CONVECTION FUNDAMENTALS

PREPARATION FOR
1) THE FIN EXPERIMENT - FORCED CONVECTION
2) THE OTHER FIN EXPERIMENT - FREE CONVECTION

EXTERNAL, FORCED CONVECTION

LET'S BEGIN WITH A SIMPLIFIED MODEL OF THE FLOW OVER A FIN SURFACE

ASSUME:
1) Boundary layer is initiated at separation at x=0
2) All cooling is on the upper and lower surfaces (thin leading edge)
3) Laminar flow until \( \text{Re}_x = \frac{U_{x}}{\nu} = \text{Re}_x, \text{transition} \) then fully turbulent.

Hydrodynamic boundary layer - region near the surface where wall influences the velocity.

Thermal boundary layer - region near wall where heated or cooled surface influences the temperature profile.

The "maturity" of the boundary layer is given in terms of \( \text{Re}_x = \frac{U_{x}}{\nu} \)

We expect transition to take place at \( \text{Re}_x \approx 5 \times 10^5 \).
The velocity distribution can be found as the solution to the partial differential equation

\[ \left( \frac{\partial^2 u}{\partial x \partial y} \right) = \mu \left( \frac{\partial^2 u}{\partial y^2} \right) \]

assuming

1) steady flow
2) 2-D flow
3) uniform static pressure
4) streamwise diffusion of momentum is insignificant relative to wall-normal diffusion of momentum

\[ \mu \frac{\partial^2 u}{\partial x^2} \approx \mu \frac{\partial^2 u}{\partial y^2} \]

5) laminar flow
6) uniform properties

boundary conditions

\[ u = 0 \quad \text{at} \quad y = 0 \quad \text{for all} \quad x > 0 \]
\[ u = u_0 \quad \text{at} \quad y \rightarrow \infty \quad \text{for all} \quad x \]
\[ u = u_0 \quad \text{at} \quad x = 0 \quad \text{for all} \quad y > 0 \]

The solution is \( u(x, y) \)

Energy equation

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \lambda \frac{\partial^2 T}{\partial x^2} \]

\[ \frac{\partial^2 T}{\partial x \partial y} \]

no new assumptions from above

boundary conditions

\[ T_w = \text{uniform} \]
\[ T = T_w \quad \text{at} \quad y = 0 \quad \text{for all} \quad x > 0 \]
\[ T = T_0 \quad \text{as} \quad y \rightarrow \infty \quad \text{for all} \quad x \]
\[ T = T_0 \quad \text{at} \quad x = 0 \quad \text{for all} \quad y > 0 \]
The solution gives $T(x, y)$ combining with

$$
T = -\frac{k \frac{\partial T}{\partial y}}{\frac{y}{y} = 0} = \frac{\frac{q}{k}}{T_w - T_m} = \frac{q}{T_w - T_m}
$$

we find that when we non-dimensionalize $T$ as $N_u = \frac{1}{\nu} x$ and $x$ as $Re_x = \frac{\nu_x}{D}$

the solution can be cast as:

$$
N_u = 0.332 Pr^{\frac{1}{3}} Re_x^{\frac{1}{2}}
$$

(confirmed by experiments)

we can find a similar relationship for this situation but where the near-wall flow is turbulent.

$$
St_x = N_u x = 0.02940 \frac{Re_x}{Pr}^{\frac{1}{3}} Pr^{\frac{1}{3}}
$$

or

$$
N_u = 0.02940 Re_x^{\frac{1}{3}}
$$

if we may assume that the turbulent boundary layer begins its growth at $x = 0$

Methods exist for combining the laminar and turbulent solutions to give an average Nusselt number for the plate of length, $l$.

$$
N_u = \frac{h L}{\nu} = Pr^{\frac{1}{3}} (0.037 Re_x^{\frac{1}{3}} - 0.871)
$$
The total heat transfer from the plate is:

\[ Q = hA(T_w - T_a) \]

\[ Q = k \cdot Pr^{1/3} \left( 0.037 \cdot Re^{0.8} \cdot 8.71 \right) \cdot L \cdot W \cdot (T_w - T_a) \text{ Watts} \]

So, what may be wrong with applying this to the fin?

1) The leading edge may lead to flow separation.

The heat transfer rate would be reduced for \( x < x_{reattach} \).

The heat transfer rate may be higher for \( x > x_{reattach} \) and a short distance downstream of \( x_{reattach} \).

2) The wall temperature may not be uniform.

The leading edge high heat transfer coefficient may suppress the leading edge fin temperature.

3) The boundary layers of neighboring plates may merge.

This would reduce \( h \) and would raise "Too downstream where the non-boundary layer core is gone".
4) The fluid flow might bypass the fin array. This would make it colder, thus reducing the cooling.

What is different about the natural convection fin experiment?

1) It's easier:
The flow is driven by buoyancy so we needn't try to figure out what velocity, \( u_m \), to assign (as we did with forced convection).

2) It's harder:
The flow path is not straight. The standard analyses are not applicable, directly.

I made a steady flow symmetric sketch but I have yet to see a natural convection problem that is either.
One might crudely model this as follows:

$$\begin{align*}
\text{Modified} \\
\text{Acceleration} \quad g' = g \cos \alpha \\
\end{align*}$$

Where $L$, $a$, and $b$ come from the sketch on the previous page.

This gives the proper viscous length and gravity component (but is still very approximate).

Let's first look at the isolated plate.

Solve for the temperature field:

Assume:

- Constant properties (except for the density change which drives the flow).

Force balance for from the plate:

$$\frac{dP}{dx} = -\rho' g'$$  \hspace{1cm} (1)

Momentum Eqn near the plate:

$$\rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{dP}{dx} - \rho g' + \mu \frac{\partial^2 u}{\partial y^2}$$

We know $dP/dx$ from (1).

By continuity (still, assume $\rho$ constant): \hspace{1cm} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0

We can express the density difference that drives the flow:

$$g (\rho_0 - \rho)$$ with the thermal expansion coeff...
\[ \beta = \frac{1}{T} \frac{\partial}{\partial T} \left( \frac{\rho a - F}{\rho (T - T_0)} \right) = \frac{\partial}{\partial T} \left( g \frac{\rho a - F}{\rho (T - T_0)} \right) \]

for an ideal gas, we can show \( \beta = \frac{1}{T} \) (try it)

so the momentum eqn. becomes

\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \rho \left( \frac{\partial u}{\partial T} \right)_T + \mu \frac{\partial^2 u}{\partial y^2} \]

The final eqn. in our bag-o-tools is the energy eqn.

\[ \rho C_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} + u \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} \]

Now, we solve this set of equations (continuity, momentum and energy) to get the thermal field \( T(x, y) \).

Boundary conditions:

\[ \begin{align*}
  &\text{as } y \to \infty \quad U = 0 \quad T \to T_\infty \\
  &\text{at } x = 0 \quad U = 0 \quad T = T_0 \\
\end{align*} \]

There are various techniques for getting this solution (see pp. 516-518 of Haiman, 9th ed.).

cise \( T(x, y) \) to compute

\[ \dot{h} = \frac{\partial}{\partial x} \left( \frac{y}{T_{\infty} - T_0} \right) \]

non-dimensionalize this solution to get

\[ \text{Nu}_x = \dot{h} \frac{x}{y} = \frac{0.508 Pr^2}{(0.752 + Pr)^{1/3}} \frac{Gr_x}{K_f} \]

where \( Gr_x = g \beta (T_{\infty} - T_0) x^3 \) remember: \( \beta = \frac{1}{T} \)

Integrate this to get an average \( \dot{h} \) value \( \dot{h} \)

\[ \dot{h} = \frac{\dot{h}}{x = L} \text{ where } \dot{h} \text{ is the } \dot{h} \text{ from } \text{Nu}_x \text{ above } \]

evaluated at \( x = L \).
This might be a good guess of \( \bar{h} \) for the case of an isolated plate. I am still recommending \( L \) and \( g' \) from Fig. 13.2 and discussion p. 5.

Now, suppose you are dealing with the flow between plates.

The correlation to this "chimney" problem is taken from: "Thermal Analysis and Control of Electronic Equipment" A. D. Kirous and A. Bar-Cohen, McGraw-Hill

\[
\text{Nu}_o = \text{Nu}_o \left[ \frac{\text{Gr}_b \text{Pr} b}{L} \right]
\]

as given below.

\[
\text{Nu}_o = \left( \frac{q}{A} \right) \frac{b}{(T_W - T_0) K_f} K_f
\]

\[
\text{Gr}_b \text{Pr} = \frac{C_{Pf} P^2 \beta}{24 \mu_f K_f L} (T_W - T_0) = \text{Ra} \quad \text{(the channel Rayleigh #)}
\]

I am still recommending \( L \) and \( g' \) from the Fig. 13.2 and discussion on p. 5.

**FIG. 13.3** Nusselt number variation for symmetric isothermal plates [4].