DATA ACQUISITION AND NYQUIST SAMPLING THEOREM

ME 4231
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PC Based Control System

Loop
{
  Read voltages from sensors
  Compute voltage to be sent to actuator
  Send voltage to actuator
}

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PC Based Control System

Example: Automotive Cruise Control
Loop
{
    Read voltages from wheel speed sensors, determine speed of vehicle
    Compute difference between desired speed and actual speed
    Compute whether throttle angle should be increased or decreased and by how much
    Compute voltage to be sent to throttle actuator
    Send voltage to throttle actuator
}
DATA ACQUISITION CARD

Common Tasks
- Read voltages
  - a2d (analog to digital conversion)
  - digital inputs
    The signal from a sensor can be analog or digital
- Send out voltages
  - d2a (digital to analog conversion)
  - digital outputs
    The voltage to be sent to an actuator can be analog or digital

DATA ACQUISITION

- Weighted-resistor D2A (digital 2 analog conversion)

```
V_{ref}
```

```
<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LSB</td>
<td>R/2</td>
<td>R/4</td>
</tr>
<tr>
<td>MSB</td>
<td>R/8</td>
<td></td>
</tr>
</tbody>
</table>

Electronic switches

Summing Amplifier

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**DATA ACQUISITION**

- Analog-to-digital conversion

Diagram:

```
Analog signal → Sample and hold → Analog to digital conversion → Digital signal
```

**DIGITIZATION**

- Two types of digitization
  - **Digitization in time, called “sampling”**
    - Depends on speed and complexity of real-time program
    - Depends on speed of data acquisition card
  - **Digitization in value**
    - Depends on resolution of data acquisition card
    - (12 bit, 16 bit, etc)
DIGITIZATION

- Digitization in time ("sampling")

![Diagram showing digitization in time]

- Sampling time $T$

DIGITIZATION

- Digitization in value

![Diagram showing digitization in value]

- Depends on resolution of data acquisition system
DATA ACQUISITION

- Successive Approximation A2D

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DATA ACQUISITION

- Data acquisition card from Sensoray in lab
  - Model 626 PCI Multi-function I/O board
  - 16 differential analog inputs (16-bit)
  - 4 analog outputs (14-bit)
  - 48 digital I/O channels
  - 6 24 bit up/down counters
The **sampling theorem** states that for a limited bandwidth (band-limited) signal with maximum frequency $f_{\text{max}}$, the equally spaced sampling frequency $f_s$ must be greater than twice of the maximum frequency $f_{\text{max}}$, i.e.,

$$f_s > 2f_{\text{max}}$$

in order to have the signal be uniquely reconstructed without **aliasing**.

The frequency $2f_{\text{max}}$ is called the **Nyquist sampling rate**.

Sampling theorem articulated by Nyquist in 1928
Mathematically proved by Shannon in 1949.

Some books use the term "Nyquist Sampling Theorem", and others use "Shannon Sampling Theorem".

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**Under Sampling**

When the sampling rate is lower than or equal to the **Nyquist rate**, a condition defined as **under sampling**, it is impossible to rebuild the original signal according to the **sampling theorem**.
Suppose we are sampling a sine wave. How often do we need to sample it to figure out its frequency?

If we sample at 1 time per cycle, we can think it's a constant.
If we sample at 1.5 times per cycle, we can think it's a lower frequency sine wave.

However, phase mismatches will distort the signal.
For loss-less digitization, the sampling rate should be at least twice the maximum frequency responses. Indeed many times more the better.
ANTIALIAS FILTERS

Therefore, an analog filter is typically applied before sampling to ensure that signal components with frequencies greater than half the sample frequency are highly reduced. This is called an anti-aliasing filter.

The quality of analog-to-digital-converters (A/D-Converters) depends critically upon that filter, since a poor filter causes phase distortion and other difficulties.

- Demonstration of the effects of aliasing using a real world music signal

![Frequency components in music signal](image)
ANTI-ALIAS FILTERS

- Original audio signal sampled at 44kHz (CD quality audio)

- Same audio signal sampled at 4 kHz

ANTI-ALIAS FILTERS

- Audio signal sampled at 4 kHz

- The sampling frequency is again at 4 kHz, but the audio signal is first processed with an anti-aliasing filter.
A2D Conversion

\[\begin{array}{cccc}
1 & 0 & \cdots & 0 & 1 \\
\end{array}\]

2\(^n\) possible combinations, other than zero

n bit binary number

e.g.

\[\begin{array}{c}
1 & 0 \\
\end{array}\]

3 possible combinations

- 3 non-zero numbers can be represented

\[\begin{array}{cccc}
1 & 0 & \cdots & 0 & 1 \\
\end{array}\]

2\(^{n-1}\) positive numbers

2\(^n\) negative numbers can be represented

A2D Conversion

For a 16-bit a2d, set to operate between -10 to 10 V, what voltages do 15,000 and 15,001 correspond to?

\[2^{n-1} - 1 = 32767\]

Hence numbers from -32768 to 32767 can be represented

15,000:

\[\frac{V}{V_{\text{max}}} = \frac{15,000}{32,767} \Rightarrow V = \frac{15,000}{32,767} \times 10 = 4.5778V\]

15,001:

\[\frac{V}{V_{\text{max}}} = \frac{15,001}{32,767} \Rightarrow V = \frac{15,001}{32,767} \times 10 = 4.5781V\]

\[z : V = \frac{z}{32,767} \times 10V \quad \text{for } z > 0\]

\[V = \frac{z}{32,768} \times 10V \quad \text{for } z < 0\]
**D2A Conversion**

For a 14-bit d2a, set to operate between -10 to 10 V, what voltages do 4,000 and 4,001 correspond to?

\[ 2^{n-1} - 1 = 8191 \]

Hence numbers from -8192 to 8191 can be represented

4,000:

\[ \frac{V}{V_{max}} = \frac{4,000}{8,191} \Rightarrow V = \frac{4,000}{8,191} \times 10 = 4.8834V \]

4,001:

\[ \frac{V}{V_{max}} = \frac{4,001}{8,191} \Rightarrow V = \frac{4,001}{8,191} \times 10 = 4.8846V \]

\[ z : \quad V = \frac{z}{8,191} \times 10V \quad \text{for } z > 0 \]

\[ V = \frac{z}{8,192} \times 10V \quad \text{for } z < 0 \]

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**PC Based Control System**

Loop

{  
  Read voltages from sensors  
  Compute voltage to be sent to actuator  
  Send voltage to actuator  

  **Wait until sampling time has been reached**  
}

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Lab 3

Task 1

Demonstration of the A2D Converter
- Set the power supply to different voltages
- Check and see if your a2d program can read those voltages correctly
- Change the voltage range settings in your program and repeat

Task 2

Demonstration of the D2A Converter
- Send out different voltages from channel 0 by writing a program that takes a user input from the screen
- The user enters an integer number, e.g. 6000
- Check the output voltage on a multimeter to see if your program works correctly
Lab 3

- Task 3
- Demonstration of Aliasing with the A2D Converter
  - Set the sampling frequency to be 1000 Hz
  - Change input signal frequency on the function generator to vary from 50Hz to 1500 Hz according to the given table
  - For each input frequency write down the estimated frequency of the output signal from the oscilloscope.