Lecture 2

• Your own goal in this course
• Office hour: M-Tu 11:00-12:00
• Textbooks
• Feedback on labs so far

• Review – momentum, continuity, least squares
• Bernoulli Equation: \( \Rightarrow \) orifice equation
• Flow control valves:
  • Needle valve (lab 3a)
  • Pressure compensated flow control valve (lab 3b)
• Pressure control valves (lab 4):
  • Direct acting relief valve
  • Pilot operated relief valve
• Simulation of hydraulic components and circuits - Example

Course Goal Survey

• What is your goal for this course (what you hope to learn)?

  • Familiarity of fluid power systems (17)
    • Principles and applications
  • Motion control and dynamic systems (8)
  • Learn a new technology (2)
**Data Fitting, Model Discovery**

Example: Find a Delta P vs Q relationship for a needle valve
- Suppose you measure Delta P and Q for a needle valve
- Data: (DP1, Q1), (DP2, Q2), …
- Procedure:
  - plot data – use physical insights if possible (e.g. orifice equation)
  - Verify validity of the hypothesized function
  - Modify hypothesis
  - Find parameters from plot using least squares

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**Least Squares Analysis**

- This is a general function fitting technique
- Need model and data

Example:
- Fitting an orifice equation and a flow meter with bias
  - Model: \( Q = k\sqrt{\Delta P} + Q_0 \)
    - \( Q_0 \) = unknown bias in flow meter
    - \( k \) = unknown constant of orifice
  - Data = \((\Delta P_1, Q_1), (\Delta P_2, Q_2), \ldots, (\Delta P_n, Q_n)\)
  - Find \( Q_0 \) and \( k \) to best explain data
Least Squares

- Data: $y_1, y_2, ... , x_1, x_2, etc.$
- Formulate into matrix equation
  
  $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

- A is known, $a_1, a_2$ unknown
- Question: Find $a_1, a_2$ that best explain $y_1, y_2, y_3, y_4$
- Best solution that minimizes $E = e_1^2 + e_2^2 + e_3^2 + e_4^2$

\[
\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} - \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \\ \bar{A}_{31} & \bar{A}_{32} \\ \bar{A}_{41} & \bar{A}_{42} \end{bmatrix} \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \end{bmatrix}
\]

Least Square Solution

- Best solution that minimizes $E$ is:

  \[
  \hat{a} = \left( A^T A \right)^{-1} A^T y
  \]

- It is called the normal solution
- Can be easily obtained in Matlab!
- Weighted least squares?
Weighted Least Squares

- Regular least squares assumes all data are equally accurate and minimizes:
  \[ E = e_1^2 + e_2^2 + e_3^2 + e_4^2 \]

- If some data points are known to be more accurate, the error due to that error can be emphasized: \((w_i > 0)\)
  \[ E_w = w_1^2 e_1^2 + w_2^2 e_2^2 + w_3^2 e_3^2 + w_4^2 e_4^2 \]

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22} \\
A_{31} & A_{32} \\
A_{41} & A_{42} \\
\end{bmatrix} \begin{bmatrix}
a_1 \\
a_2 \\
\end{bmatrix} = \begin{bmatrix}
w_1 & 0 & 0 & 0 \\
0 & w_2 & 0 & 0 \\
0 & 0 & w_3 & 0 \\
0 & 0 & 0 & w_4 \\
\end{bmatrix} \begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{bmatrix} - \begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{bmatrix}
\]

Weighted Least Squares

- The weighted error can be minimized and put into regular least squares form:
  \[ E_w = e_{w1}^2 + e_{w2}^2 + e_{w3}^2 + e_{w4}^2 \]

- Modify the data set accordingly:
  \[
  \begin{bmatrix}
y_{w1} \\
y_{w2} \\
y_{w3} \\
y_{w4} \\
\end{bmatrix} =
  \begin{bmatrix}
w_1 & 0 & 0 & 0 \\
0 & w_2 & 0 & 0 \\
0 & 0 & w_3 & 0 \\
0 & 0 & 0 & w_4 \\
\end{bmatrix} \begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{bmatrix} =
  \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22} \\
A_{31} & A_{32} \\
A_{41} & A_{42} \\
\end{bmatrix} \begin{bmatrix}
a_1 \\
a_2 \\
\end{bmatrix} = A_w \begin{bmatrix}
a_1 \\
a_2 \\
\end{bmatrix}
\]
**Weighted Least Squares**

- Apply solution of regular least squares to modified problem:

\[
\hat{\mathbf{a}} = \left( A_w^T A_w \right)^{-1} A_w^T y_w
\]

\[
\hat{\mathbf{a}} = \left( A^T W^2 A \right)^{-1} A^T W^2 y
\]

- What to set \( W_i \)?
  - Large \( W_i \) when error is expected to be small
  - Small \( W_i \) when error is large
  - Example:
    - \( W_i = 1 / \text{error of the data } i \)

**Least Squares Example**

- Model:

\[
y(t) = a \cdot \sin(2t) + b
\]

- \( a, b \) are unknown

- Data: \((t_1, y(t_1)), (t_2, y(t_2)), (t_3, y(t_3)), \ldots\)

\[
\begin{bmatrix}
y(t_1) \\
y(t_2) \\
\vdots \\
y(t_n)
\end{bmatrix} =
\begin{bmatrix}
\sin(2t_1) & 1 \\
\sin(2t_2) & 1 \\
\vdots & \vdots \\
\sin(2t_n) & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix}
\]

- How to formulate polynomial fit - linear, quadratic, cubic \ldots ?
**“The” ORIFICE equation**

- Most fundamental fluid power equation!

- Used in all manners of valves
  - Needle valve (lab 3a)
  - Pressure compensated flow control valve (lab 3b)
  - Pressure relief valve (lab 4)

- Derived from Bernoulli equation

**Bernoulli Equation**

- When fluid is not at rest, Pascal’s law is not strictly applicable

- If energy is conserved, as fluid speeds up, pressure decreases

\[
P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2
\]

- Velocities related by continuity equation …..
How small does a restriction have to be for Bernoulli to be important?

- Q = 30lpm (liters/min) = 5x10^-4 m³/sec
- D = 2cm
  - Pressure variation = 3.6 kPa = 0.52 psi
- D = 0.2cm
  - Pressure variation = 36 MPa = 5220 psi !!

\[ \frac{\rho v^2}{2} = \left( \frac{8 \rho}{\pi} \right) \frac{Q^2}{D^4} \]

Very sensitive to changes in diameter!!!
Needle Valve as Flow Control

- Flow equation is generally given by the orifice equation:

\[ Q = C_d A_0 (\text{turns}) \sqrt{\frac{2}{\rho}} (P_i - P_o) \]

- A needle controls the opening of the flow channel (effective orifice area)

- Needle valve controls the resistance to flow, not flow directly

- Can be characterized by P-Q relationship either graphically, as an equation or as computer code

Matlab code:

```matlab
function Q = orifice1(DP, turn)
a0=0.1; a1=0.2; a2=0.3;
K = a0+a1*turn+a2*turn^2;
Q = K*sqrt(abs(DP))*sign(DP);
end

function DP=orifice2(Q,turn)
a0=0.1; a1=0.2; a2=0.3;
K = a0+a1*turn+a2*turn^2;
DP=(Q/K)^2*sign(Q);
end
```

Which needle valve is better flow control?

- Which is which?
- Small orifice:
  - \( dQ/dP_{\text{load}} \)?
  - \( P_s \)?
- Large orifice:
  - \( dQ/dP_{\text{load}} \)?
  - \( P_s \)?
Flow Control Valves: Needle Valve

- The goal of flow control valves is to maintain the flow rate at the specified setting.
- Needle valve is essentially an orifice.
- If pressure difference is fixed, flow will be constant.
- Sensitivity depends on dQ/dP.
  - You can determine from your experiment.
- Needle valve can also be used as pressure control valve.
  - Then flow must be constant.

- Is a needle valve a better flow control valve at lower pressures or at higher pressure?
- How about as a pressure control valve?

\[ Q = C_d A_d \sqrt{\frac{2}{\rho} (P_1 - P_2)} \]

Pressure Compensated Flow Control Valve (PCFC)

- Motivation: Needle valve cannot maintain flow if pressure varies.
- PCFC uses feedback to ensure that pressure across an orifice is constant.
- As pressure across fixed orifice varies, the adjustable orifice tries to compensate - how?
- How does spring rate affect accuracy in flow control?
- Why must there be a minimum working pressure?
- Poor energy efficiency - where is the energy lost??
### PCFC Continued

**Restricter type**

- How does spring constant affect flow control accuracy?

**What happens here?**

### PCFC - Operation

- Identify all components
- What is the flow path?
- What is the P-Q relationship when the control spool is stuck open?
- What are the forces and pressures that cause the control spool to move?
- How does spring rate affect the shape of the P-Q curve?
- Comment on efficiency
- What happens when pressure is too low?
Pressure control valve:  
Direct Acting Relief Valve

- Poppet is seated against a spring
- As pressure in main line increases above the preset spring force, poppet is moved up
- Orifice to drain is opened
- Relief flow occurs
- Cracking pressure versus full flow pressure?
- Questions: Formula for cracking pressure??

Relieve valves are Non-passing valve

A relief valve is a non-passing pressure control valve
- Force balance on the spool?
- Cracking pressure versus spring rate, spring adjustment screw?
- Over-ride versus spring rate?
**Direct Acting Relief Valve**

- Inlet pressure ($P_1$) versus spool displacement ($x$) is related by spring constant and spool area.
- How to draw the pressure ($P_1$) versus flow ($Q$) curve?
- How does the spring constant ($K$) affect the shape of the ($P_1$ versus $Q$) curve? What is an ideal relief valve?
- For sensitivity: Stiffer or softer spring?
  - If spring is too soft, there may be stability problems.
- Cracking pressure is set by: Preload in spring = $K x_0$ (long springs)
- **Conflict** between high cracking pressure and sensitivity
- **Open question:** how does outlet pressure $P_2$ affect $P$ versus $Q$ curve?

**Pilot Operated Relief Valve**

- Pilot operated relief valves are supposed to be more sensitive than direct acting relief valves

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*M.E., University of Minnesota (updated 12.2010)*
**Pilot Operated Relief Valve - Schematic**

- Consists of a main stage and a pilot stage
- Main stage is lightly spring loaded
- Pilot stage dart is seated by a preloaded spring

**Main spool and Pilot**

- **Main Spool** - high flow, low pressure
- **Pilot** - high pressure, low flow
**Operation of Pilot Operated Relief**

- Pilot stage limits pressure above the spool (set by pilot spring)
- When pilot is closed, main spool is seated by the light spring
- When pilot is open, main spool sees more force from lower chamber than from upper chamber and spring - hence spool opens
- It is an avalanche effect!

**Pilot Operated Relief Valve**

- This solves the dilemma of directing acting relief valve by separating the roles of sensitivity and cracking pressure setting.
- Cracking pressure determined by pilot dart (poppet) and spring
  - stiff spring and small area can be used for high cracking pressure
- As pilot cracks, pressure above main spool is rapidly released.
- An avalanche results, as the main spool is held by a light spring.
- Sensitivity is controlled by the soft spring in the main spool
Remote Operated Pilot

Ideal P-Q curve for relief valves

Which one is better??

How do spring rates affect this curve
• For direct acting, or
• Pilot operated relief valve?