shows a ducting system of the type frequently used for distributing conditioned air for either heating or cooling residential spaces.

If a duct is sufficiently long, a fully developed flow will occur regardless of the shape of the duct cross section. The conditions for fully developed flow are the same both for round pipes and non-circular ducts. They are: (a) a pressure drop characterized by a constant slope and (b) velocity profiles which maintain a given shape. The magnitude of the pressure gradient and the shape of the velocity profile depend on the nature of the duct cross section. Furthermore, the distance from the duct inlet at which fully developed conditions are first encountered also depends on the cross-sectional shape. Two commonly encountered non-circular cross sections are the rectangular and the annular ducts which are pictured in Fig. 9.2.

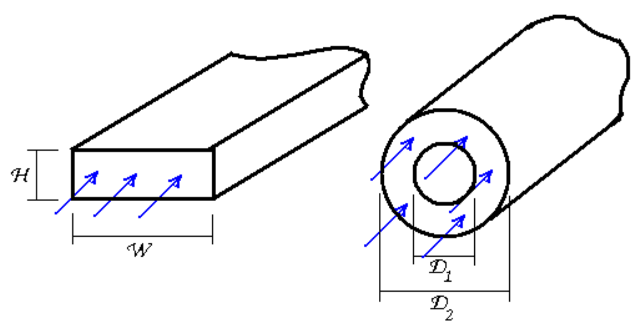

Fig. 9. 2 Examples of commonly encountered non-circular ducts, rectangular and annular

It is clear from Fig. 9.2 that more than one dimension is needed for the description of the cross-sectional shapes of non-circular ducts. This fact leads to a dilemma with regard to the Reynolds number. In the definition of the Reynolds number, there is space to accommodate a single dimension. For a round pipe, that dimension is clearly the diameter. The difficulty with regard to the Reynolds number for non-circular ducts is the need to find a single dimension that characterizes the non-circular cross section.

Guidance for the selection of the appropriate single dimension can be taken by consideration of Newton's Second Law for fully developed flow in pipes and ducts. To implement the discussion, attention may be turned to Fig. 9.3 which shows control volumes specific to circular and non-circular ducts. For fully developed flow, the rate at which x-directed momentum enters the control volume is precisely equal to the rate at which the x-directed momentum leaves the control volume. Therefore, the application of Newton's Second Law reduces to a force balance between the pressure and shear forces.

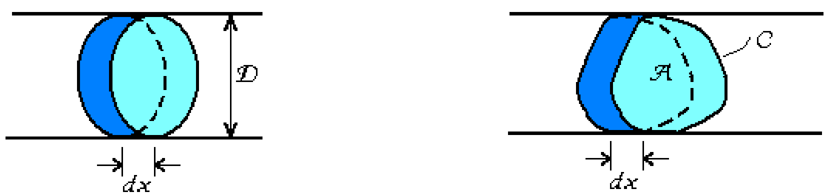

Fig. 9. 3 Control volumes for the analysis of fully developed flow in circular pipes and non-circular ducts

For the case of the circular cross section, Eq. (9.1) expresses the fully developed force balance.

$$-\frac{dp}{dx} = \frac{4\tau_w}{D} \tag{9.1}$$

On the other hand, for the force balance for the case of the non-circular cross section, it is necessary to recognize that the resulting skewed velocity profile gives rise to a wall shear stress that varies around the circumference. For the execution of the force balance, it is convenient to let $\overline{\tau_w}$ denote the average wall shear. With this notation, the force balance for the non-circular case is:

$$-\frac{dp}{dx} = \frac{\overline{\tau_w}\, C}{A} \tag{9.2}$$

To bring the force balances of Eqs. (9.1 and 2) into congruence,

$$\frac{4}{D} = \frac{C}{A} \tag{9.3}$$

or,

$$D = \frac{4A}{C} \tag{9.4}$$

The quantity D represented by Eq. (9.4) is the equivalent diameter of a round pipe which yields the same force balance as that for a non-circular duct. It is common practice to denote this quantity as the *hydraulic diameter* D_H, so that,

$$D_H = \frac{4A}{C} \tag{9.5}$$

It is important to note the result of applying this definition to a round pipe, which gives

$$D_H = \frac{4\,\pi\frac{D^2}{4}}{\pi D} = D \tag{6.6}$$

Therefore, the definition of the hydraulic diameter is equally applicable to circular and non-circular ducts.

For the rectangular duct that is pictured in Fig. 9.2,

$$D_H = \frac{4HW}{2(H+W)} = \frac{2H}{1+\frac{H}{W}} \tag{9.7}$$

The quantity W/H is called the *aspect ratio*. For ducts that are wide, relative to their height, the aspect ratio approaches infinity, and H/W approaches zero. Such ducts are called *parallel-plate channels* and are characterized by hydraulic diameters $D_H = 2H$.

For the case of the annular duct, which is also shown in Fig. 9.2,

$$D_H = \frac{4\pi\frac{D_2^2-D_1^2}{4}}{\pi\,(D_2+D_1)} = D_2 - D_1 \tag{9.8}$$

Therefore, the hydraulic diameter is equal to the difference in the diameters of the outer and inner bounding walls.