Lecture 9

• Coming week labs:
  • Lab 16 – System Identification (2\textsuperscript{nd} or 2 sessions)
  • Lab 17 – Proportional Control

• Today:
  • Systems topics
  • System identification (ala ME4232)
    • Time domain
    • Frequency domain
  • Proportional Control


**Systems Analysis**

- **Transfer functions**
  - Relationship between input and output of a linear system
  - Compact way of writing a differential equation with input
  - Laplace variable “s” = time differentiation operator

- What is a linear system?

- What are the solutions to:
  - $\dot{x} + 5x = u_1(t) + u_2(t); \quad x(t = 0) = x_0$
  - $\dot{x} + 5x = u_1(t); \quad x(t = 0) = 0$
  - $\dot{x} + 5x = u_2(t); \quad x(t = 0) = 0$
  - $\dot{x} + 5x = 0; \quad x(t = 0) = x_0$
Transfer Functions

- Block diagram manipulations:
  - Series
  - Parallel
  - Feedback

- Three uses of transfer functions:
  - **Prediction:**
    - Given input, find output (Everything!)
  - **Control:**
    - Given desired output, find input to achieve it (Labs 17-22)
  - **System Identification:**
    - Given input and output, find the system (Labs 15-16)
Basic Skills

- Convert signals between Time domain and Laplace domain
- System description from differential equation to transfer function
- Find system response to inputs
- Block diagram simplification
- Pole locations and characteristic responses
- Frequency response
Frequency Response

• Given a stable transfer function $G(s)$ such that $Y(s) = G(s) R(s)$

• If the input is a sinusoid, $r(t) = A \sin (\omega t)$, then the output is given, after the transient has died down, by

$$y(t) = B \sin(\omega t + \phi)$$

with $B = A \ | G(j \omega) |$ and $\phi = \text{phase of } G(j \omega)$ in radians

• This allows sinusoidal response to be easily characterized
Fourier series

- Any periodic time function, $r(t)$ of frequency $\Omega$ can be represented by a sum of sinusoids of harmonics of $\Omega$

$$r(t) = \sum_{k=0}^{1} M_k \sin(k \Omega t + \phi_k)$$

- Thus the steady state response of system $G(s)$ to a periodic input is:

$$y(t) = \sum_{k=0}^{1} B_k \sin(k \Omega t + \Phi_k)$$

where

$$B_k = M_k |G(j k \Omega)| \text{ and } \Phi_k = \phi_k + \angle G(j k \Omega)$$
Fourier Transform / Frequency content

- An arbitrary signal (with finite power) can be considered to be a periodic signal with a very long period, i.e. $T = 1/\Omega \rightarrow 1$

- Then, $\Omega \rightarrow 0$, and the harmonics

$$0, \Omega, 2\Omega, \ldots, k\Omega, \ldots.$$ 

Become a continuum …..

$$r(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(j\omega)e^{j\omega t} d\omega$$

For some $R(j\omega)$ which turns out to be $R(s = j\omega)$ where $R(s)$ is the Laplace transform

What do high and low frequency signals look like in time domain?
Input/Output Frequency Spectra

- Roughly speaking:
  - \( Y(jw) = G(jw)U(jw) \)

- System \( G(s) \) amplifies or attenuates different frequencies differently

- Equalizers in audio system

- If \( u(t) \) is a noise, e.g. 60Hz, we can design \( G(s) \) to “notch” out 60Hz

- If \( u(t) \) is a low frequency command signal, \( G(s) \) should be close to 1 in the range of frequency expected.
Frequency Response (Bode Plot)

- Analytical method (learn this first)
  - $|G(jw)|$ and Phase ($G(jw)$)
  - Program this in Matlab

- Matlab tool - know what to expect first!!! (Garbage in/garbage out)
  - “Bode” plot – $20 \log_{10}(|G(jw)|)$ vs $\log_{10}(w)$
  - “Freqresp”

- $\text{Sys} = \text{tf}($num,den$)$;
  - $\text{Bode}($sys$)$
  - $[H, w] = \text{freqresp}($sys$)$
**EH Labs Overview**

- To develop control systems

1. Determine a system model
   - Lab 15 – Simple open loop approach
   - Lab 16 – System identification (time domain, frequency domain)

2. Determine controller structure for desired properties
   - Stability
   - Performance (tracking);
   - Disturbance rejection;
   - Noise immunity;
   - Robustness to model uncertainty

- Sophistication (analysis/design) $\rightarrow$ better tradeoffs
  - Lab 17 Proportional; Lab 18 Proportional-integral
  - Lab 19 Feedforward; Lab 20: Internal model control;
  - [ Lab 20 Adaptive control ]

- Lab 21: Force control (integrative lab)
System Identification

- Determine $G(s)$ by testing the system
  - Probe with input $U(s)$
  - Measure output $Y(s)$

- Often you have choice of what $U(s)$ to use
  - Time domain (step, impulse, …)
  - Frequency domain (sine/cosine)
System Identification Approach

In this course – we use a two-step process:

1. Determine the type and general form of system
   - 1st order, 2nd order, linear/nonlinear, delay ……
   - E.g. For a strictly proper second order system:

   \[ G(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} \]

   - Use process of pattern recognition

2. Design and conduct experiments to determine the values of the parameters: \( a_0, a_1, a_2, b_1, b_0 \)
Repertoire of System Response

(Stable) First Order System

- What does the transfer function look like?
- How many parameters does it have?
- What determines whether it is stable or not?
- What is the step response?
- What is the impulse response?
- What is the response to \( u(t) = \sin(\omega t) \)?

Time domain
Frequency domain
First Order System

- Step response
  - # of measurements = # of unknown parameters = 2
  - Not unique choice – pick the ones that are most distinct
    - Final value -
    - Initial slope -
    - (other choices – 1/e of final value ....)

- Plot a line and use least squares to find the parameters!
Impulse Response

• Ideal impulse impulse

• Approximate impulse

• How to preserve the shape of the impulse response?
  • Signal to noise ratio becomes low
First Order System

- Shape of the bode plot for first order system
- Approximate with straight lines for sketching
- What happens when $w = |\text{pole}|$?
Control Design - Objectives

\[ Y(s) = G(s) U(s) \]

**Overall goal:** choose \( U(s) \) so that \( Y(s) \) behaves as desired

1. **Stability** – closed loop system poles on left half of complex plane

2. **Performance** – how well does \( Y(s) \) follow command

3. **Disturbance rejection** – not affected by disturbance

4. **Immunity to measurement noise** – not affected by sensor inaccuracies

5. **Robustness** – not affected by uncertainty in system model – \( G(s) \)
Usefulness of Feedback

- Feedback versus dead reckoning
Proportional Control

• Simplest feedback controller

• Consider 1st order plant (note not all plants are first order!!!)

• Formulate closed loop transfer function

• Use closed loop transfer function to analyze
  • Stability – how design parameters affect stability
  • Performance
    • time domain and frequency domain
  • Disturbance rejection and noise immunity
    • Input disturbance, output disturbance
  • Robustness