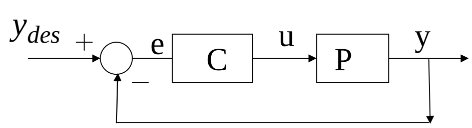
LAB 8
PID Control

Rajesh Rajamani

ME 4231
Department of Mechanical Engineering
University Of Minnesota

REVISITING P-CONTROL



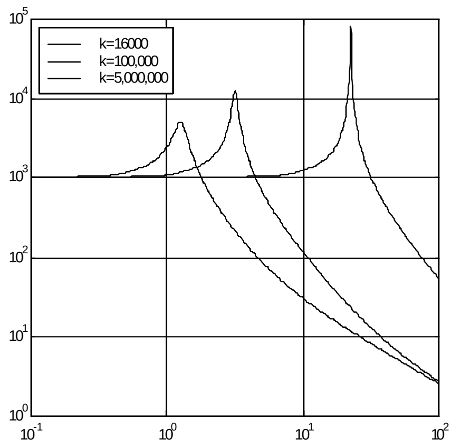
- Closed-loop transfer function is

$$\frac{Y(s)}{Y_{des}(s)} = \frac{\frac{K_p K_m}{T_m}}{s^2 + \frac{1}{T_m}s + \frac{K_p K_m}{T_m}}$$

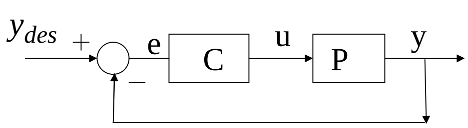
- For good damping $\quad \xi \geq 1 \;\Rightarrow\; K_p \leq \frac{1}{4K_m T_m}$

REVISITING P-CONTROL

- For good bandwidth $\omega_n = \sqrt{\dfrac{K_p K_m}{T_m}} \Rightarrow K_p$ needs to be large



PD CONTROL



$$u = K_p e + K_d \dot{e}$$

$$P = \frac{K_m}{s(T_m s + 1)}$$

$$C = K_p + K_d s$$

$$\frac{Y(s)}{Y_{des}(s)} = \frac{PC}{1 + PC} = \frac{K_p K_m + K_m K_d s}{T_m s^2 + s + K_m K_d s + K_p K_m}$$

PD CONTROL

- Draw Bode plot on blackboard

$$\frac{Y(s)}{Y_{des}(s)} = \frac{PC}{1+PC} = \frac{K_p K_m + K_m K_d s}{T_m s^2 + s + K_m K_d s + K_p K_m}$$

$$= \frac{\frac{K_p K_m}{T_m}\left(1 + \frac{K_d}{K_p} s\right)}{s^2 + \frac{1}{T_m}(1 + K_m K_d)s + \frac{K_p K_m}{T_m}}$$

PD CONTROL

- Independent control of ξ and ω_n .
  - Damping ratio $2\xi\omega_n = \frac{1}{T_m}(1 + K_m K_d)$
  - Bandwidth $\omega_n = \sqrt{\frac{K_m K_p}{T_m}}$
  - High bandwidth and good damping
- The derivative feedback (D-term) provides more damping, better stability and more speed of response.

IMPLEMENTING PD CONTROL

```
{
error = ref_position - encoder;
error_derivative = (error - previous_error)/Ts;
previous_error = error;
control_voltage =       Kp*error
                      + Kd* error_derivative ;
}
```

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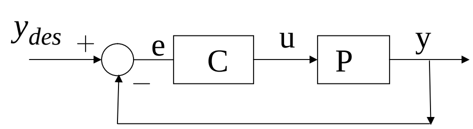
PID CONTROL

- Proportional-integral-derivative feedback

$$u = K_p e + K_i \int e \, dt + K_d \dot{e}$$



$$P = \frac{K_m}{s(T_m s + 1)}$$

$$C = K_p + K_i \frac{1}{s} + K_d s$$

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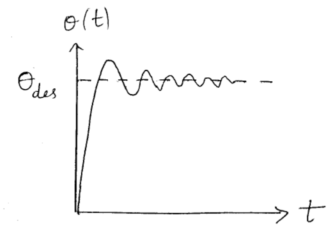
MOTIVATION FOR INTEGRAL TERM

- In the case of P-control, the steady state error is zero, if the desired signal $y_{des}(t)$ is a step signal.



- However, the steady state error is not zero, if the desired signal $y_{des}(t)$ is a ramp signal.

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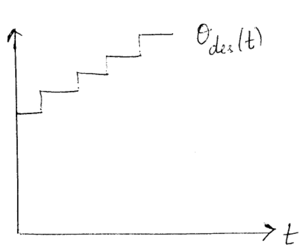
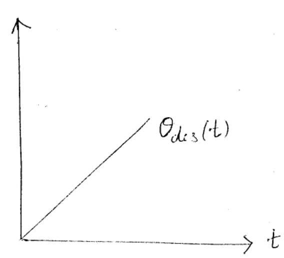
MOTIVATION FOR INTEGRAL TERM

- Consider a case where the desired value of signal keeps changing
- Alternatively, consider the case where the desired signal is a ramp signal





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FINAL VALUE THEOREM

- The Final Value Theorem can be used to calculate steady state error
- Let $Y(s)$ be the Laplace transform of y(t)
- If y(t) has a steady state value, then the steady state value is given by $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

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STEADY STATE ERROR: P-CONTROL

- Error is

$$\frac{E(s)}{Y_{des}(s)} = \frac{1}{1+PC} \quad \text{or} \quad \frac{E(s)}{Y_{des}(s)} = \frac{1}{1+\dfrac{K_m K_p}{s(T_m s+1)}}$$

- Desired value of y is a step signal

$$Y_{des}(s) = \frac{A}{s}$$

$$E(s) = \frac{1}{1+\dfrac{K_m K_p}{s(T_m s+1)}} \frac{A}{s}$$

$$\Bigg\} \quad \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{A}{1+\dfrac{K_m K_p}{s(T_m s+1)}} = 0$$

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STEADY STATE ERROR: P-CONTROL

- Desired value of y is a ramp signal

$$Y_{des}(s) = \frac{A}{s^2}$$

$$E(s) = \frac{1}{1+\frac{K_m K_p}{s(T_m s+1)}} \frac{A}{s^2}$$

Steady state error

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} \left(\frac{1}{1+\frac{K_m K_p}{s(T_m s+1)}} \right) \frac{A}{s} = \frac{A}{K_p K_m} \neq 0$$

STEADY STATE ERROR: PID CONTROL

- Desired value of y is a ramp signal

$$Y_{des}(s) = \frac{A}{s^2} \qquad \frac{E(s)}{Y_{des}(s)} = \frac{1}{1+PC}$$

$$E(s) = \frac{1}{1+PC} \frac{A}{s^2}$$

$$E(s) = \frac{1}{1+\frac{K_m}{s(T_m s+1)}\left(K_p + \frac{K_i}{s} + K_d s\right)} \frac{A}{s^2}$$

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STEADY STATE ERROR: PID-CONTROL

Steady state error

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s}{1+\frac{K_m}{s(T_m s+1)}\left(K_p + \frac{K_i}{s} + K_d s\right)} \frac{A}{s^2}$$

or

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s^3(T_m s+1)}{s^2(T_m s+1) + K_m\left(K_p s + K_i + K_d s^2\right)} \frac{A}{s^2} = 0$$

Hence steady state error is zero without requiring large K_p

MOTIVATION FOR INTEGRAL TERM

- Consider the presence of dry friction in the motor

$$E(s) = \frac{1}{1+PC} Y_{des}(s) + \frac{1}{s} F_o$$

$$Y_{des}(s) = \frac{A}{s}$$

$$E(s) = \frac{1}{1+\frac{K_m K_p}{s(T_m s+1)}} \frac{A}{s} + \frac{1}{s} F_o$$

MOTIVATION FOR INTEGRAL TERM

- Steady State Error

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{A}{1+\dfrac{K_m K_p}{s(T_m s+1)}} + F_o = F_o$$

- Hence steady state error is not zero in the presence of dry friction

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PID GAINS: ZIEGLER-NICHOLS RULES

$$C(s) = K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$$

Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	-	-
PI	$0.45K_{cr}$	$P_{cr}/1.2$	-
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

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IMPLEMENTING PID CONTROL

```
{
error = ref_position - encoder;
error_derivative = (error - previous_error)/Ts;
previous_error = error;
error_integral += Ts*error;
control_voltage =       Kp*error
                      + Kd* error_derivative
                      + Ki*error_integral;
}
```

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TASKS IN LAB

- Task 1
  - P Control. Use Kp = 0.004
  - Reference displacement is a sine wave of magnitude 250 counts
  - Various frequencies 4 Hz, 8, 10, 12, 14, 16, 20 Hz
  - Find output/input ratio
  - Estimate the bandwidth of the closed-loop controller from the experimental data
  - Estimate bandwidth from theoretical Bode plots using Matlab

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TASKS IN LAB

- Task 2
  - Implement PI control.
  - Plot experimental step response
- Task 3
  - Implement PID control.
  - Plot experimental step response

- Post-Lab
  - Compare step responses with P, PI and PID controllers
  - Comment on the steady state error value, oscillatory behavior and speed of response.

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