Measurements in High Speed Flow

Pressure

Static pressure
This is the thermodynamic pressure.
The pressure on which properties are evaluated.

Travel with the flow
The net force/unit area due to molecules colliding against the wall moving with the flow.

Stagnation pressure
Pressure measured when fluid is brought to rest against the wall.

\[ P_1 + \frac{\rho V_i^2}{2g} = P_0 \]

when isentropically reduced to zero velocity; an incompressible
Bernoulli Eqn.

Temperature

Static temperature
This is the thermodynamic temperature.
The temperature on which the thermodynamic properties are evaluated.
Travel with the flow
A measure of the molecular kinetic energy. Sensor and flow in thermal equilibrium.

Stagnation temperature
Temperature when the flow is brought to rest adiabatically

\[ T_0 = T + \frac{1}{2} \frac{V^2}{C_p} \]

Now consider an idea gas being brought to rest such that 
\[ \gamma \neq 0 \]
Start with temperature

Conservation of energy
use stagnation enthalpy
\[ h_0 = h + \frac{V^2}{2} \quad \text{adiabatic} \]

for a perfect gas, we can say
\[ h = C_p T \quad \text{where} \quad C_p = R - C_v \]

then
\[ C_p T_0 = C_p T + \frac{V^2}{2} \]

\[ T_0 = \frac{T + \frac{V^2}{2C_p}}{1} \]

\[ T_0 = \text{stagnation} \]

Using \( M = \frac{V}{c} \) and \( C = \sqrt{\frac{kR}{C_v}} \)

we can write

\[ \frac{T_0}{T} = 1 + \frac{1}{2} (k-1) M^2 \]

look at some values
Auto speed 70 mi/hr \( T = 300 K \)
\[ V = 70 \text{ mi/hr} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} \times \frac{1 \text{ ft}}{39 \text{ in}} = 31.6 \text{ m/s} \]
\[ C = \sqrt{\frac{kRT}{M}} = \sqrt{\frac{1.4 \times 8.314 \text{ KJ/kg mol} \times 300 \text{ K}}{10^3 \text{ Kg m}^2 \text{/K}} \times \frac{1 \text{ Kg mol}}{28.97 \text{ Kg / KJ}} \times \frac{1 \text{ KJ}}{10^3 \text{ sec}^2}} = 347 \text{ m/sec} \]

\[ M = \frac{V}{c} = 0.09 \]

\[ \frac{T_0}{T} = 1 + \frac{1}{2} (0.4) (0.09)^2 = 1.0016 \]
$$T_0 = 1.0016 \ (300K) = 300.5 \ K$$

$$T_0 - T = 0.5 \ K$$

Aircraft speed 650 mph

$$T = -50^\circ F = -45^\circ C = 227 \ K$$

$$v = \frac{650 \ mi}{5280 \ ft/hr} \cdot \frac{12 \ in}{1 \ hr} \cdot \frac{1 \ mi}{3600 \ sec} \cdot \frac{1 \ ft}{39.3 \ in}$$

$$= 29.3 \ m/s$$

$$C = \sqrt{\frac{kRT}{h \cdot k \cdot m}} = \sqrt{\frac{1.4 \cdot 8.314 \ KV \cdot \text{mol} \cdot 227 \ K \cdot 10^3 \text{Kg} \cdot \text{m}^2} {28.97 \text{Kg} \cdot \text{kJ} \cdot \text{sec}^2}}$$

$$= 302 \ K$$

$$M = \frac{293}{302} = 0.97$$

$$T_o = 1 + \frac{1}{2} (0.4)(0.97)^2 = 1.19$$

$$T_o = 227K(1.19) = 270K$$

$$T_o - T = 43 \ K$$

So the probe tends to be of a higher temperature than the surroundings.

In reality, the probe in the flow will see neither $T$ or $T_o$, there will be some heat transfer.

Heat transfer in the fluid tends to reduce the measured temperature.
for the adiabatic wall

\[ \frac{T_{aw} - T}{T_0 - T} = r \quad \text{recovery factor} \]

\[ r = 0.87 - 0.91 \quad \text{for subsonic flow} \]

See our thermocouple notes.

Compute for airplane

\[ T_{aw} = 0.9 (43K) + 227K = 265.7K \]

If we knew \( r \) and measured \( T_{rec} = T_{aw} \), we could compute \( T \), given \( V \).

Now look at pressure

assume isentropic deceleration to

Use the Gibbs eqn.

\[ T_{d} = c_{p} dT = d \frac{h - 0}{dP} \]

but \( d \alpha = 0 \) and \( dh = c_{p} dT \)

so

\[ c_{p} dT = \frac{dP}{\rho} = \frac{dP}{\rho} \frac{RT}{T} \]

\[ \frac{dP}{\rho} = \frac{c_{p}}{R} \frac{dT}{T} \]

Integrate

\[ \frac{P_0}{\rho} = \left( \frac{T_0}{T} \right)^{c_{p}/k} \]

but \( c_{p} = \frac{c_{p}}{R} \frac{c_{p} - c_{v}}{k-1} \)

\[ \frac{P_0}{\rho} = \left( \frac{T_0}{T} \right)^{k-1} \]
Look at some magnitudes
Auto 70 mph  \( (110.4) \)
\[
P_0 = 1.0016 \quad \frac{P}{P_0} = 1.0056
\]
\[P_a = 100.56 \text{ kPa}
\]
\[P_0 - P = 560 \text{ Pa}
\]

Plane 650 mph
\[
P_0 = 1.19 \quad \frac{P}{P_0} = 1.84
\]
\[P = 23.6 \text{ kPa}
\]
\[P_a = 43.5 \text{ kPa}
\]
\[P_0 - P = 23 \text{ kPa}
\]

The total pressure tube can measure \( P_0 \) quite well. There are considerations about:

1) probe alignment, yaw
2) viscous effects, (Low Reynolds number based upon probe size)
3) shocks (when the approach flow is supersonic)
4) near-wall effects
5) turbulence effects

For more on these, see "Instrumentation for Fluid Dynamics - Pressure Measurements," Handbook of Fluid Dynamics and Fluid Machinery, Wiley, 1996